Quantum gas microscopy of spin superdiffusion in Heisenberg chains

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Les Houches

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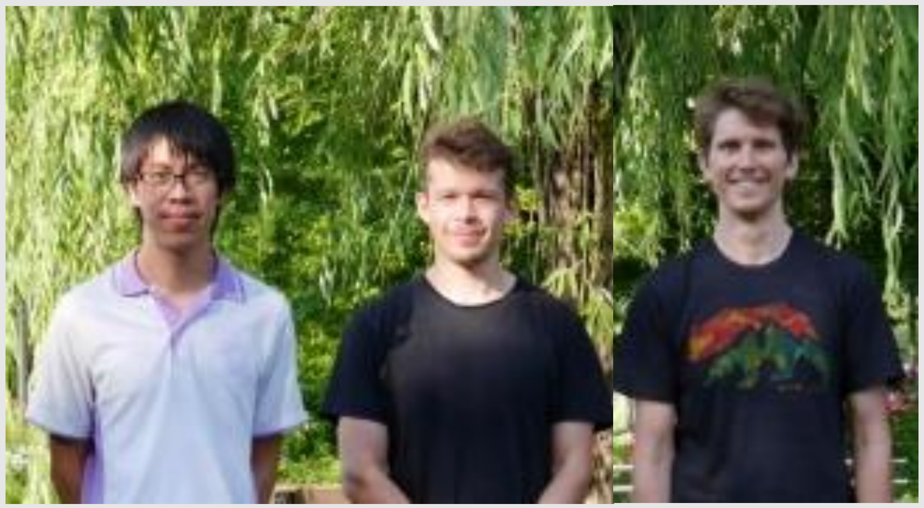
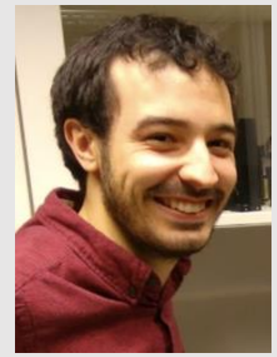
D. Wei, A. Rubio-Abadal, B. Ye, F. Machado, J. Kemp, K. Srakaew, S. Hollerith, J. Rui,
S. Gopalakrishnan, N. Y. Yao, I. Bloch, J. Zeiher, Science **376**, 716 (2022)



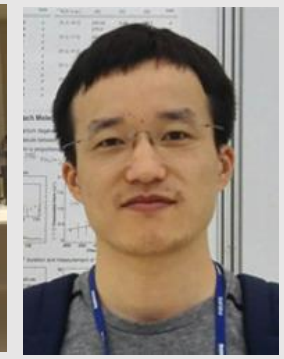
Kritsana Srakaew

Simon Hollerith

Johannes Zeiher



Antonio Rubio Abadal

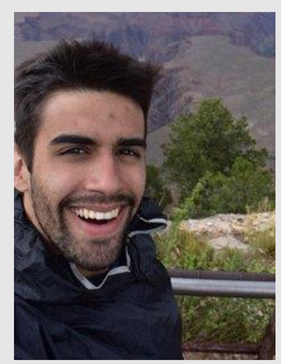


Jun Rui

Immanuel Bloch

Sarang Gopalakrishnan

Bingtian Ye



Francisco Machado

Jack Kemp

Norman Yao

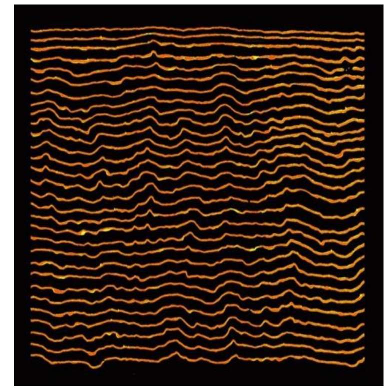
Kardar-Parisi-Zhang universality

1D interface growth

$$\frac{\partial h(x,t)}{\partial t} = \nu \frac{\partial^2 h(x,t)}{\partial x^2} + \frac{\lambda}{2}\left(\frac{\partial h(x,t)}{\partial x}\right)^2 + \sqrt{\Gamma}\eta(x,t)$$

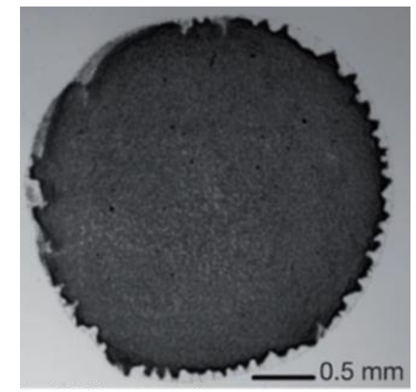
M. Kardar, G. Parisi, and Y.-C. Zhang, PRL **56,** 889 (1986)

Burning of paper



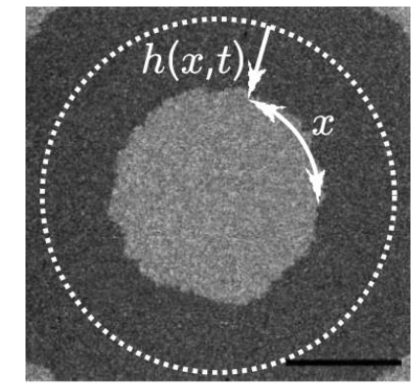
M. Myllys, et al.,
PRE **64**, 036101 (2001)

Coffee stains



P. J. Yunker, et al.,
Nature **476**, 308 (2011)

Liquid crystals



Y. T. Fukai, et al.,
PRL **119**, 030602 (2017)

Universal scaling exponents

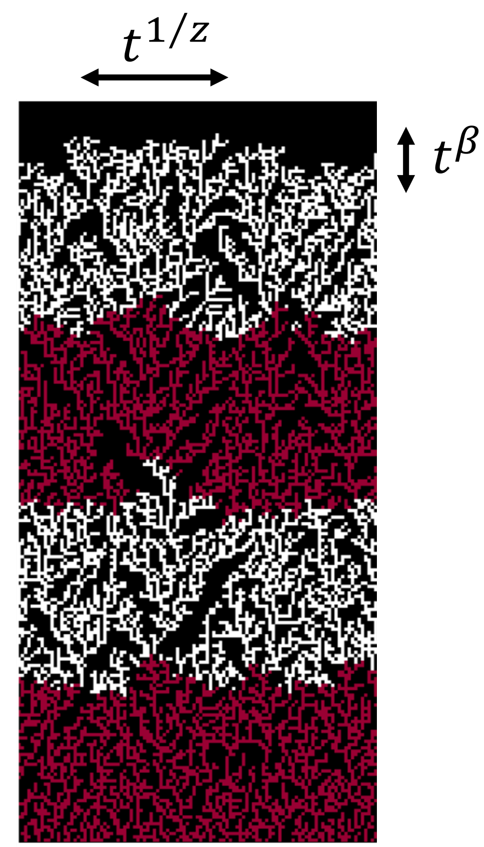
$$\alpha = \frac{1}{2}, \beta = \frac{1}{3}, z = \frac{\alpha}{\beta} = \frac{3}{2}$$

Universal correlation functions

$$\langle \partial_x h(x,t) \partial_x h(0,0) \rangle = \frac{1}{t^{2/3}} f_{\mathrm{KPZ}}\left(\frac{x}{t^{2/3}}\right)$$

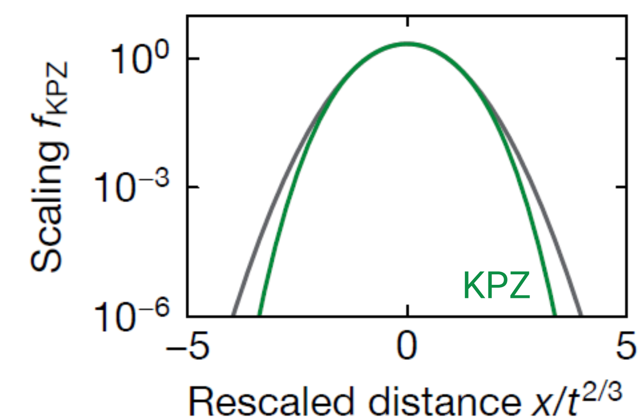
Universal distributions of height fluctuations

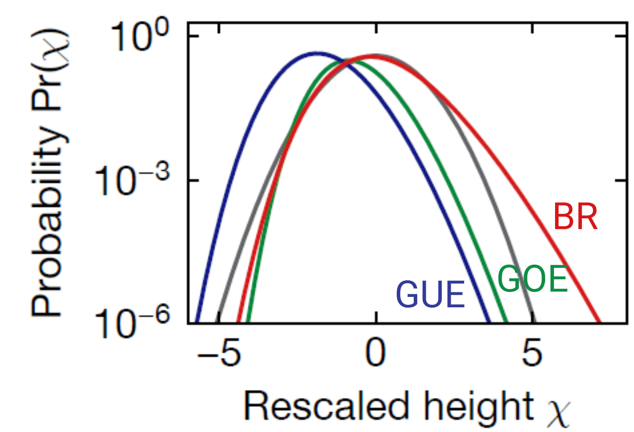
$$\delta h(x,t) \sim t^{1/3} \chi(x,t)$$



Ballistic deposition

I. Corwin, Not. AMS **63**, 230 (2016)



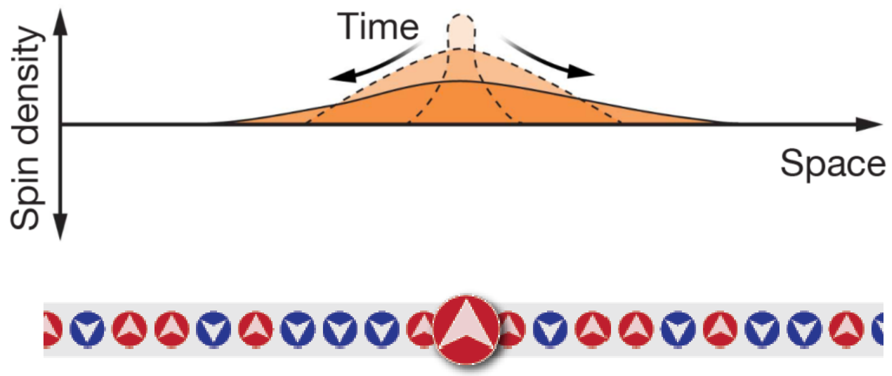


M. Prähofer, H. Spohn, PRL **84**, 4882 (2000)

High-temperature spin hydrodynamics

Late-time spin transport

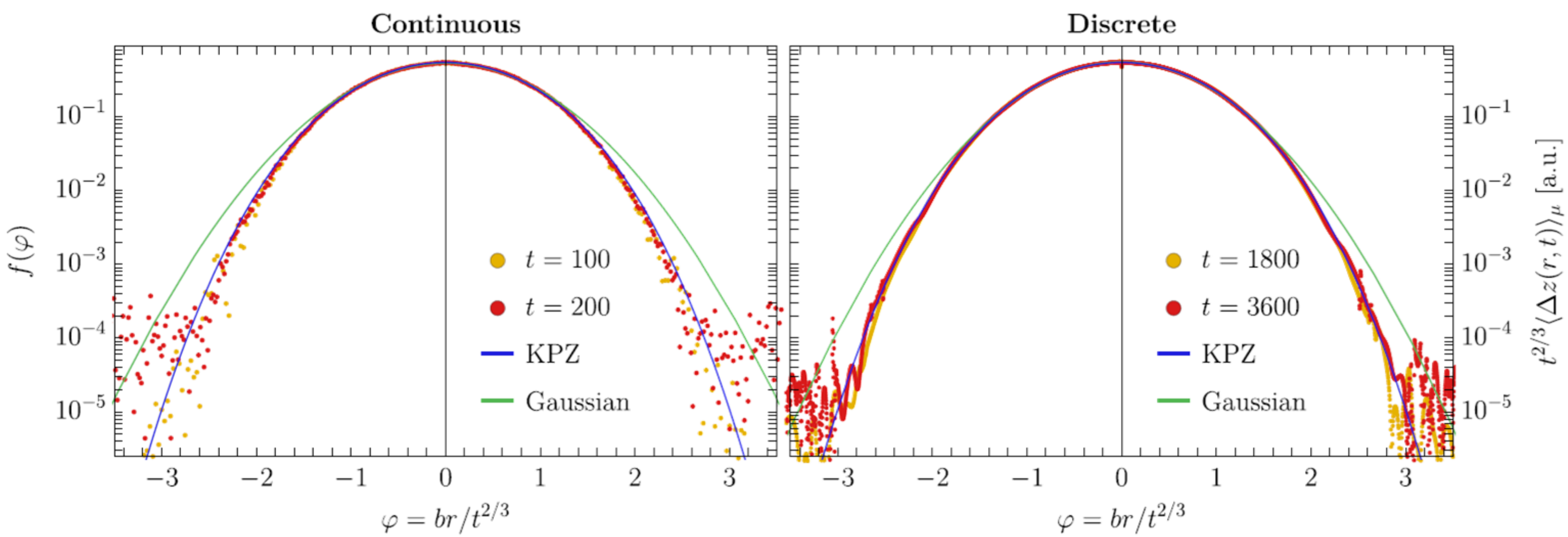
$$\langle S^z(x,t)S^z(0,0)\rangle = \frac{1}{t^{1/z}} f\left(\frac{x}{t^{1/z}}\right)$$



Heisenberg chain: $\hat{H} = J\sum_i \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_{i+1}$

Numerics in linear-response regime at infinite temperature:

- Anomalous dynamical exponent $z = 3/2$
- Non-Gaussian scaling function f_{KPZ}

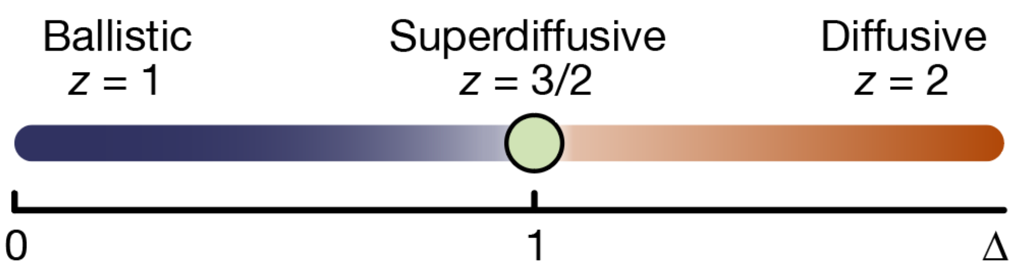


M. Ljubotina, M. Žnidarič, T. Prosen, Phys. Rev. Lett. **122**, 210602 (2019)

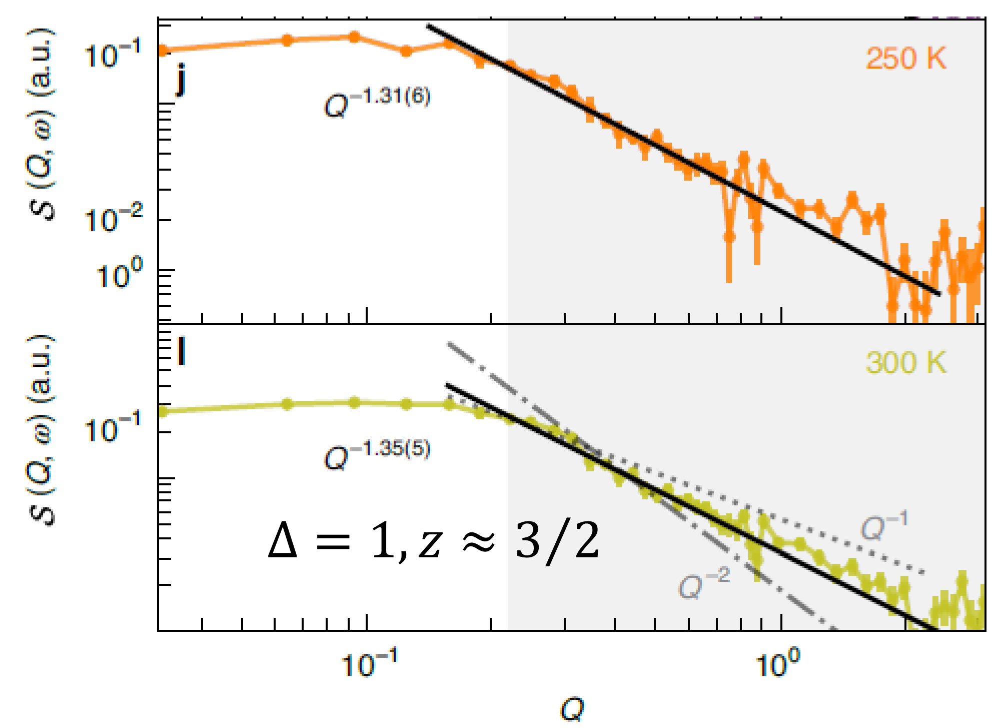
Transport experiments in XXZ chain

1D XXZ model

$$H = -J \sum_i \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z \right)$$

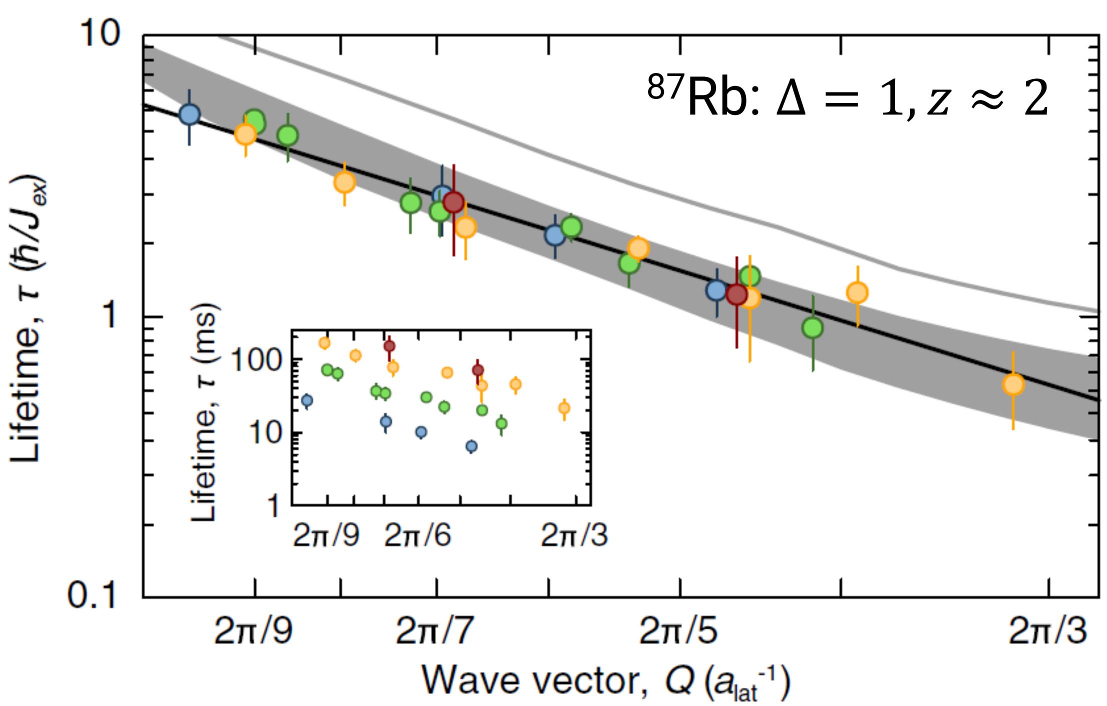


Neutron scattering in $KCuF_3$: $S \sim q^{-z}$

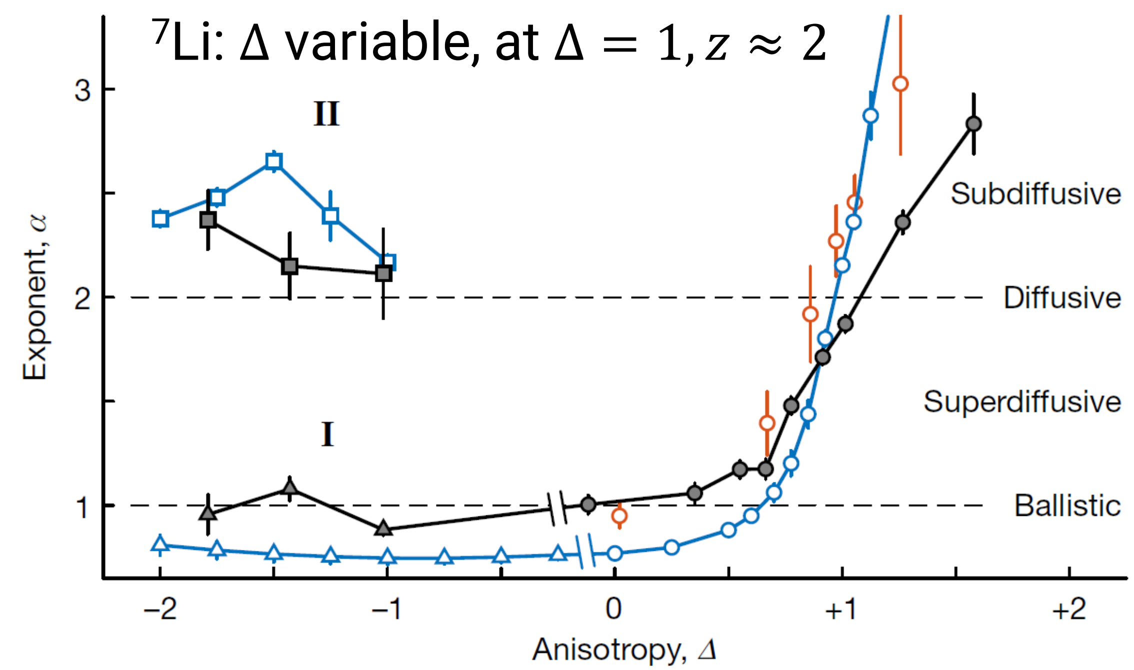


A. Scheie, et al., Nat. Phys. **17**, 726 (2021)

Cold atoms, lifetime of spin helices: $T \sim q^{-z}$



S. Hild, et al., Phys. Rev. Lett. **113**, 147205 (2014)



P. N. Jepsen, et al., Nature **588**, 403 (2020)
P. N. Jepsen, et al., Phys. Rev. X **11**, 041054 (2022)

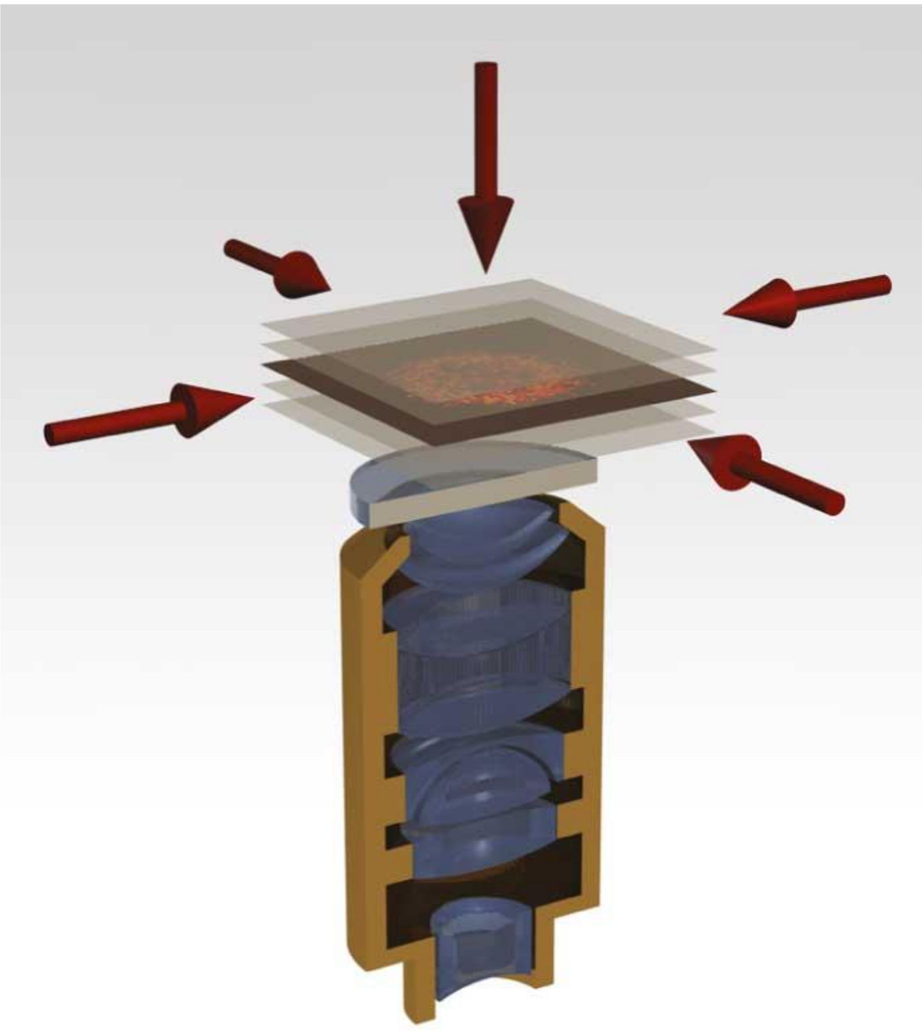
LMU

Outline

- Experimental platform
- Measuring dynamical exponents
- Probing microscopic requirements
- Characterizing transport fluctuations

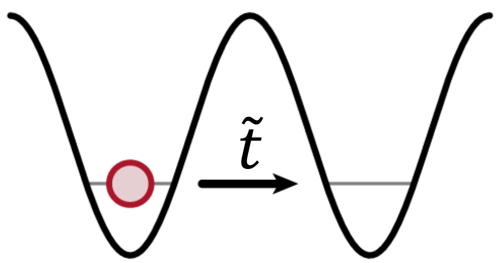
Quantum gas microscope

^{87}Rb atoms in optical lattice

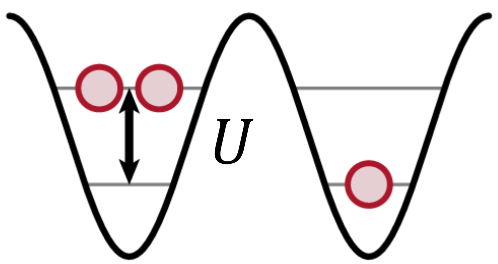


realize Bose-Hubbard model:

$$\hat{H} = -\tilde{t}\sum_{\langle i,j\rangle}\hat{c}_i^{\dagger}\hat{c}_j + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i + 1)$$

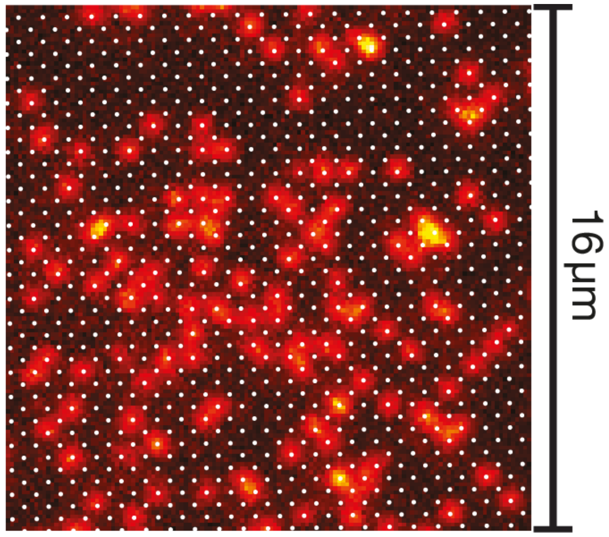


Hopping

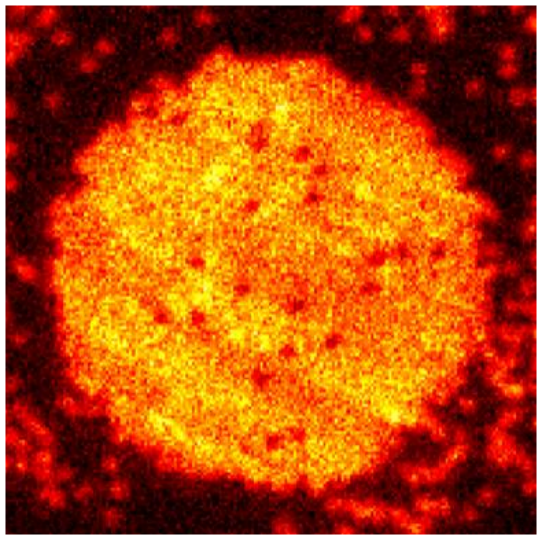


On-site interaction

Single-site resolved fluorescence imaging



Mott insulators with 2000 atoms



J. F. Sherson, C. Weitenberg, et al., Nature **467**, 68 (2010)
W. Bakr, et al., Nature **462**, 74 (2009)

Implementing the Heisenberg model

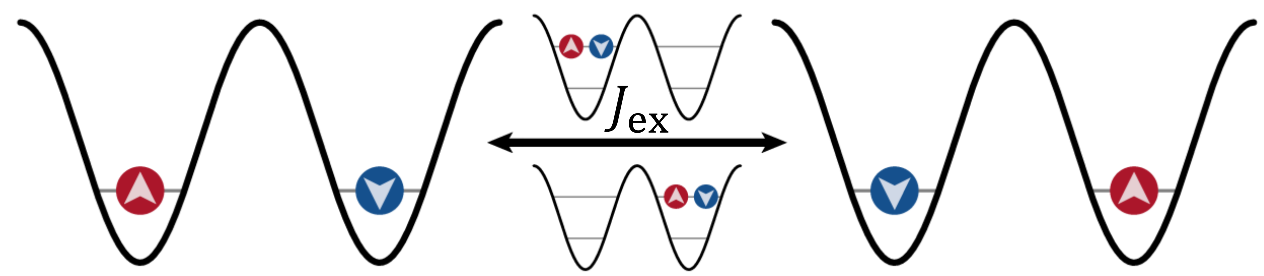
Spin encoding in hyperfine ground states

$|F = 1, m_F = -1\rangle$

$|F = 2, m_F = -2\rangle$

Heisenberg model in **atomic limit** $\tilde{t} \ll U$

Second-order spin exchange $J_{\text{ex}} = 4\tilde{t}^2/U$

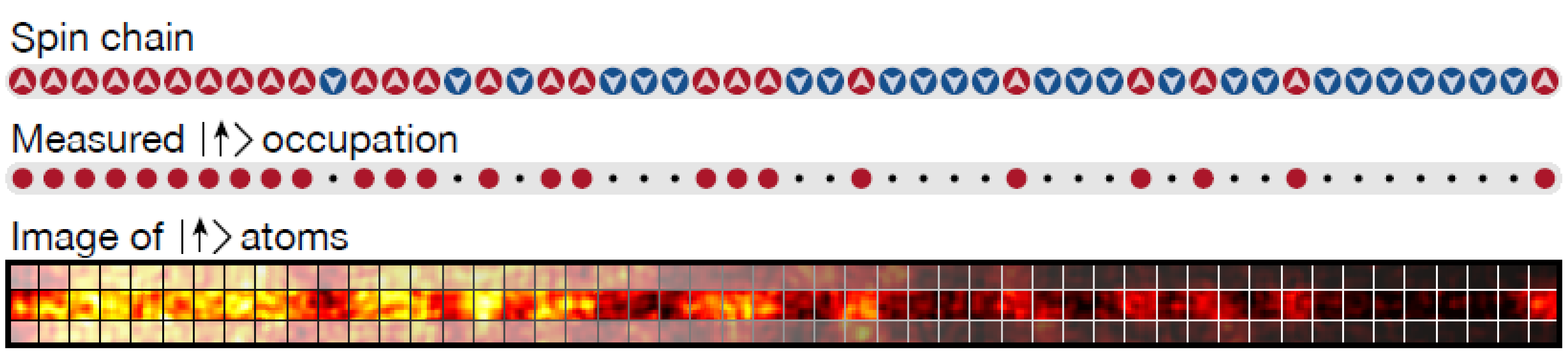


A. B. Kuklov, et al., Phys. Rev. Lett. **90**, 100401 (2003)
L.-M. Duan, et al., Phys. Rev. Lett. **91**, 090402 (2003)
S. Trotzky, et al., Science **319**, 295 (2008)

Isotropic Heisenberg model for ^{87}**Rb**

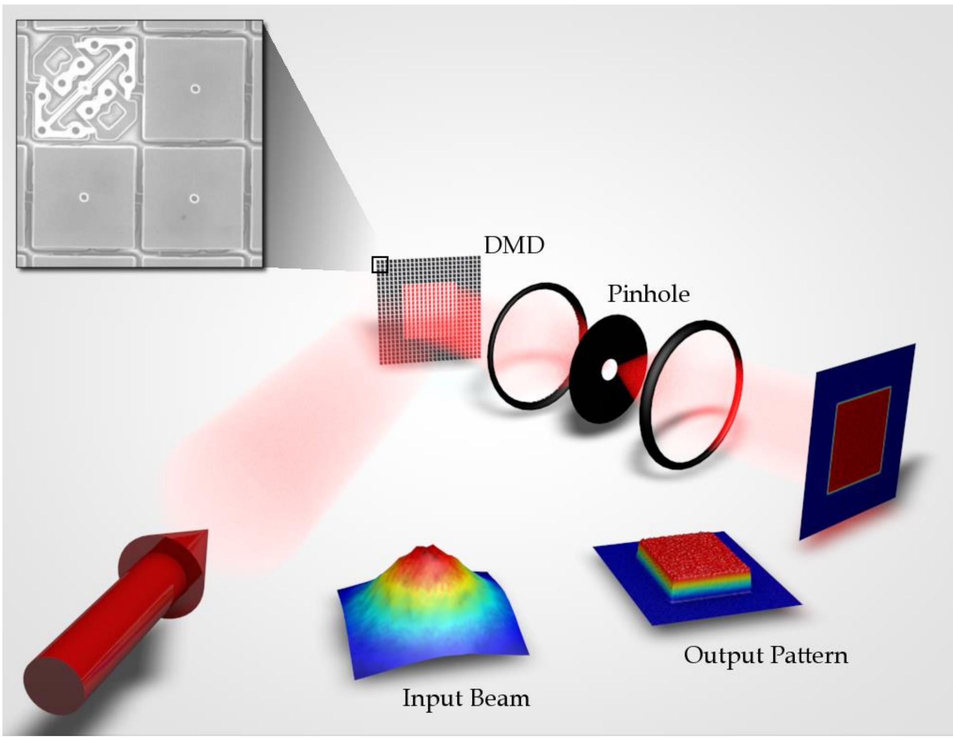
$$H = -J_{\text{ex}} \sum_i \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z \right), \Delta \approx 0.99$$

Spin detection

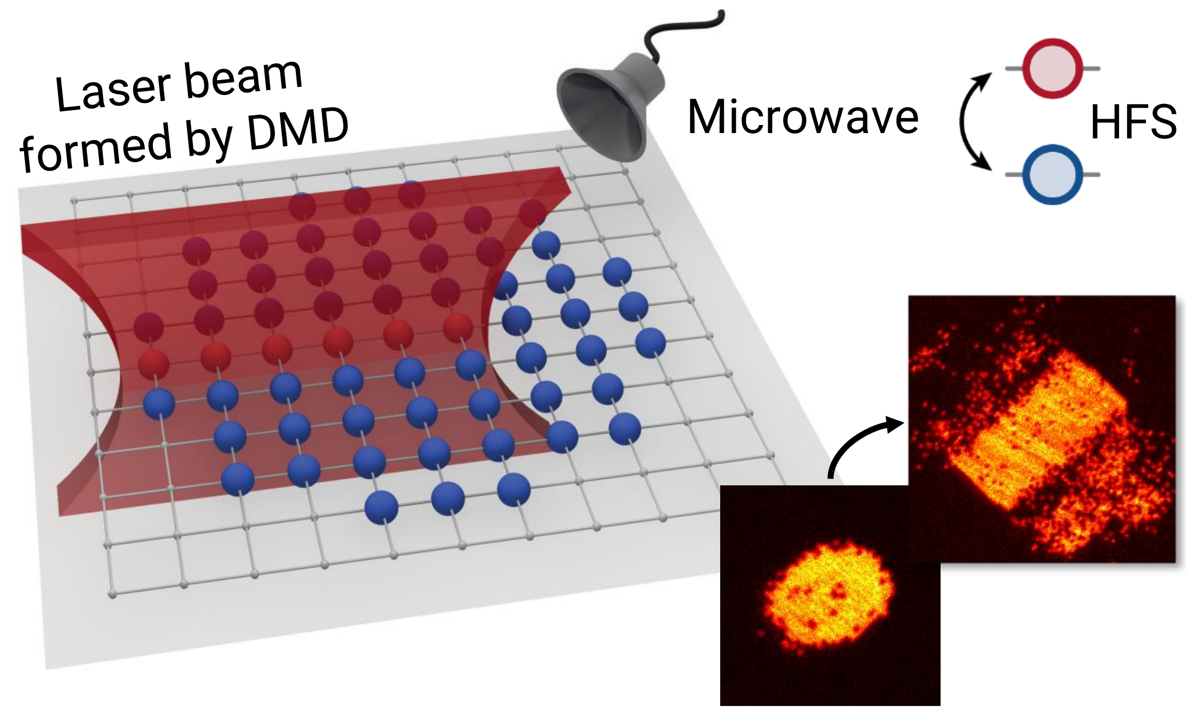


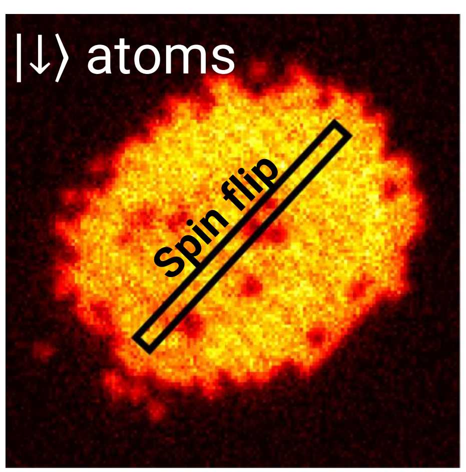
Site-resolved addressing

Digital micromirror device (DMD)



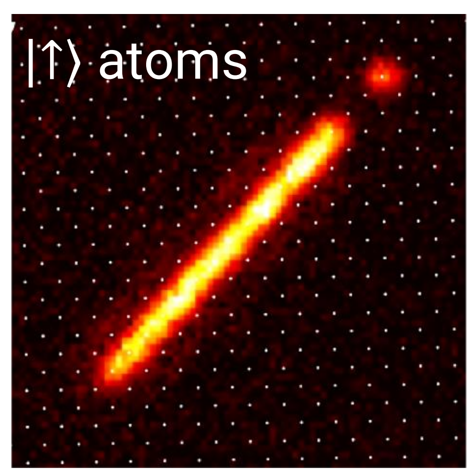
Potential shaping and local spin flips

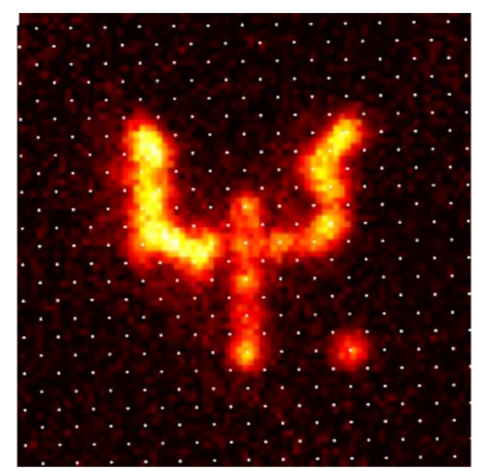




Single-site resolved

addressing





Arbitrarily programmable patterns

C. Weitenberg, et al., Nature **471**, 319 (2011)
T. Fukuhara, et al., Nat. Phys. **9**, 235 (2013)

Outline

- Experimental platform
- Measuring dynamical exponents
- Probing microscopic requirements
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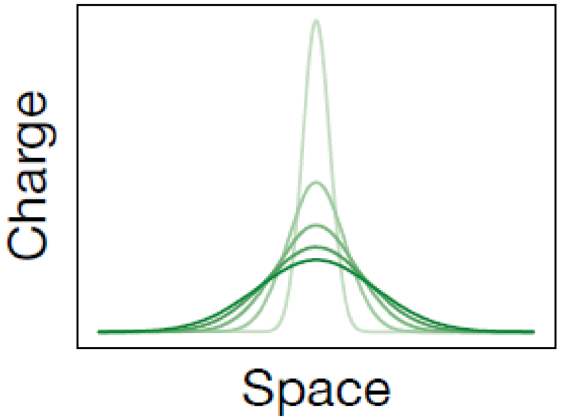
Measuring scaling functions

Measurement protocols for **experiments with spatiotemporal readout**:

$$\langle S^z(x,t)\rangle - \langle S^z(x,0)\rangle_{\text{eq}} \sim \int dx' \langle S^z(x',0)\rangle \langle S^z(x,t) S^z(x',0)\rangle_c$$

Local initial state: $S^z(x,0) \sim \delta(x)$

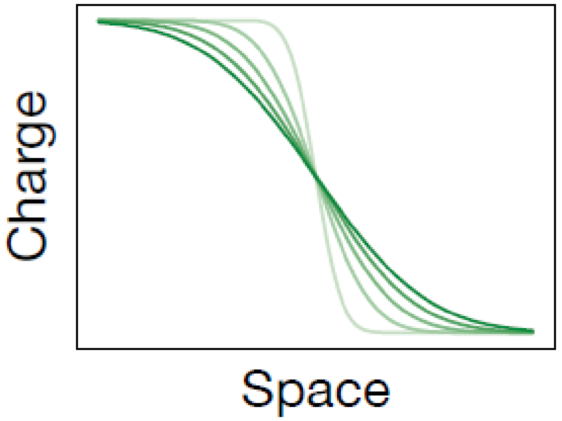
$$\langle S^z(x,t)\rangle \sim \langle S^z(x,t) S^z(0,0)\rangle_c \sim \frac{1}{t^{1/z}} f\left(\frac{x}{t^{1/z}}\right)$$



Domain-wall initial state: $S^z(x,0) \sim \Theta(x)$

$$\langle S^z(x,t)\rangle \sim \int dx\, \langle S^z(x,t) S^z(0,0)\rangle_c \sim F\left(\frac{x}{t^{1/z}}\right),$$

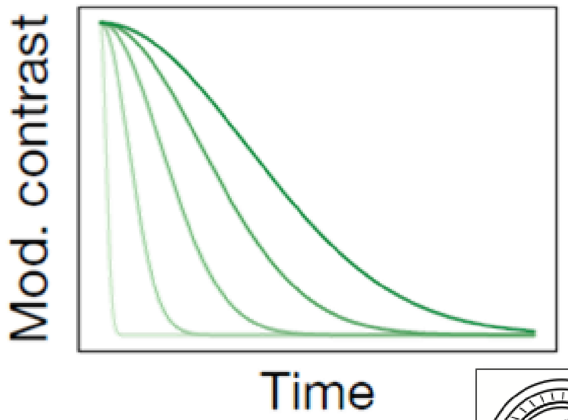
$$F'(\zeta) = f(\zeta)$$



Sinusoidal initial state: $S^z(x,0) \sim \cos kx$

$$\langle S^z(0,t)\rangle \sim \int_{-\infty}^{\infty} dx\, \langle S^z(x,t) S^z(0,0)\rangle_c \sim \tilde{f}\left(k t^{1/z}\right),$$

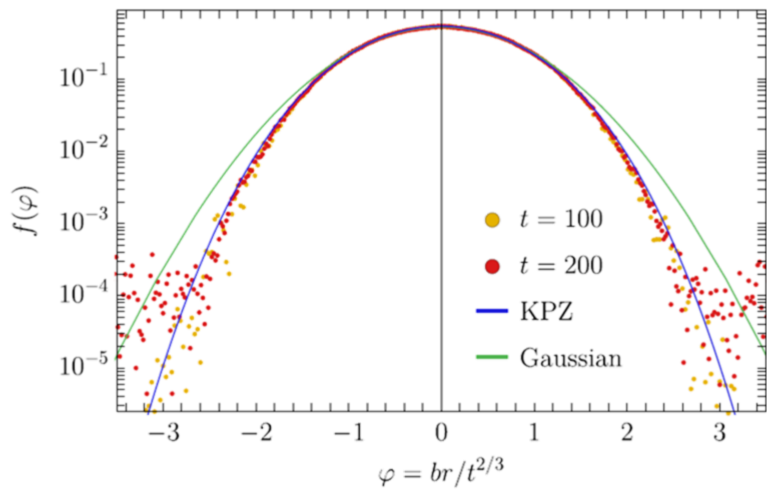
$$\tilde{f}(\nu) = \int d\zeta\, e^{i\nu\zeta} f(\zeta)$$

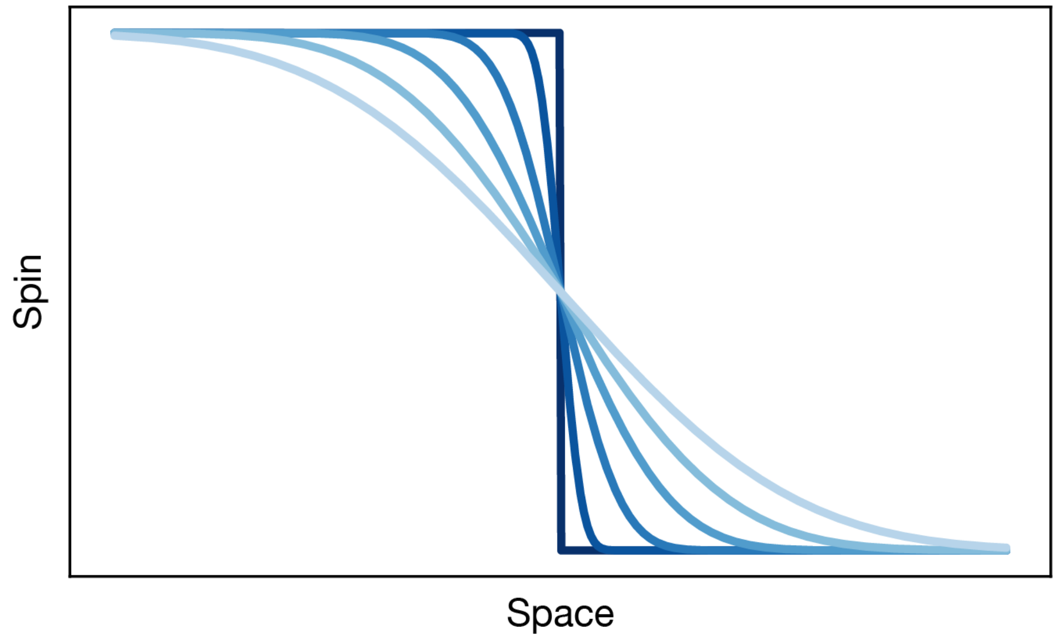


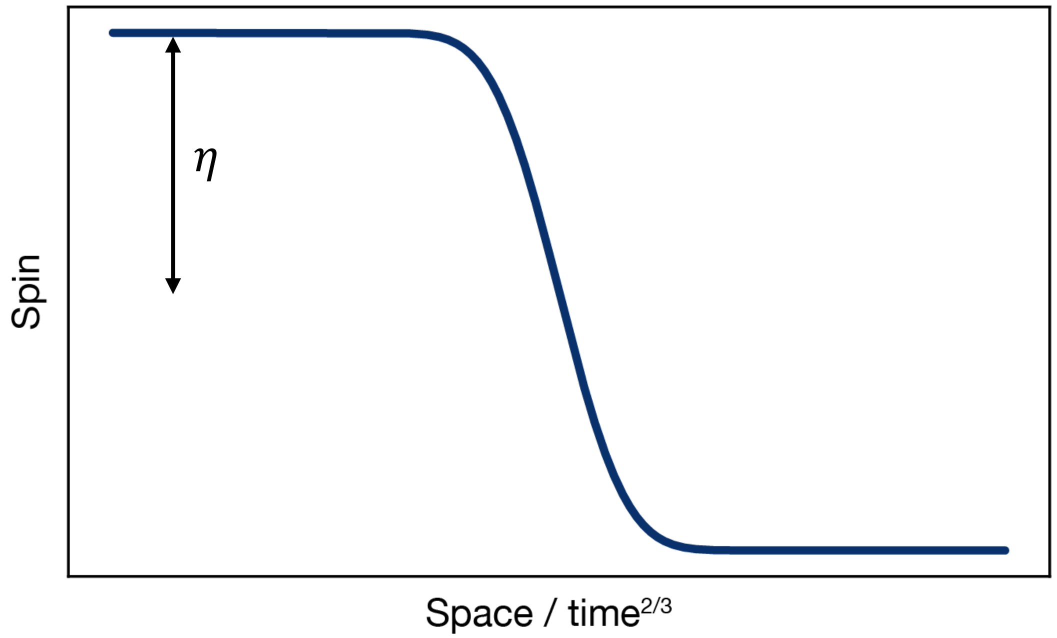
B. Bertini et al., Rev. Mod. Phys. **93**, 025003 (2020)

Measuring scaling functions

Realize high-temperature initial state in the linear-response regime: $\eta \to 0$
→ **Domain wall protocol** with highest **experimental fidelity**:

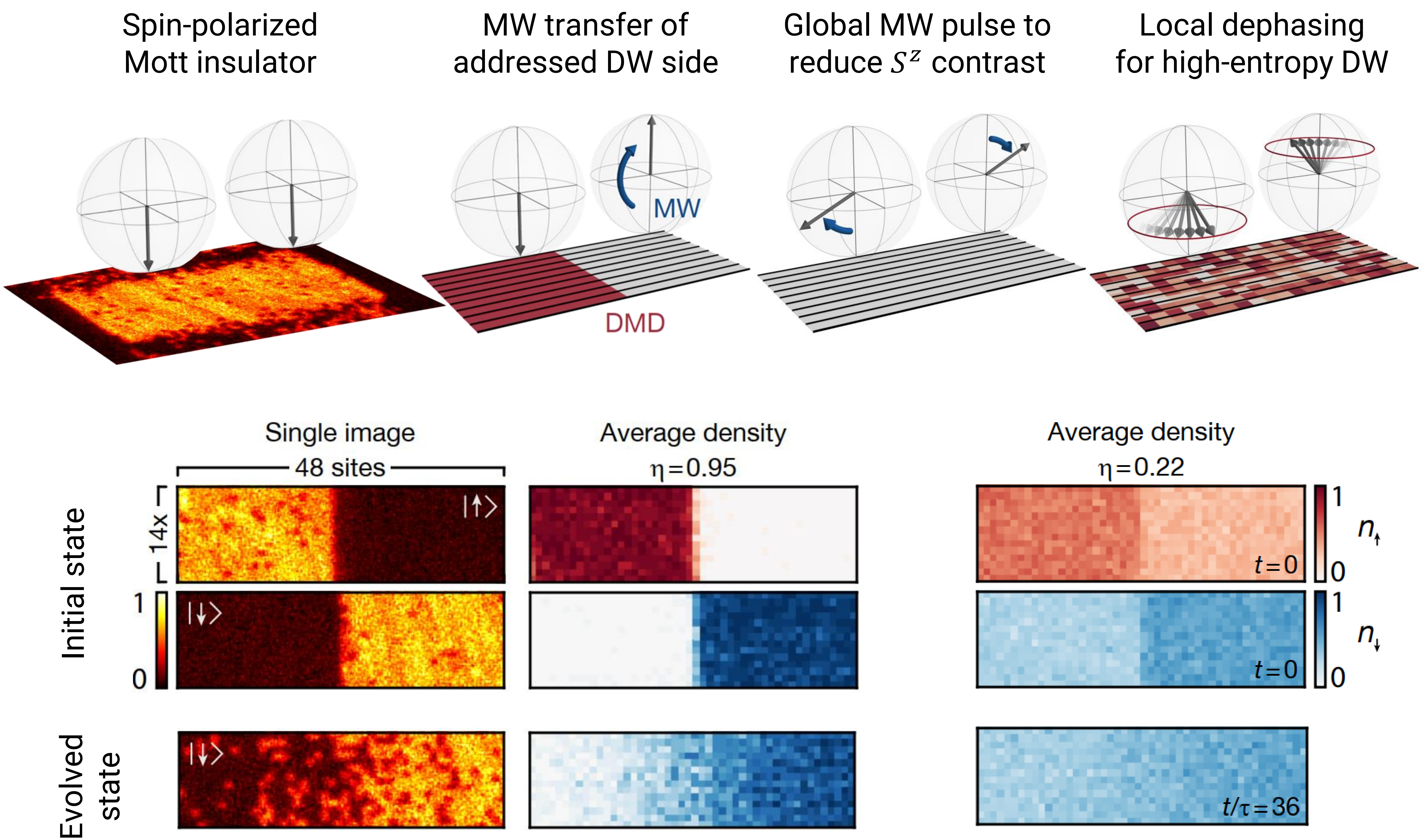




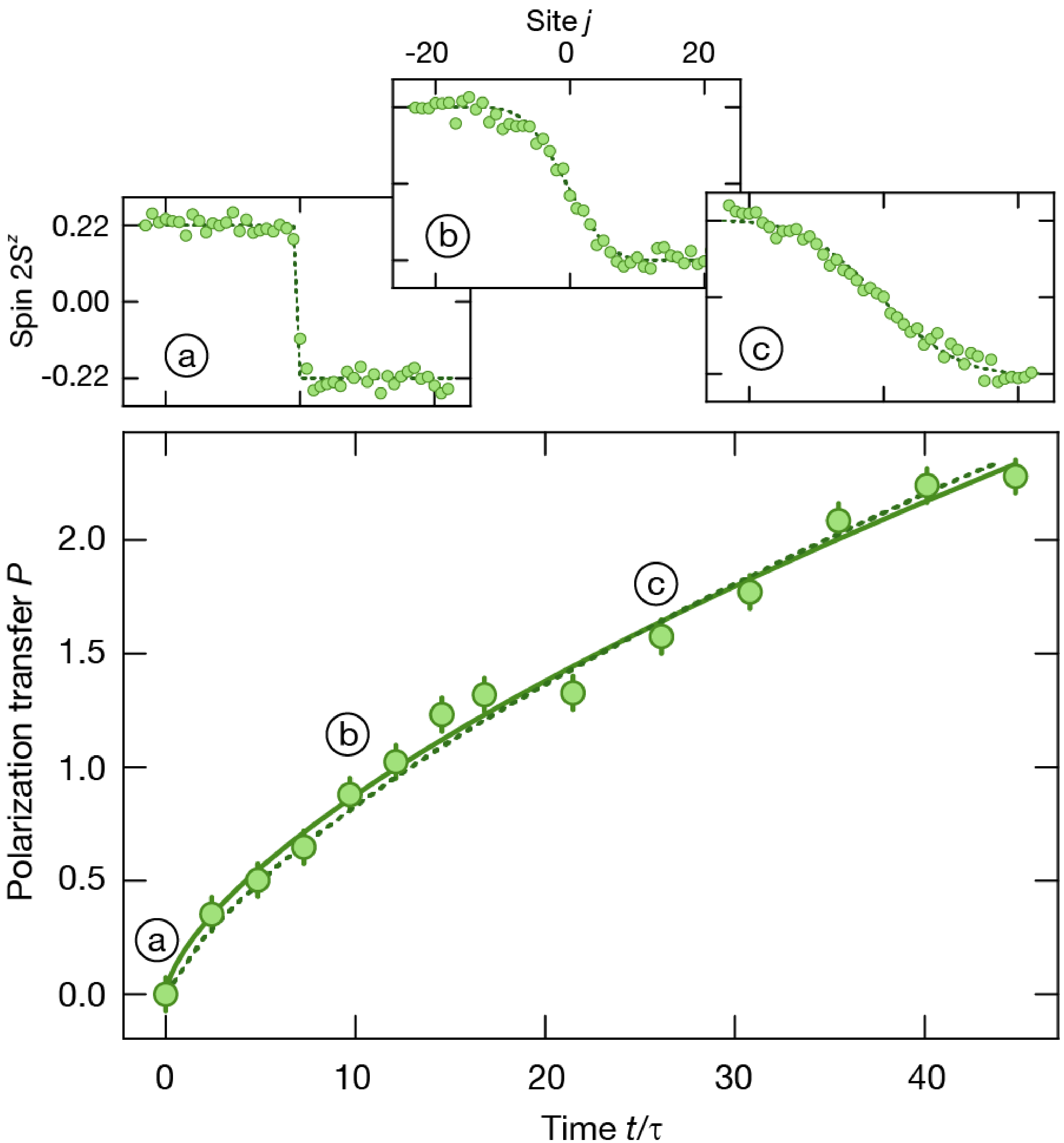


M. Ljubotina, et al., Nat. Comm. **8**, 16117 (2017)
M. Ljubotina, et al., Phys. Rev. Lett. **122**, 210602 (2019)

Experimental sequence

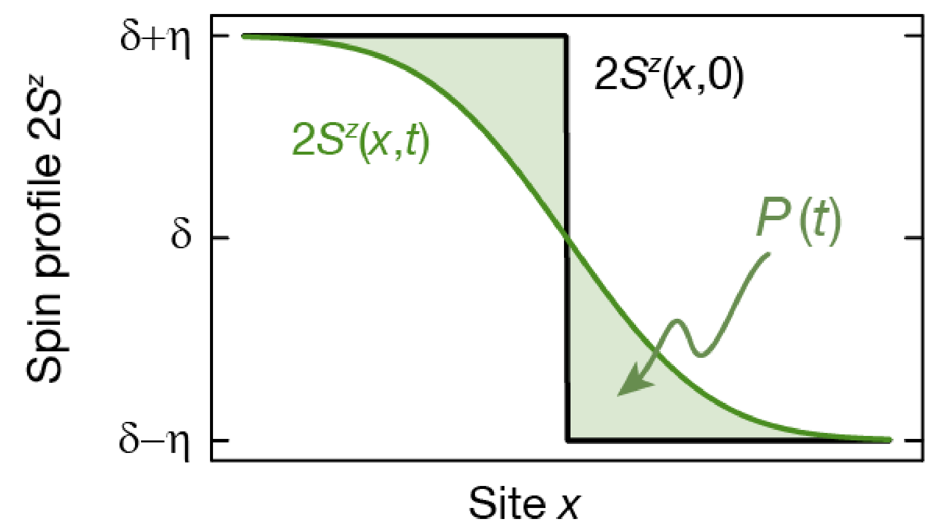


Superdiffusive polarisation transfer with $z = 1.54(7)$

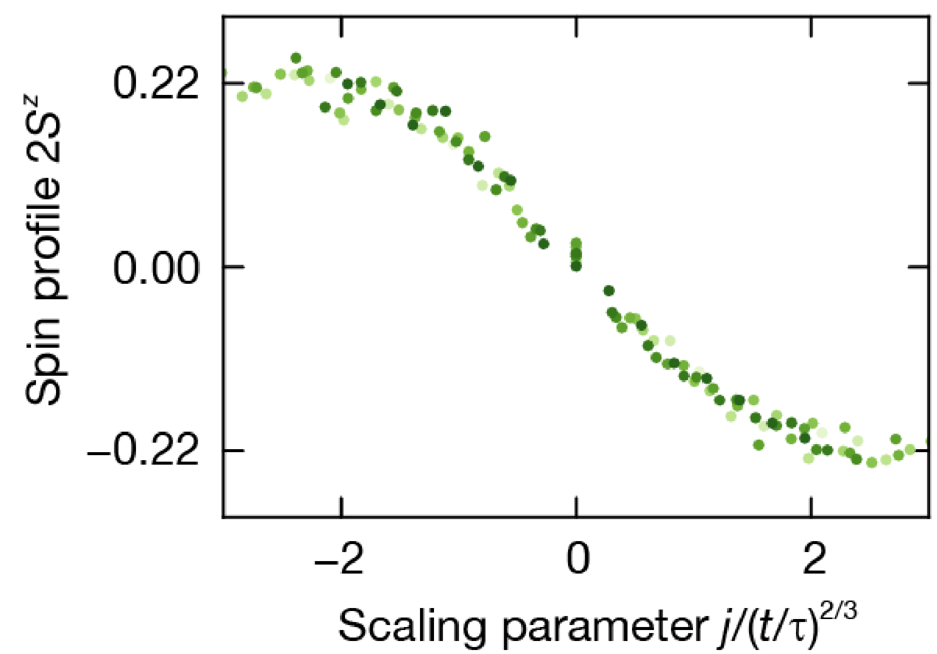


Evaluate **polarization transfer**

$$P(t) \sim \int_0^\infty \left(S^z(x,t) - S^z(x,0)\right)dx \sim t^{1/z}$$



Rescaled profiles collapse onto KPZ scaling form



Outline

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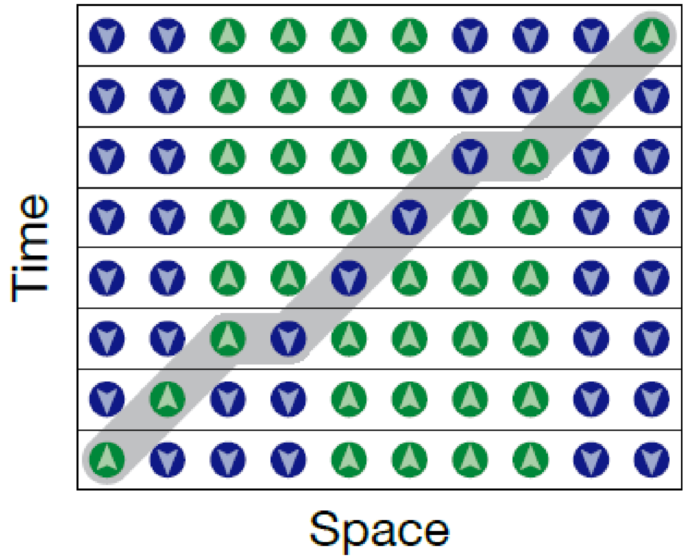
Microscopic origin of superdiffusion

Generalized hydrodynamics:

- Spin transport as result of **ballistically propagating quasiparticles**
- Quasiparticle **kinematics** governed by thermodynamic quantities like density, magnetization, velocity

$$\int dx\, \langle j(x,t) j(0,0)\rangle_c \sim \sum_s j_s^2 \rho_s e^{-\frac{t}{\tau_s}}$$

In Ising limit:

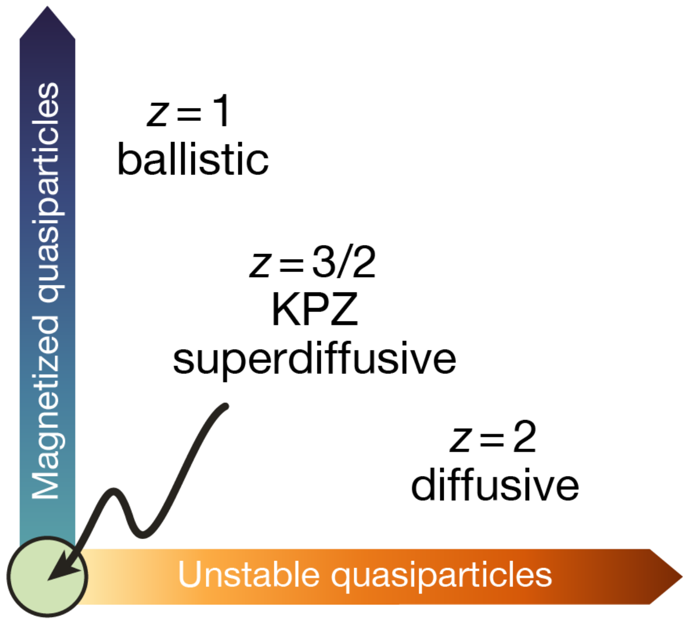


Consequence at **isotropic point**:

- Drude weight: $\mathcal{D} \sim m^2 |\log m|$
- Diffusion constant: $D \sim t^{1/3} \Rightarrow z = \frac{3}{2}$

B. Bertini, et al., Phys. Rev. Lett. **117**, 207201 (2016)
O. A. Castro-Alvaredo, et al., Phys. Rev. X **6**, 041065 (2016)
B. Bertini, et al., Rev. Mod. Phys. **93**, 025003 (2020)
V. B. Bulchandani, et al., J. Stat. Mech. 084001 (2021)

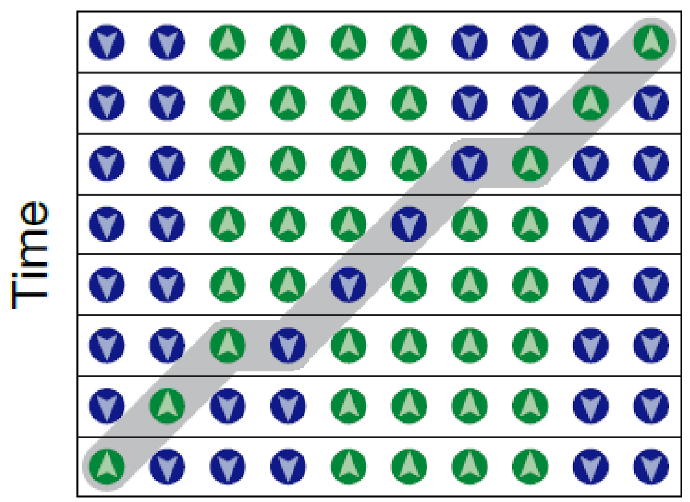
Microscopic origin of superdiffusion



Requirements for Heisenberg superdiffusion:

- Integrability
- SU(2) symmetry

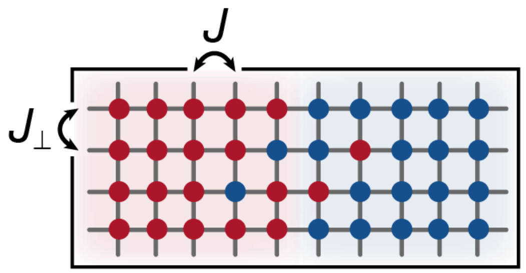
In Ising limit:

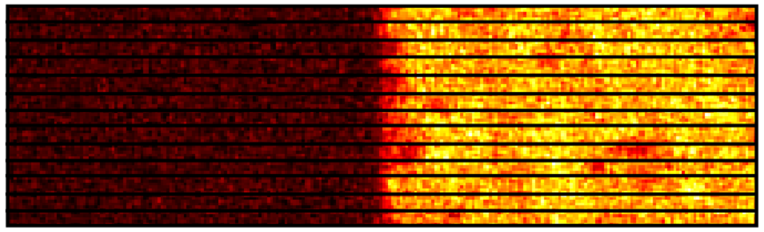


B. Bertini, et al., Phys. Rev. Lett. **117**, 207201 (2016)
O. A. Castro-Alvaredo, et al., Phys. Rev. X **6**, 041065 (2016)
B. Bertini et al., Rev. Mod. Phys. **93**, 025003 (2020)
V. B. Bulchandani, et al., J. Stat. Mech. 084001 (2021)

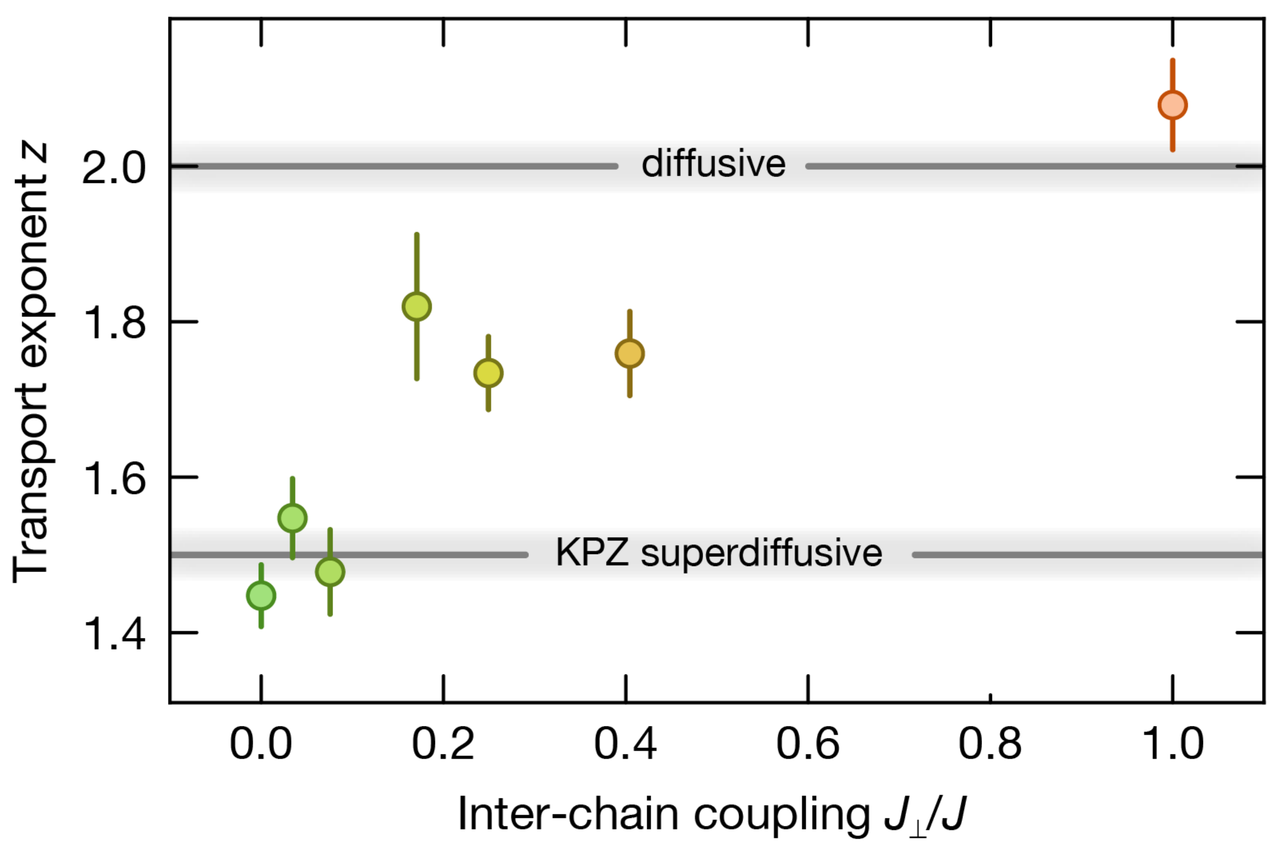
Breaking integrability: Dimensionality

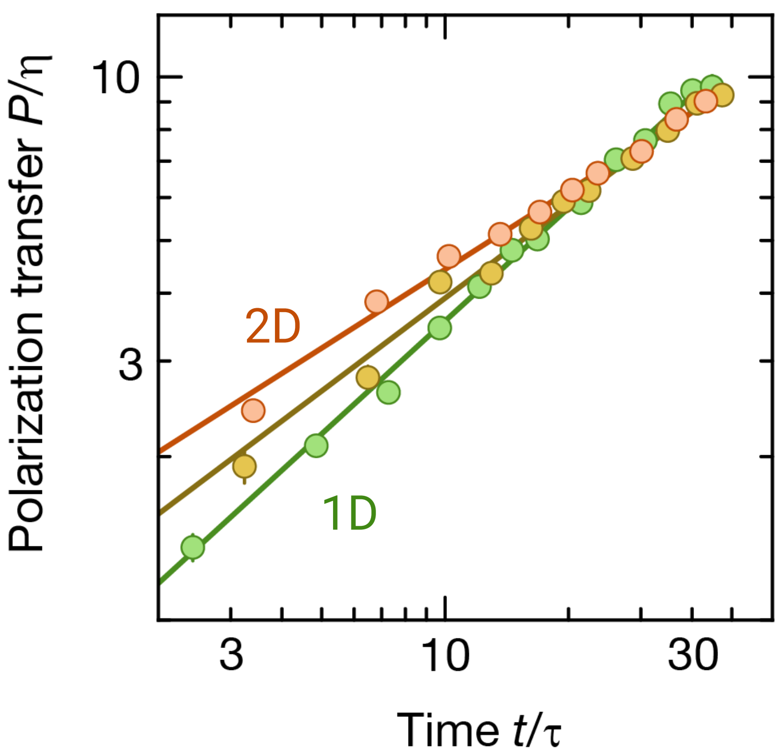
Use inherently 2D system to couple chains
Vary coupling between 1D and (non-integrable) 2D Heisenberg model





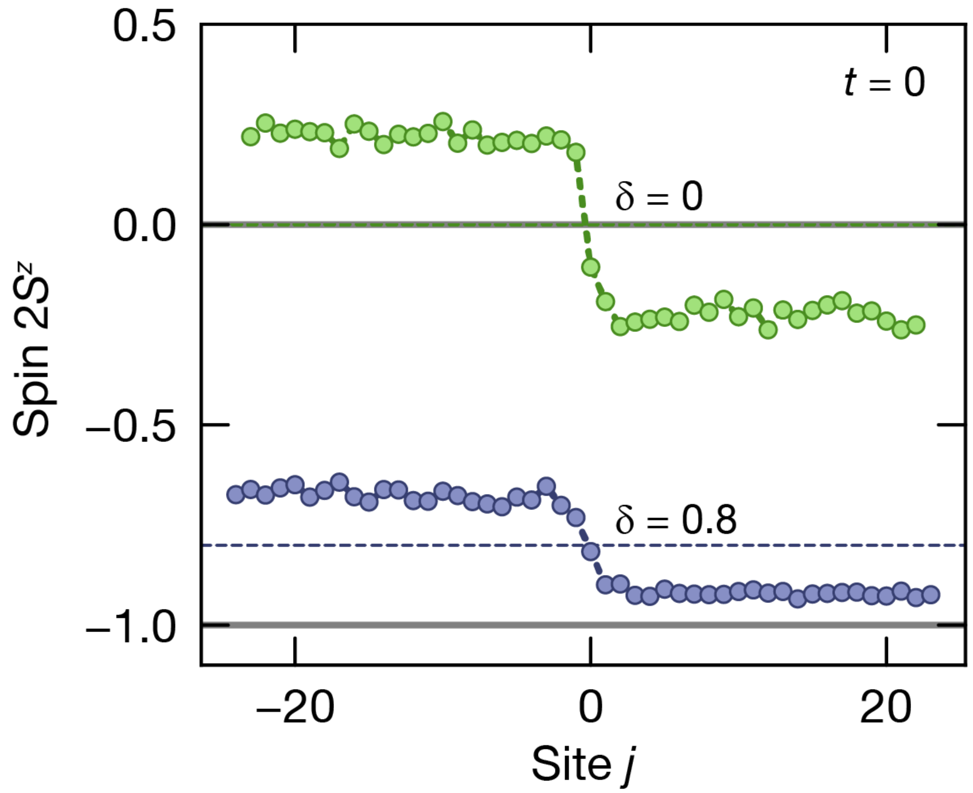
Breakdown of superdiffusion under integrability breaking

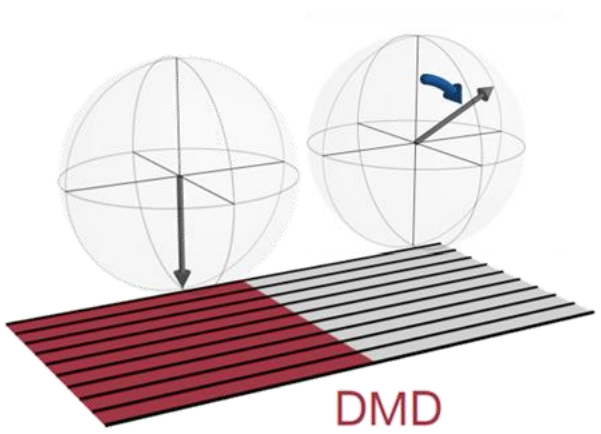


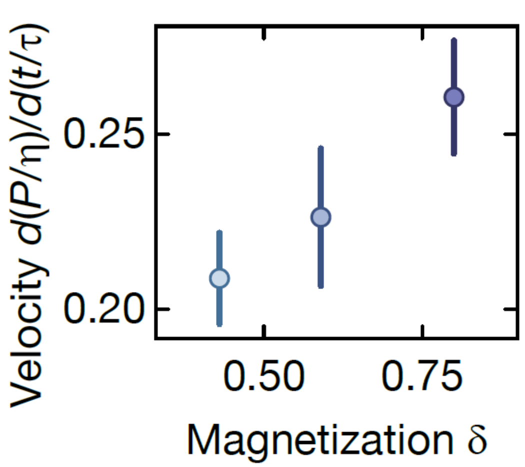


Breaking symmetry: Magnetization

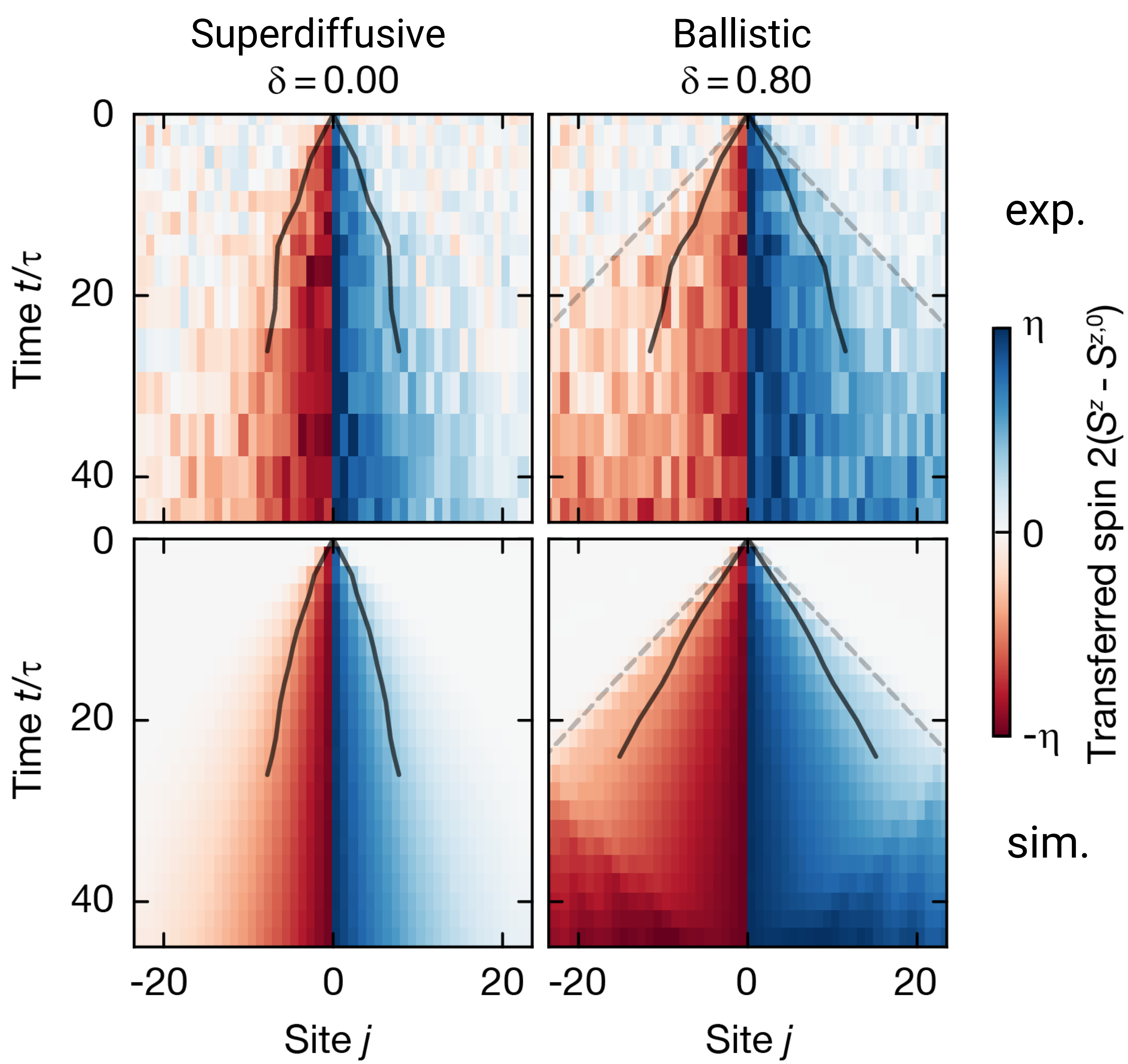
Prepare net magnetized DW $\delta \neq 0$



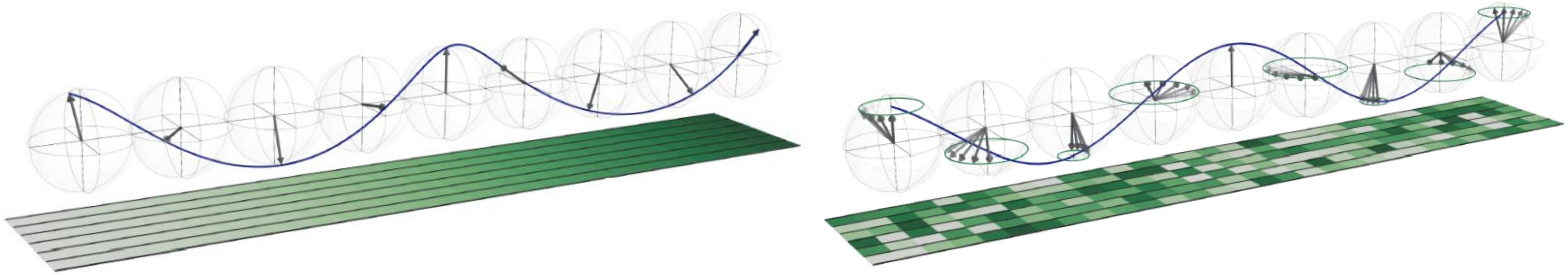




Qualitative difference to unmagnetized case



Spin helix initial states



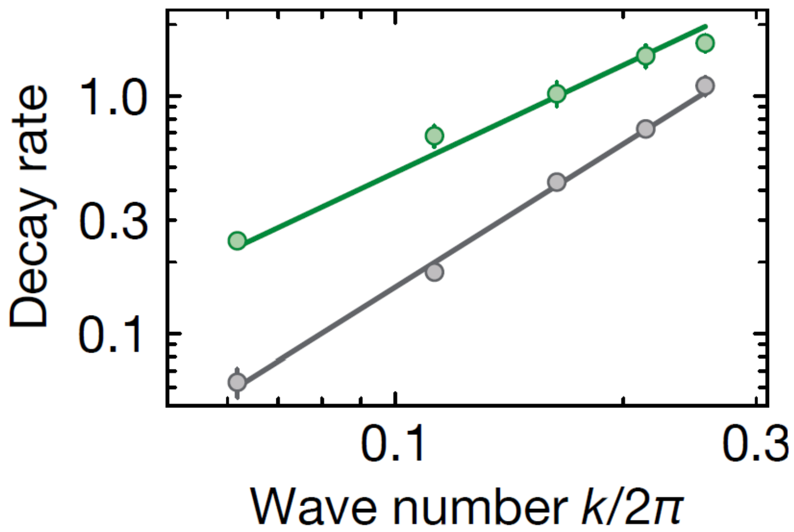
Spin helix preparation with magnetic gradient

Decay rate: $\gamma \sim k^z$ with $\mathrm{z} = 1.9(1)$ and $2.3(2)$

S. Hild, et al., PRL **113**, 147205 (2014)
P. N. Jepsen, et al., Nature **588**, 403 (2020)

Dephasing DMD pulse after helix preparation

Pure helix: $z = 2.1(1)$
Dephased: $z = 1.4(1), \eta \sim 0.3$
$z = 1.5(1), \eta \sim 0.9$



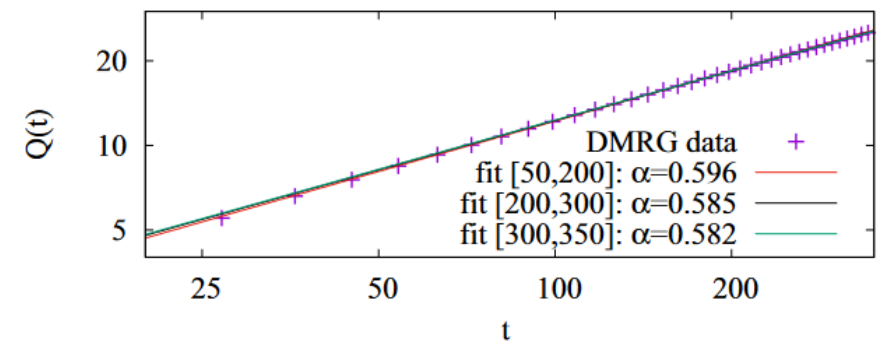
Indication of non-uniform population of quasiparticles for full-contrast helix

Full-contrast domain walls

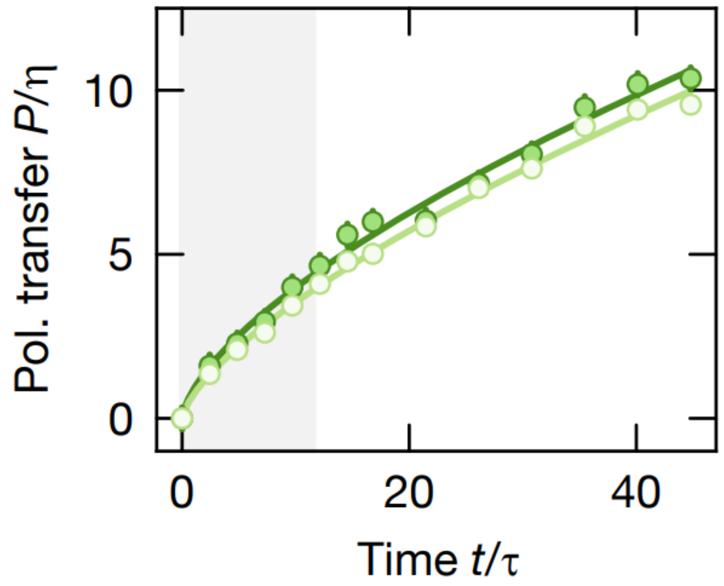
Domain walls without contrast-reducing MW pulse

Expect **diffusion for full contrast** at very late times

M. Ljubotina, et al., Nat. Commun. **8**, 16117 (2017)
G. Misguich, et al., Phys. Rev. B **96**, 195151 (2017)



Superdiffusive fitted exponents within experimentally **accessible times** (here 50τ)



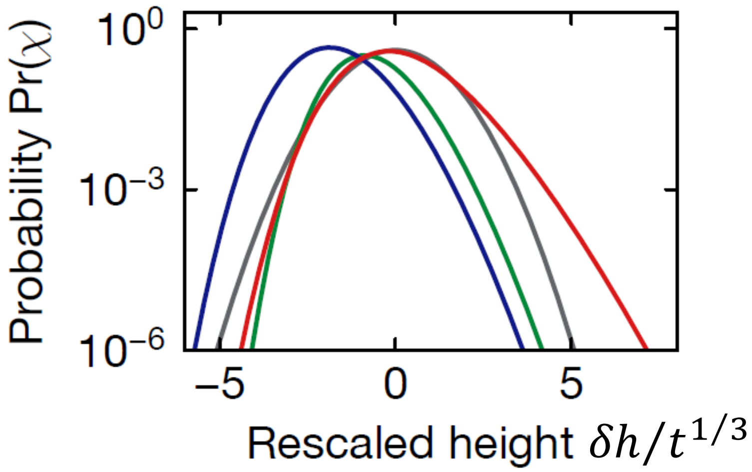
$\eta = 0.22, z = 1.54(7)$
$\eta = 0.93, z = 1.45(5)$

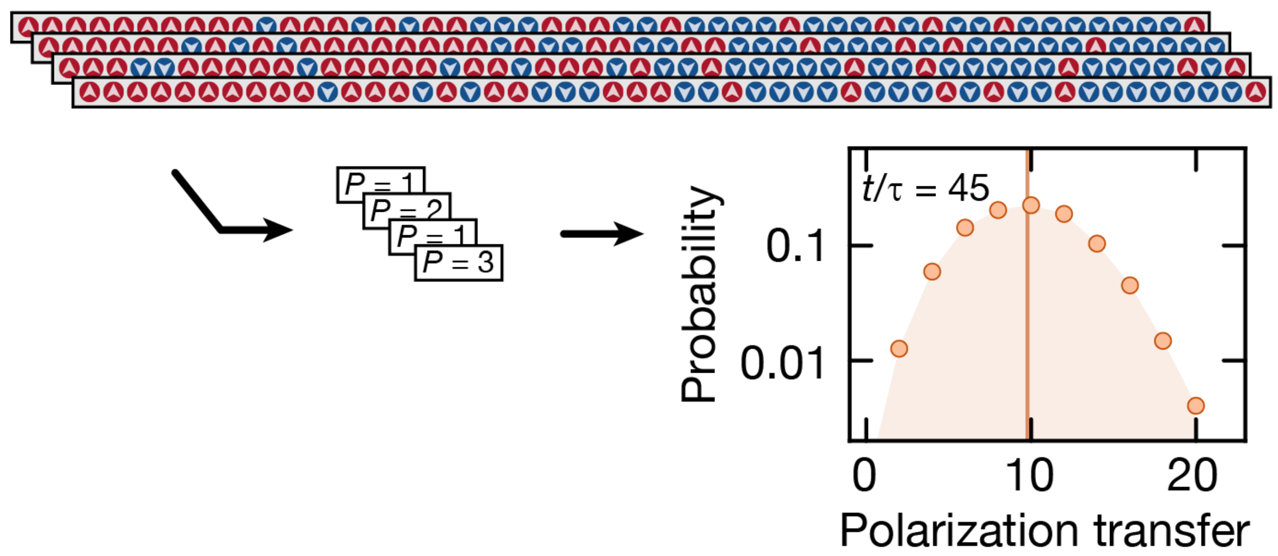
KPZ-like behavior at early times?

Outline

- Experimental platform
- Measuring dynamical exponents
- Probing microscopic requirements
- Characterizing transport fluctuations

Fluctuations as KPZ signature





- Signature for universality class **beyond dynamical exponent**

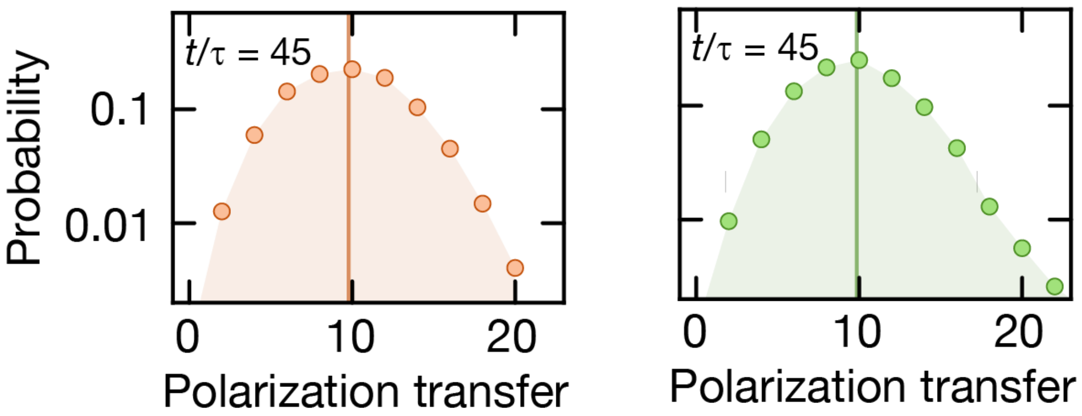
- Numerics: $\langle \partial_x h(x,t) \partial_x h(0,0) \rangle \sim \langle S^z(x,t) S^z(0,0) \rangle$
- **Assume** as effective mapping: $\partial_x h \sim S^z$? $\Rightarrow$ $\mathrm{Pr}(\delta h) \sim \mathrm{Pr}(\delta P)$?

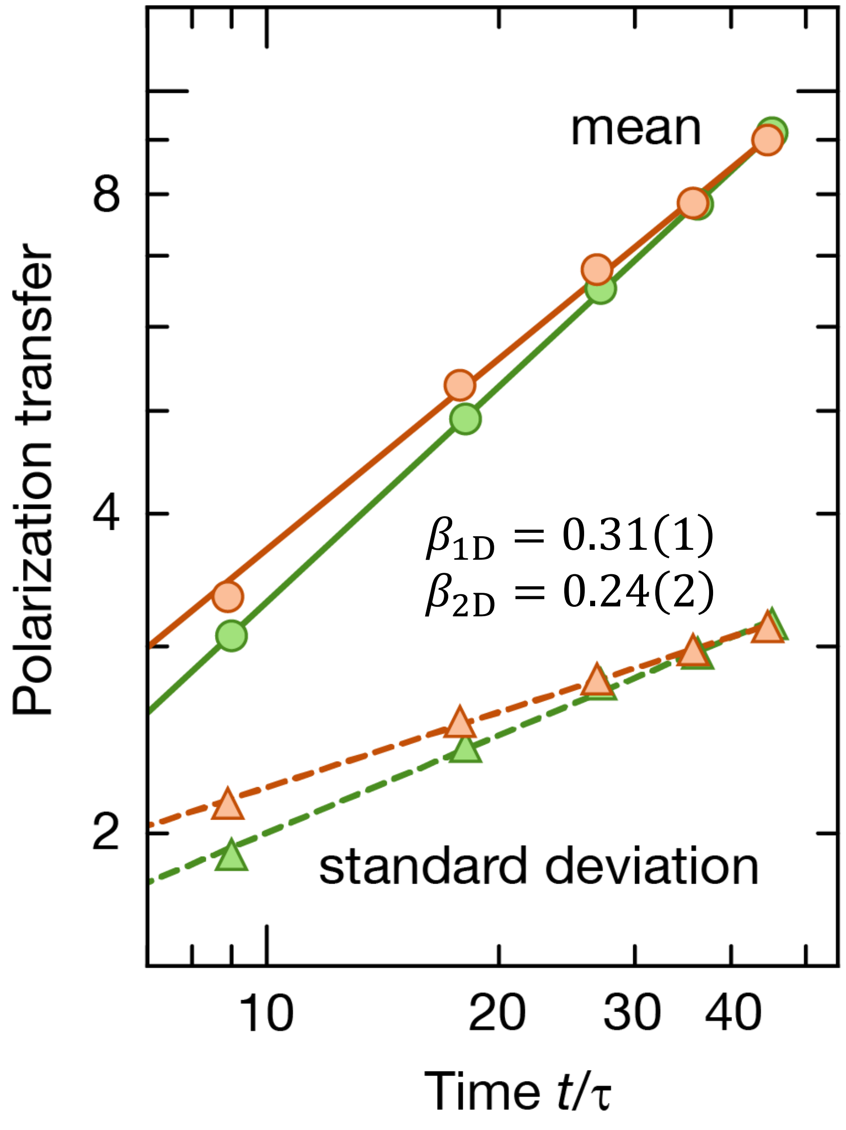
- Quantum gas microscope single-spin sensitive
- Access to full counting statistics $\mathrm{Pr}(P;t)$ at $\eta \sim 1$

Moments of polarization transfer

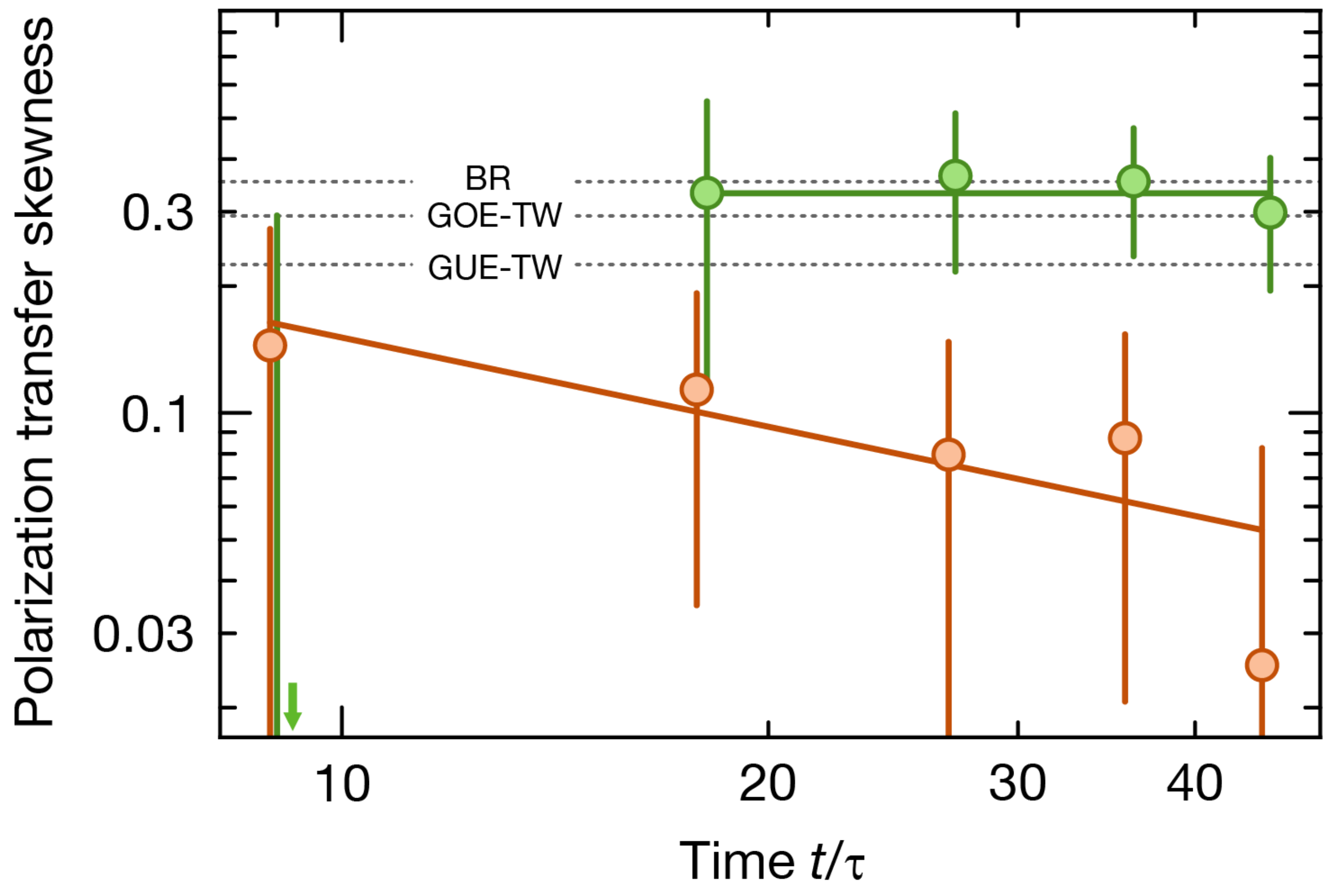
Compare **1D** to non-KPZ **2D**

Mean: dynamical exp. $z_{\text{KPZ}} = 3/2$
Std. dev.: temporal exp. $\beta_{\text{KPZ}} = 1/3$



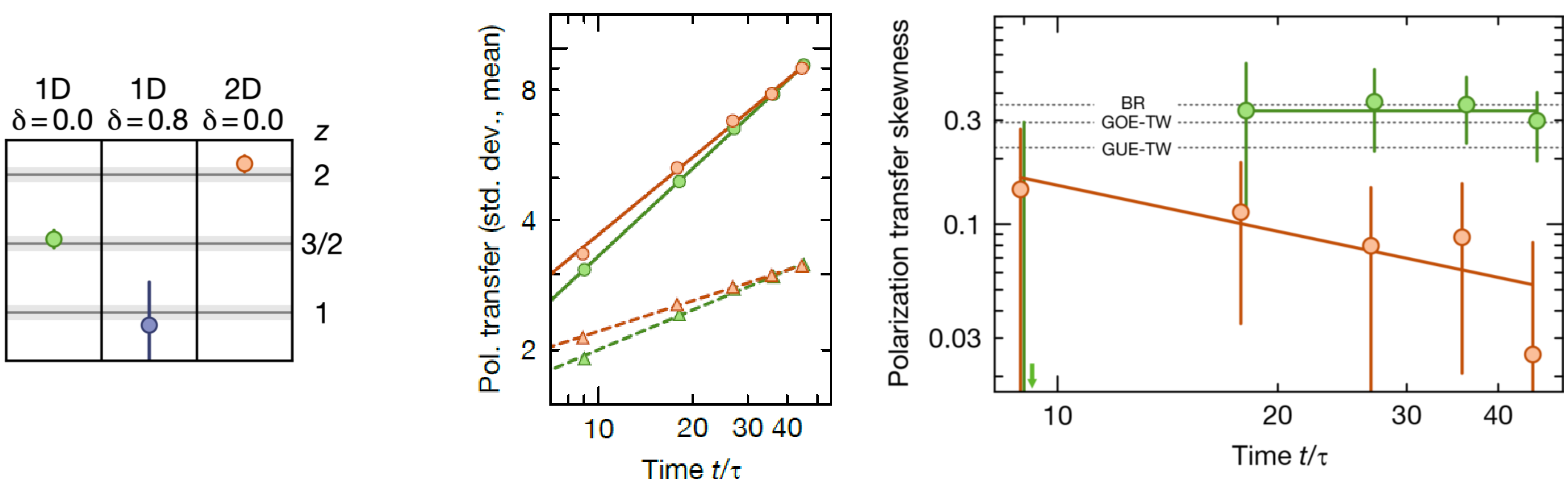


KPZ: asymmetric distr. → skewness $\mu_3/\sigma^3 = 0.33(8) \neq 0$



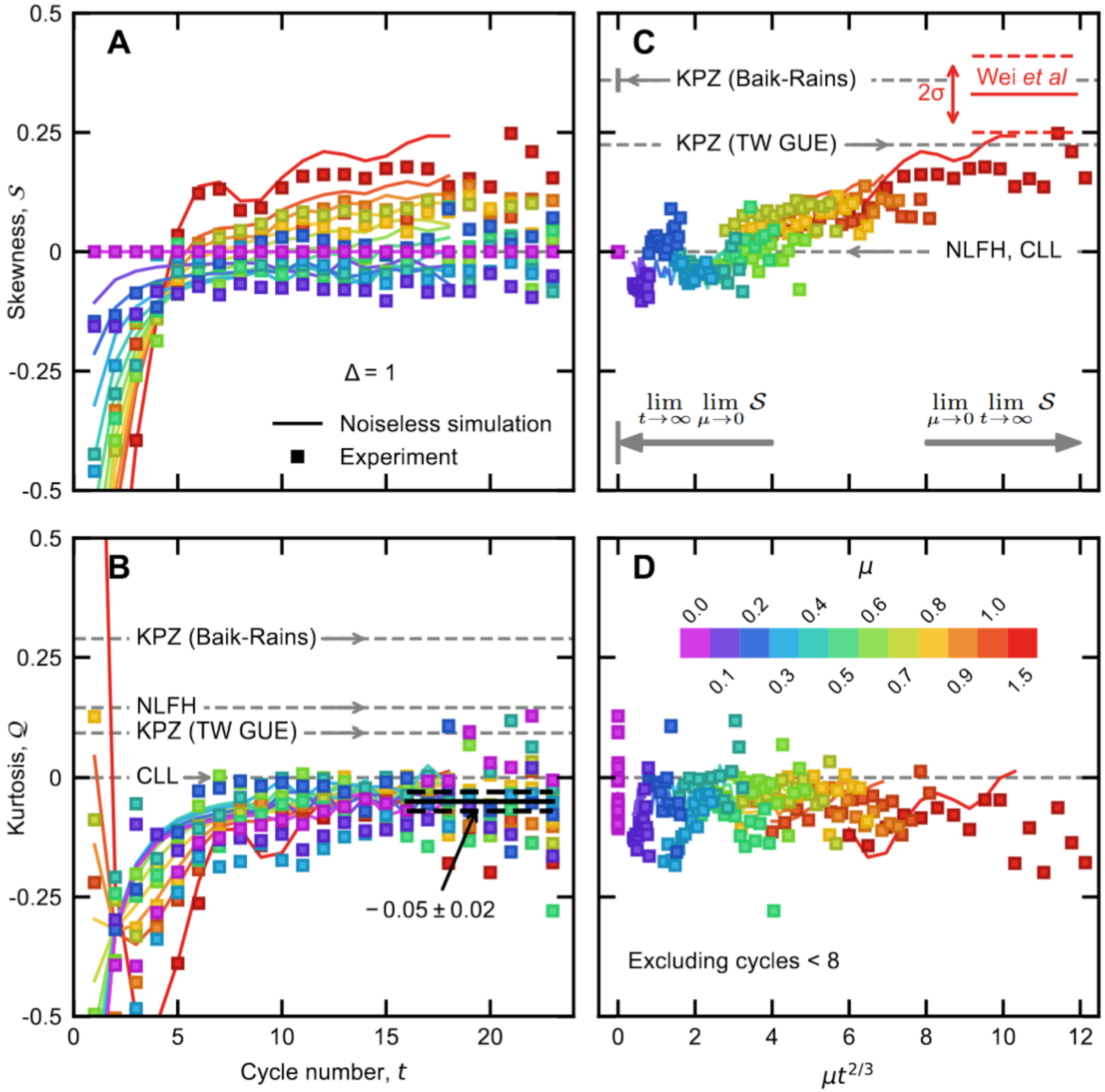
Conclusion

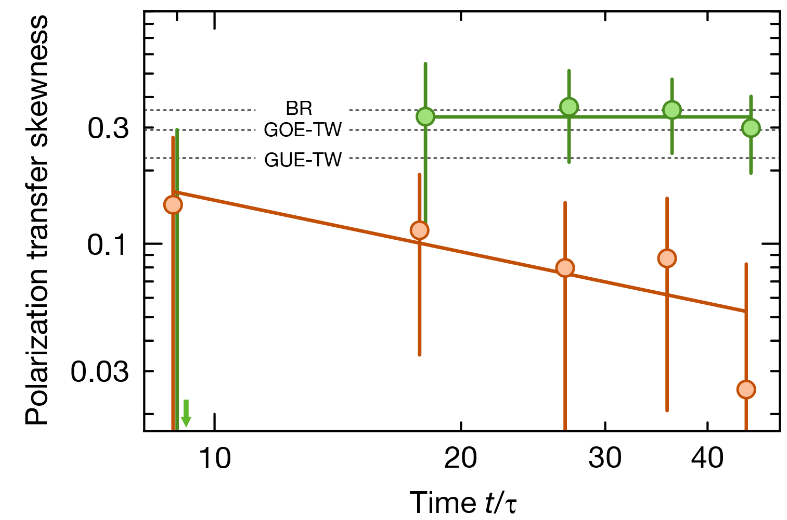
- Spin **superdiffusion** with $z = 1.54(7)$ in 1D Heisenberg model
- **Diffusive/ballistic** when breaking **integrability/SU(2) symmetry**
- Polarization transfer statistics at $\eta \sim 1$ showing **nonlinear transport** with growth exponent $\beta = 0.31(1)$ and distribution skewness $0.33(8)$



- Fundamental relation to KPZ?
- Skewness consistent with KPZ in transient non-equilibrium regime
  → near-equilibrium statistics?

Near-equilibrium fluctuations



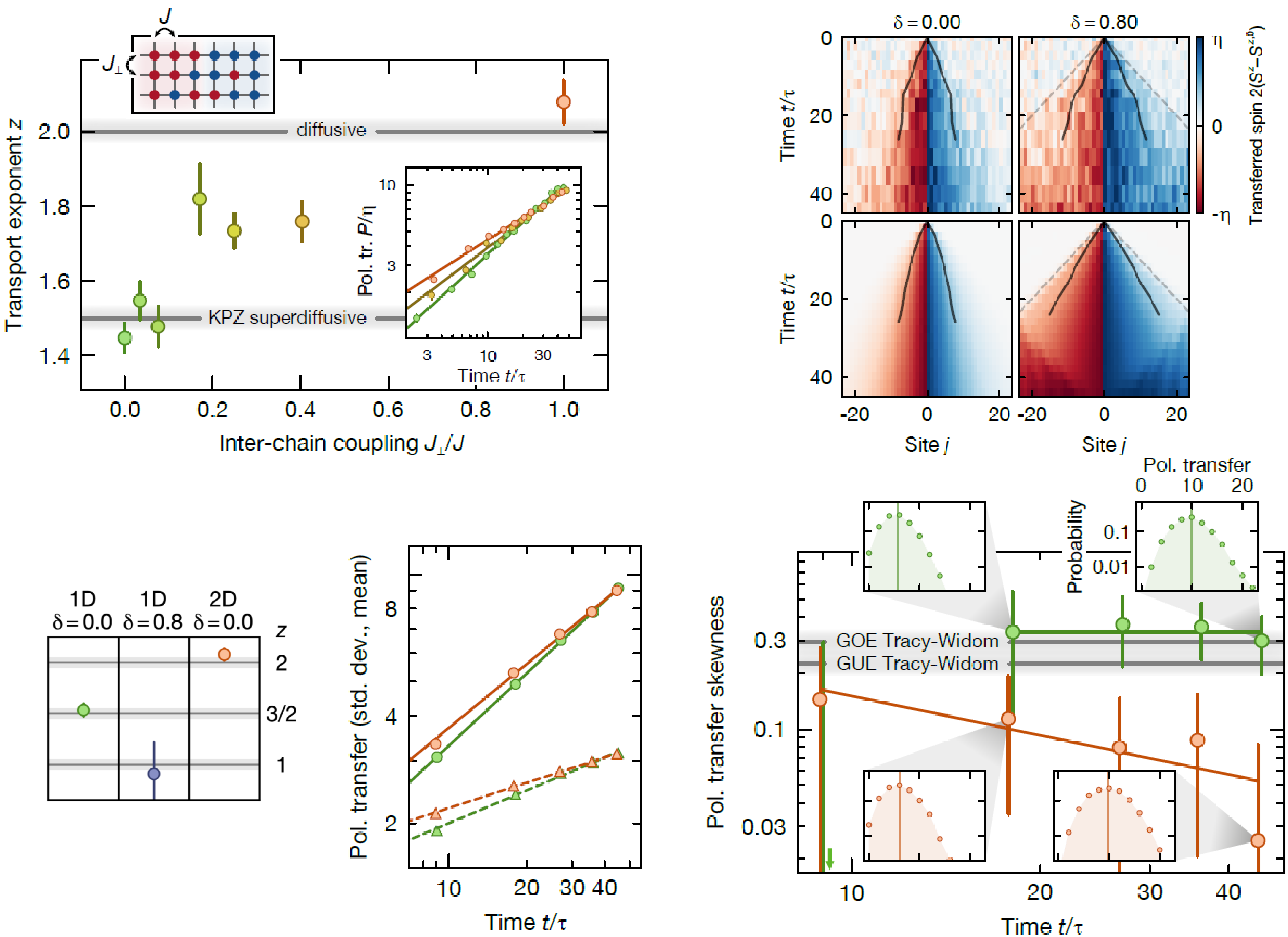


Consistent with
recent experiment using
superconducting qubits

Full counting statistics
with variable $\eta = \tanh \mu$

Near-equilibrium statistics:
Deviation from KPZ,
new universality class?

E. Rosenberg, et al., Science **384**, 48 (2024)



J
J⊥
diffusive
KPZ superdiffusive
Transport exponent z
2.0
1.8
1.6
1.4
0.0
0.2
0.4
0.6
0.8
1.0
Inter-chain coupling J⊥/J
Pol. tr. P/η
10
3
Time t/τ
30
δ = 0.00
δ = 0.80
Time t/τ
0
20
40
η
-η
Transferred spin 2(S^z−S^{z,0})
-20
Site j
1D δ = 0.0
1D δ = 0.8
2D δ = 0.0
z
2
3/2
1
Pol. transfer (std. dev., mean)
8
4
2
10
20
30
40
Time t/τ
Pol. transfer skewness
0.3
0.1
0.03
GOE Tracy-Widom
GUE Tracy-Widom
Pol. transfer
Probability
0.1
0.01