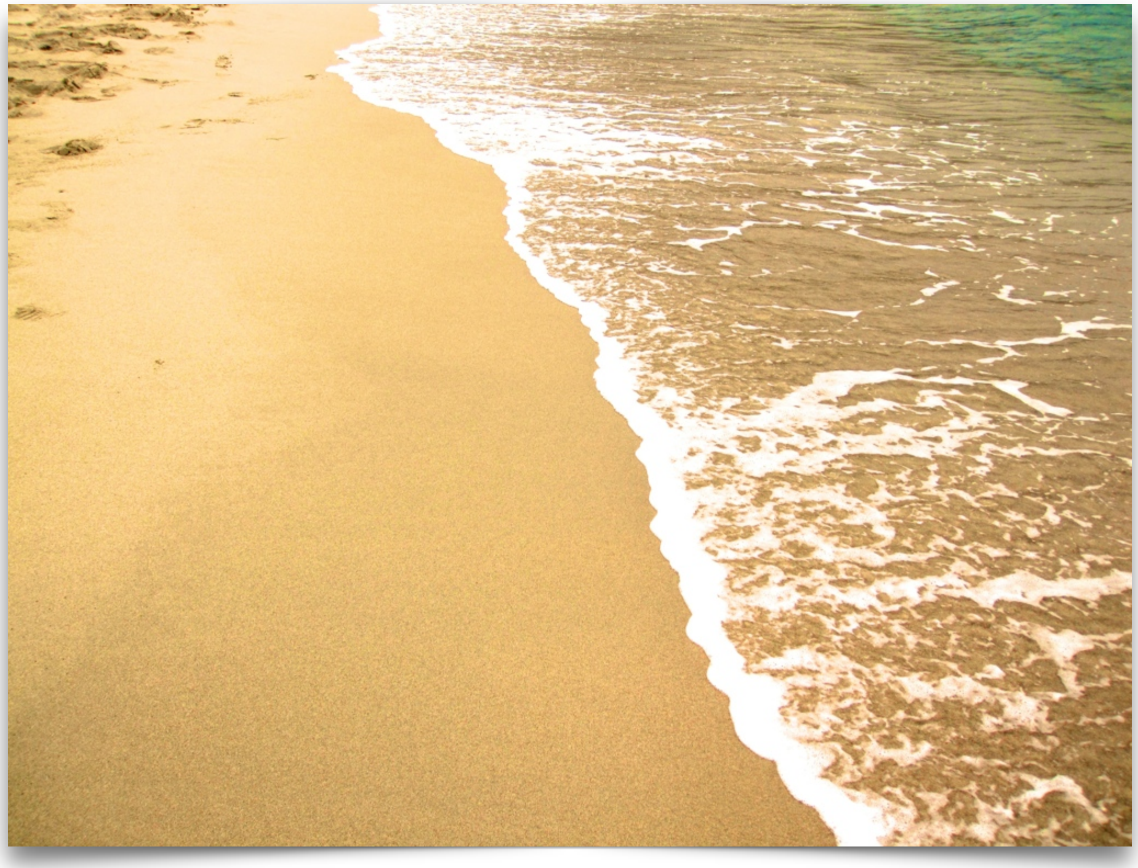
Doctoral Training School *Kardar-Parisi-Zhang equation: new trends in theories and experiments*
April 15-26, 2024 — Ecole de Physique des Houches (France)

Interfaces in disordered systems and directed polymer

Elisabeth Agoritsas



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Interfaces in disordered systems and directed polymer


Disordered medium
Interface
Role of disorder
on the shape & dynamics of
the interface?
What can we infer on disorder
from the interface shape?
Can we engineer given
geometrical/dynamical properties
by tuning disorder?

Interfaces can be found everywhere…

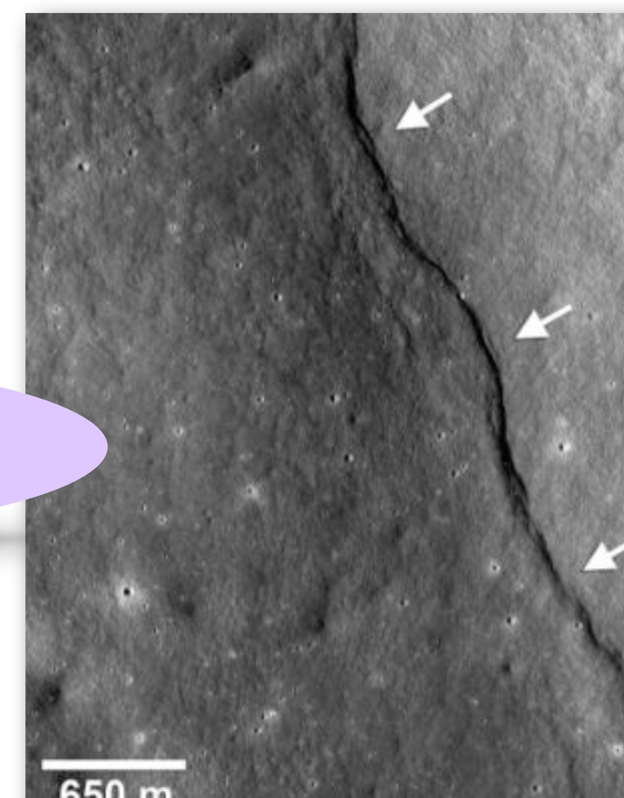
Lengthscale matters!

⬅ Scotland coastline

⬅ Crack in a pavement

Scale invariance?

Crack at the Moon's surface ➡

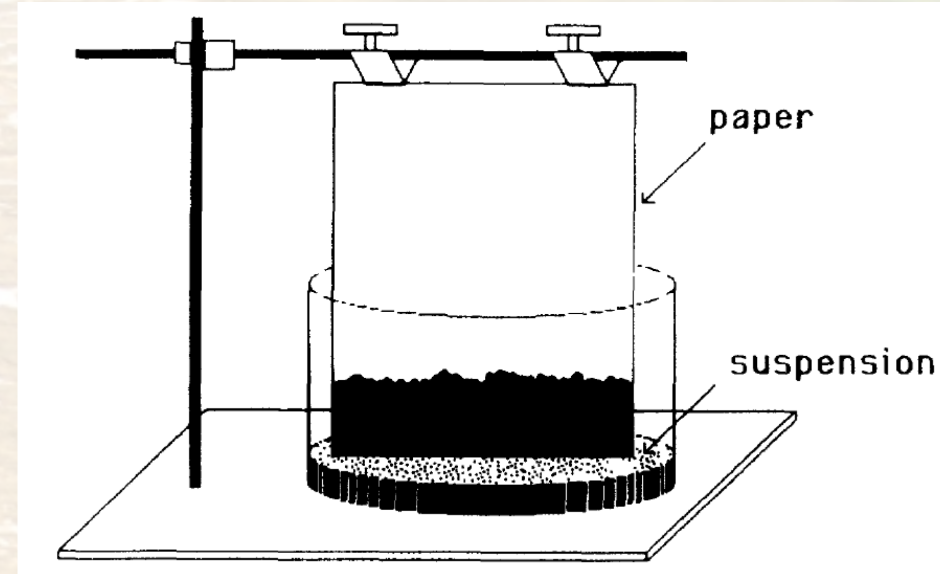


Experimental examples of 'disordered elastic systems'

- Contact line

Moulinet et al., *Eur. Phys. J. E* 8, 437 (2002).

- Imbibition front



Buldyrev et al., *Phys. Rev. A* 45, 8313 (1992).

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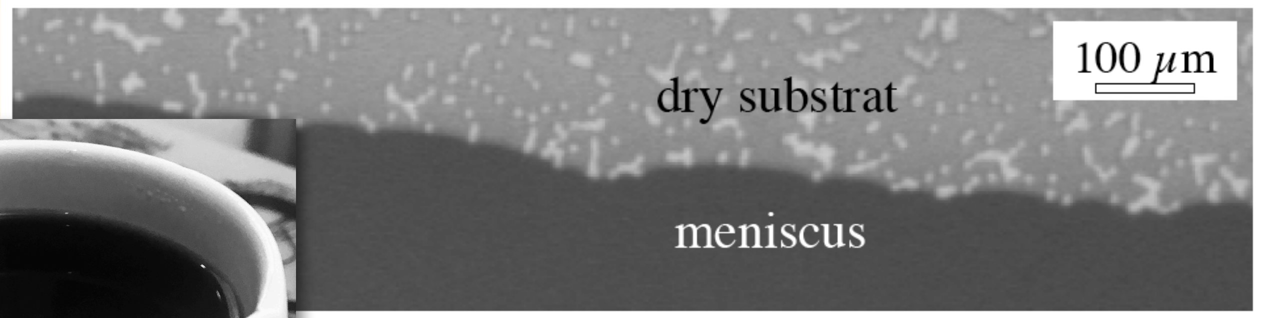

Moulinet et al., *Eur. Phys. J. E* 8, 437 (2002).

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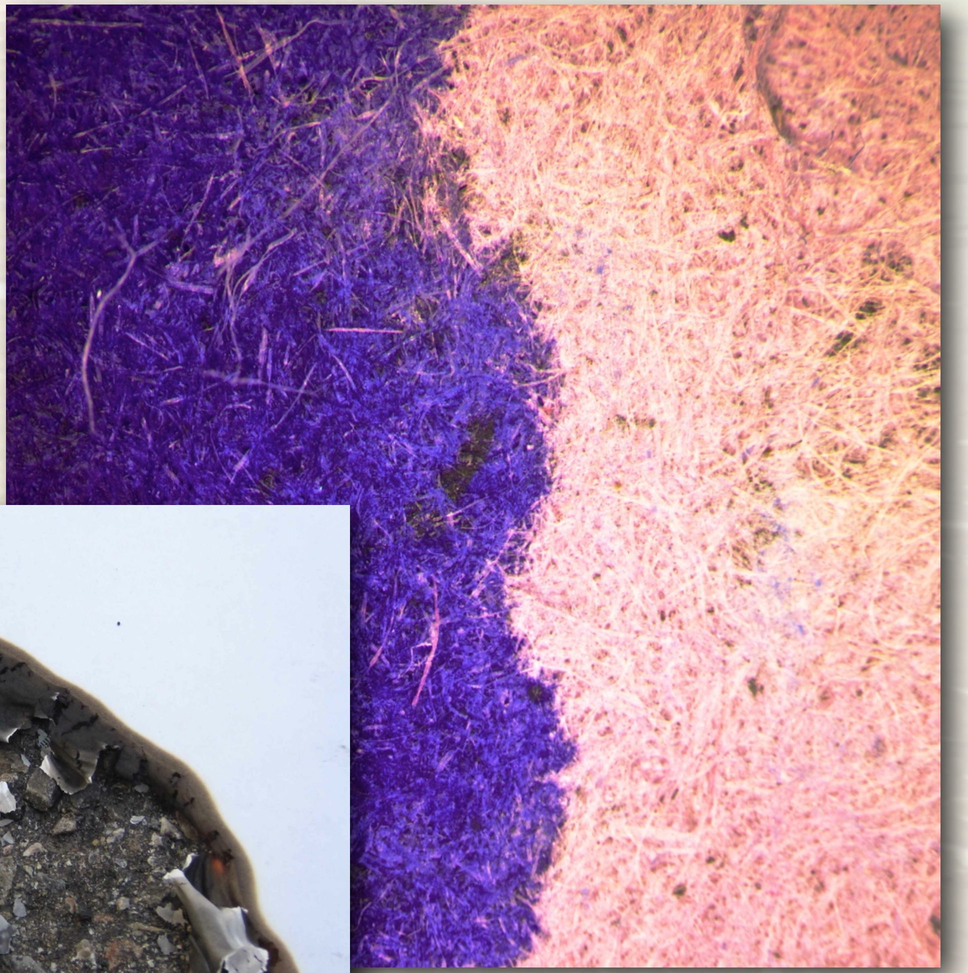

- Burning front

Experimental examples of 'disordered elastic systems'

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Moulinet et al., *Eur. Phys. J. E* 8, 437 (2002).

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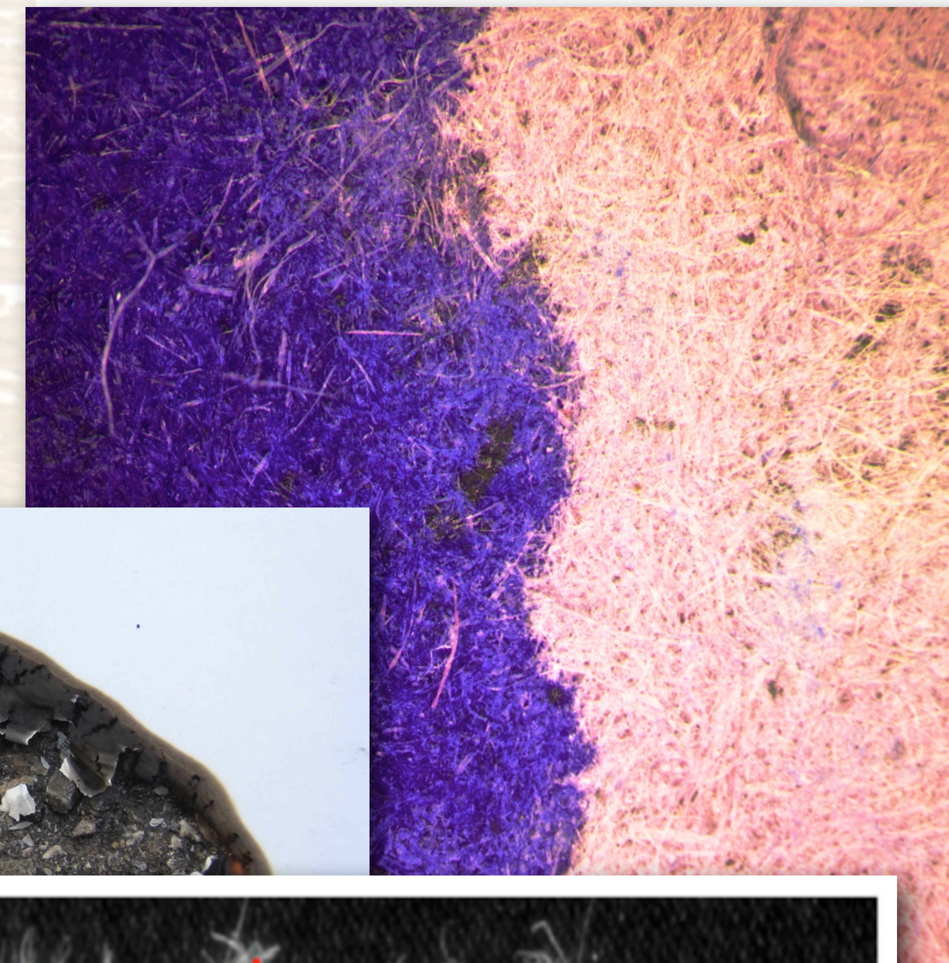
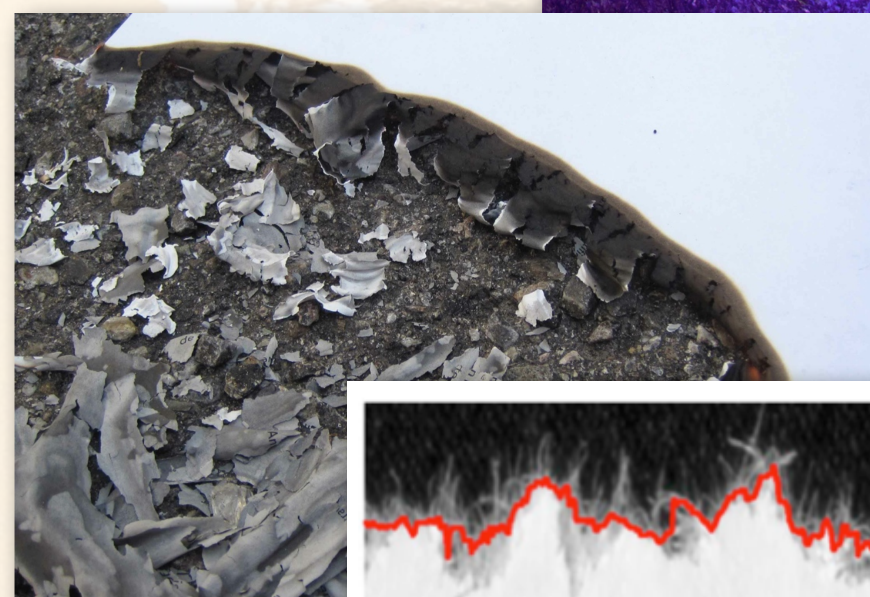
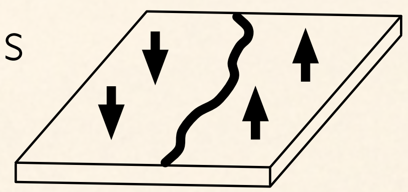


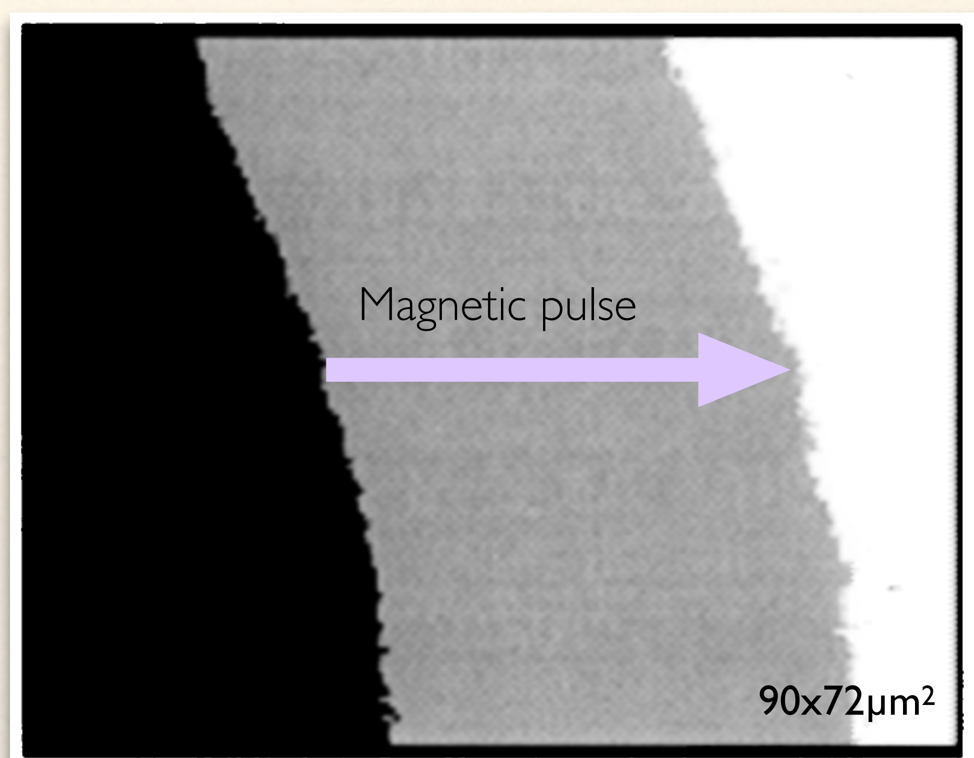
- Burning front



- Ferromagnetic domain walls

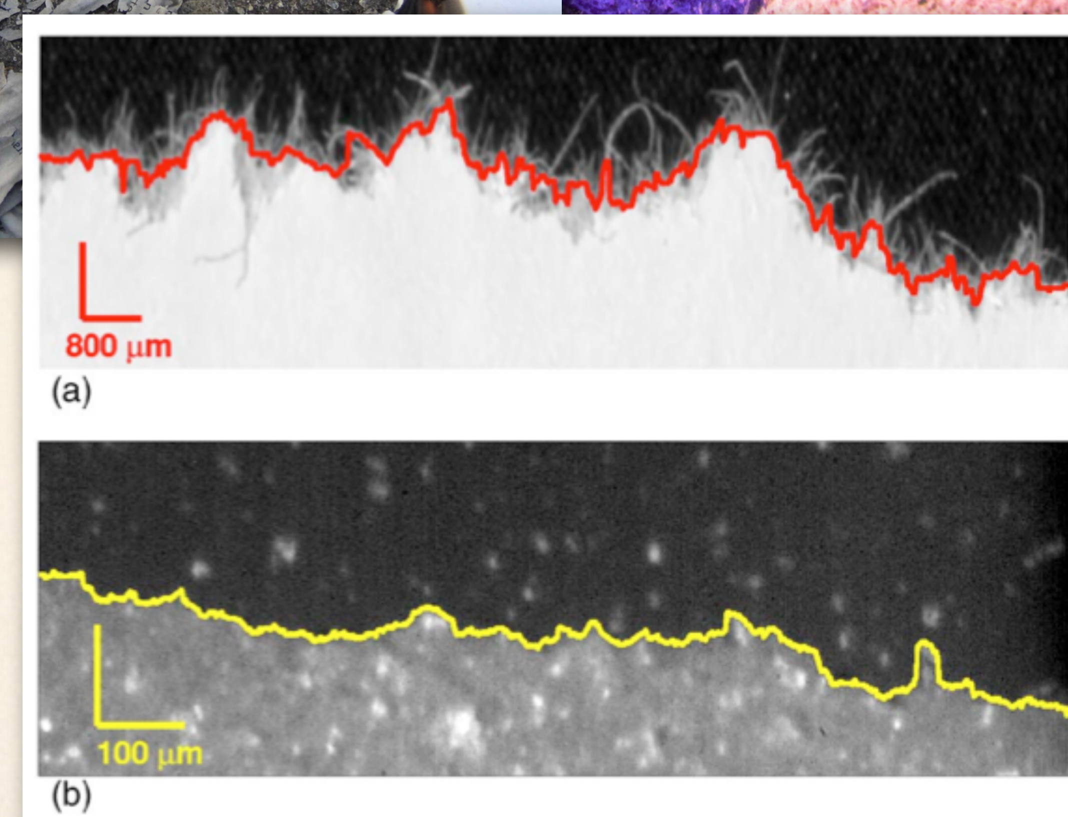


(Ultrathin film of Pt/Co/Pt)



Lemerle et al., *Phys. Rev. Lett.* 80, 894 (1998).

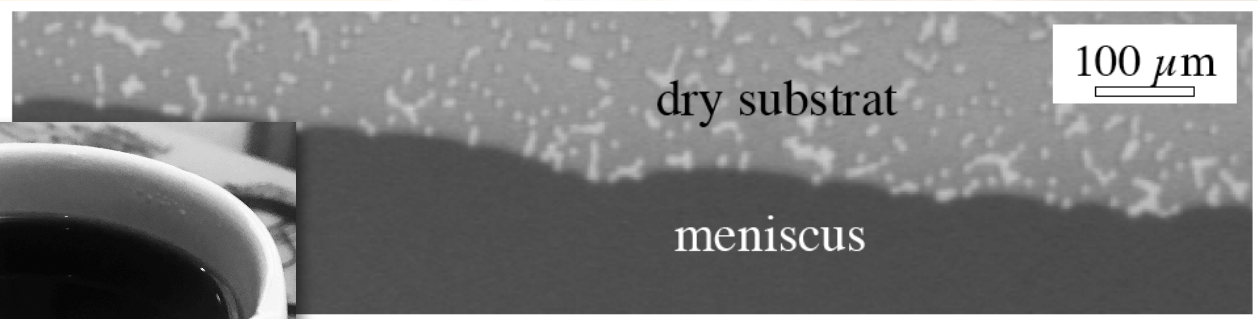
- Crack fronts



Santucci et al., *Phys. Rev. E* 75, 016104 (2007).

Experimental examples of 'disordered elastic systems'

- Contact line



Moulinet et al., *Eur. Phys. J. E* 8, 437 (2002).

- Imbibition fronts in porous media
- Burning fronts in paper/forest fires
- Fracture/cracks fronts
- Vortices in high-Tc supraconductors
- Proliferating bacteria/cell fronts
- etc.

- Ferromagnetic domain walls

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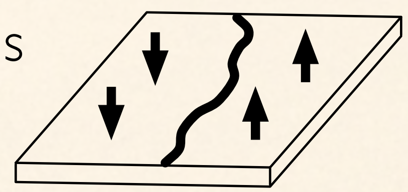
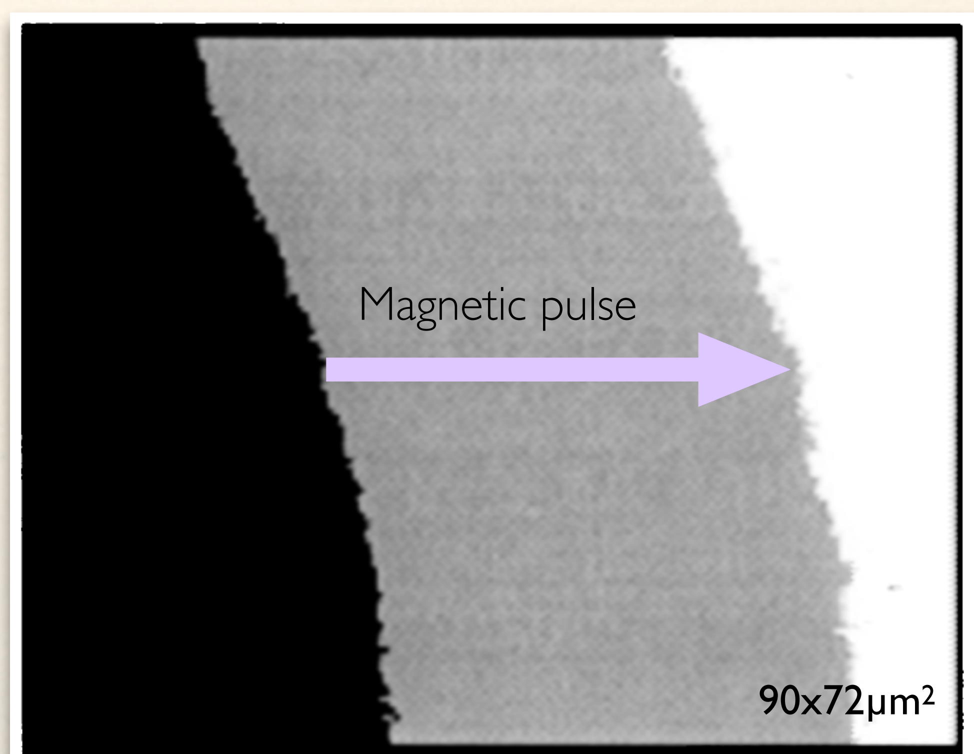
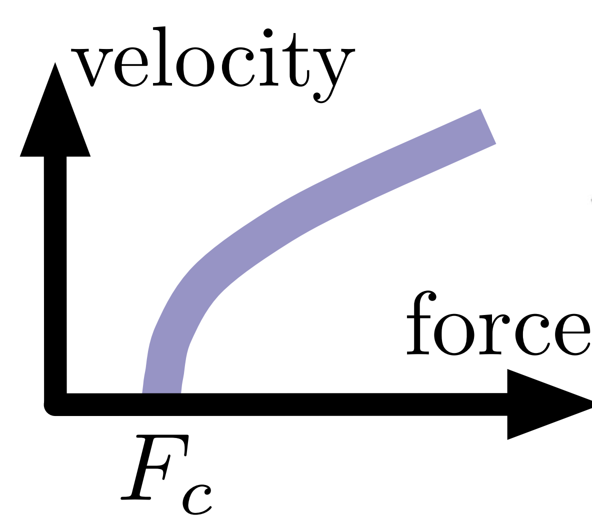
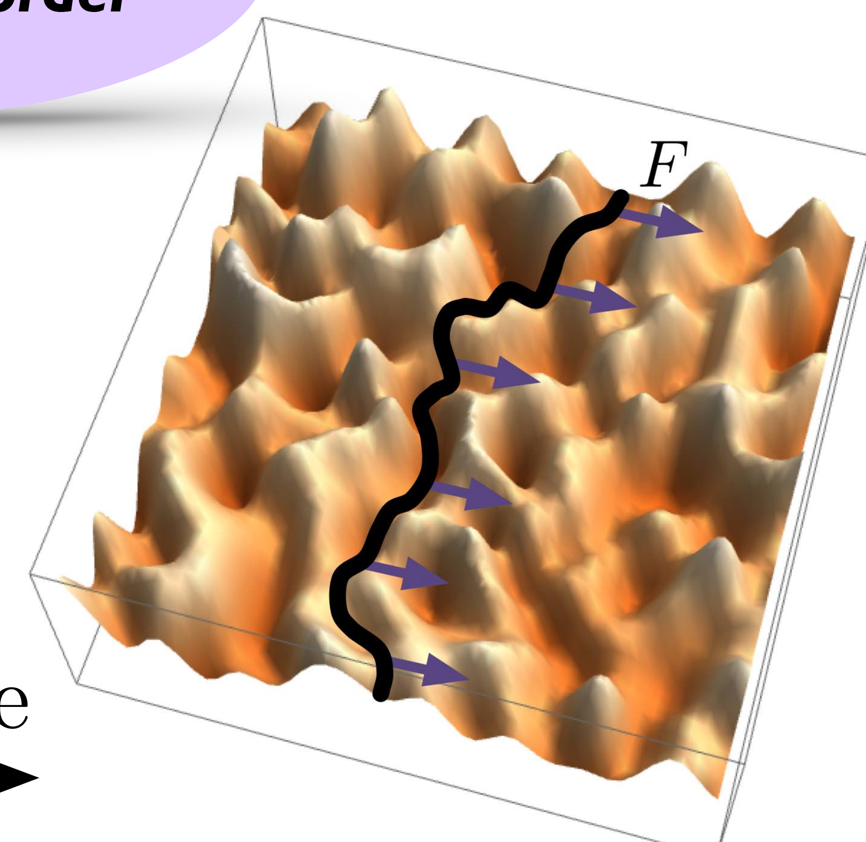




Lemerle et al., *Phys. Rev. Lett.* 80, 894 (1998).

Two competing ingredients:
elasticity & disorder





Mini-review: E. Agoritsas, V. Lecomte, T. Giamarchi, *Physica B* 407, 1725 (2012).

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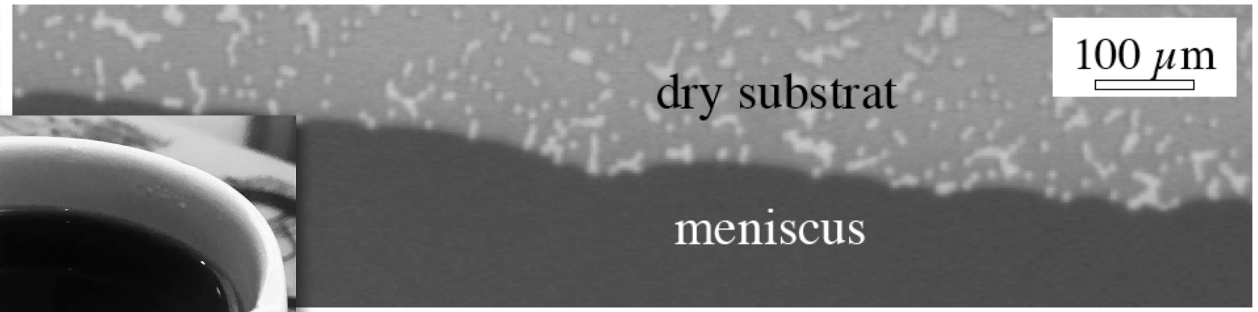

Moulinet et al., *Eur. Phys. J. E* 8, 437 (2002).

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- Burning fronts in paper/forest fires
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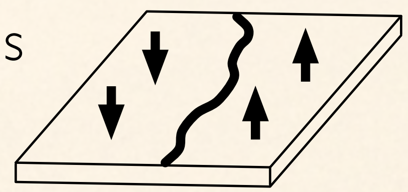
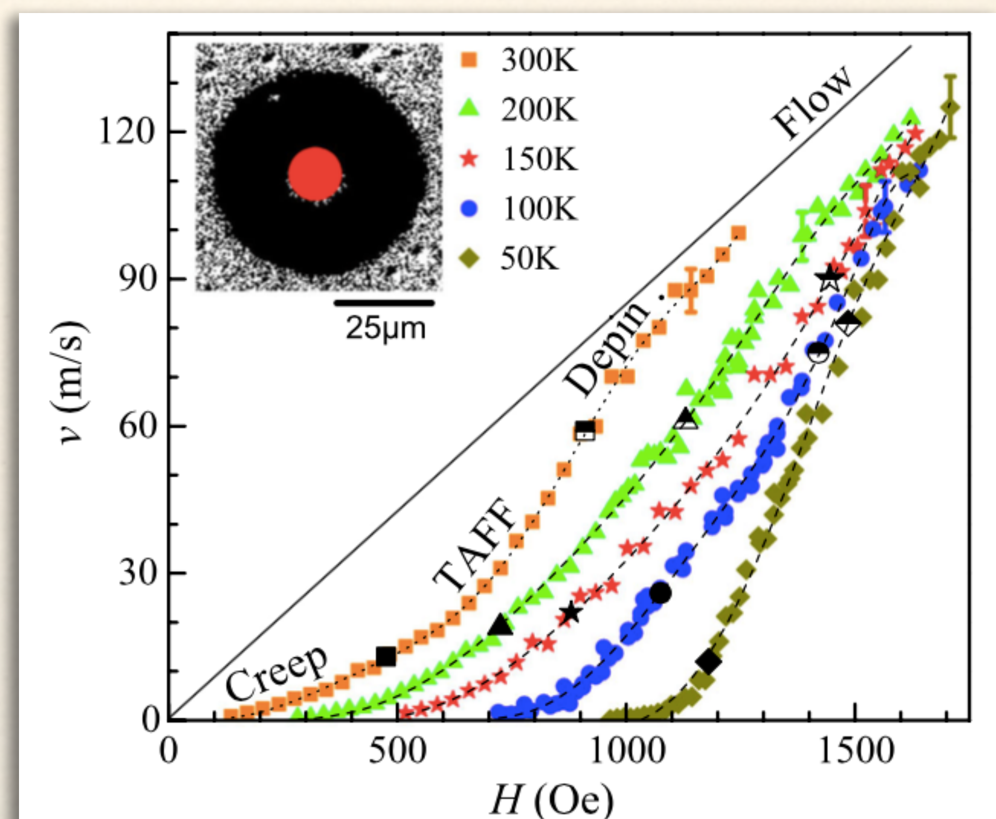
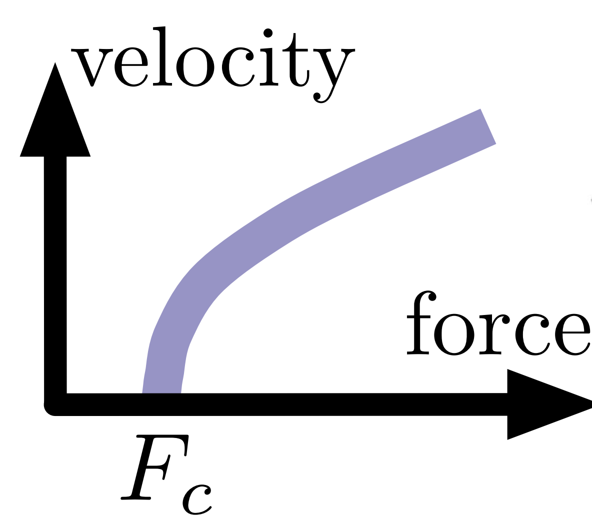
(Ultrathin film of Pt/Co/Pt)

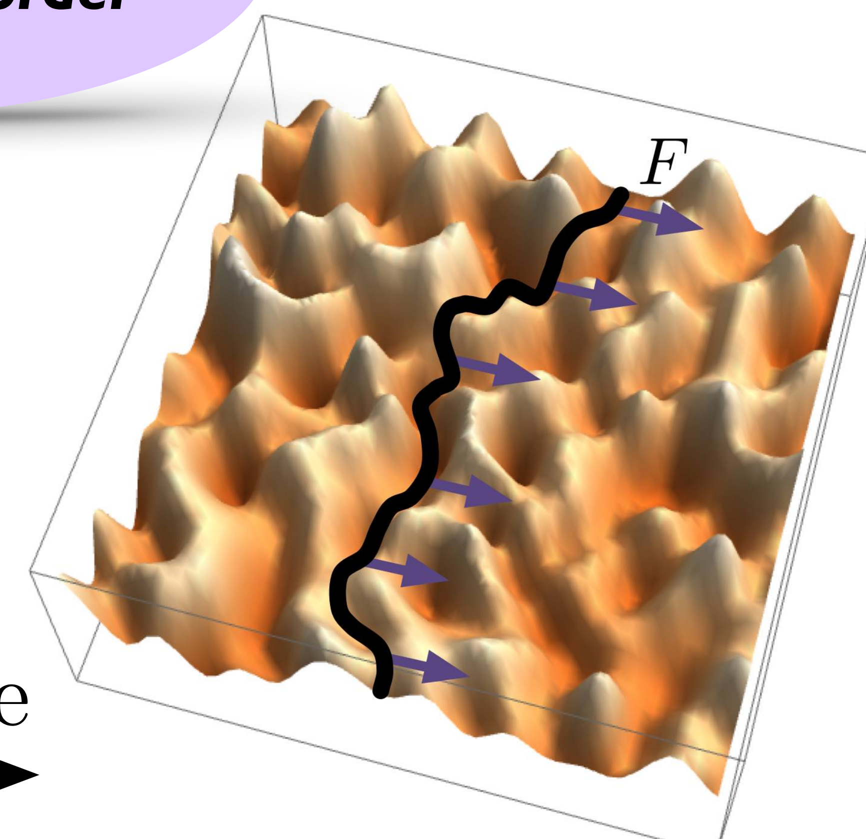




J. Gorchon *et al.*, *Phys. Rev. Lett.* 113, 027205 (2014).

Two competing ingredients:
elasticity & disorder





Mini-review: E. Agoritsas, V. Lecomte, T. Giamarchi, *Physica B* 407, 1725 (2012).

Statistical-physics approach: effective mesoscopic description

- Ubiquitous in nature, large variety of lengthscales & microphysics.

BUT do they share nevertheless common (universal?) features?

Review: A.-L. Barabàsi & H. E. Stanley, *Fractal Concepts in Surface Growth*, Cambridge University Press, 1995.

- Increasing complexity starting from a microscopic 'bulk' description.

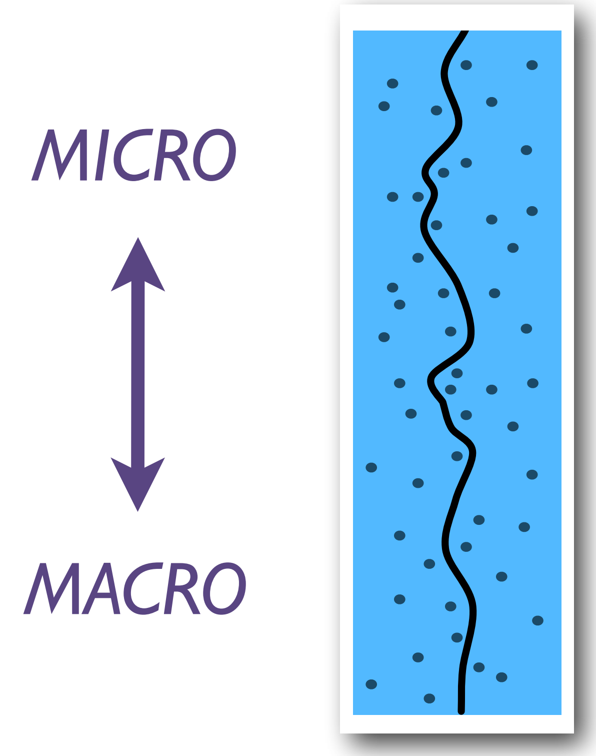
⇒ Need of a mesoscopic effective starting point

- Emergent structures, supported by a disordered underlying medium.

⇒ Only statistical characterization of disorder

- Effective description depending on the lengthscale considered.

⇒ Expectation of some kind of scale invariance



How do they look like?

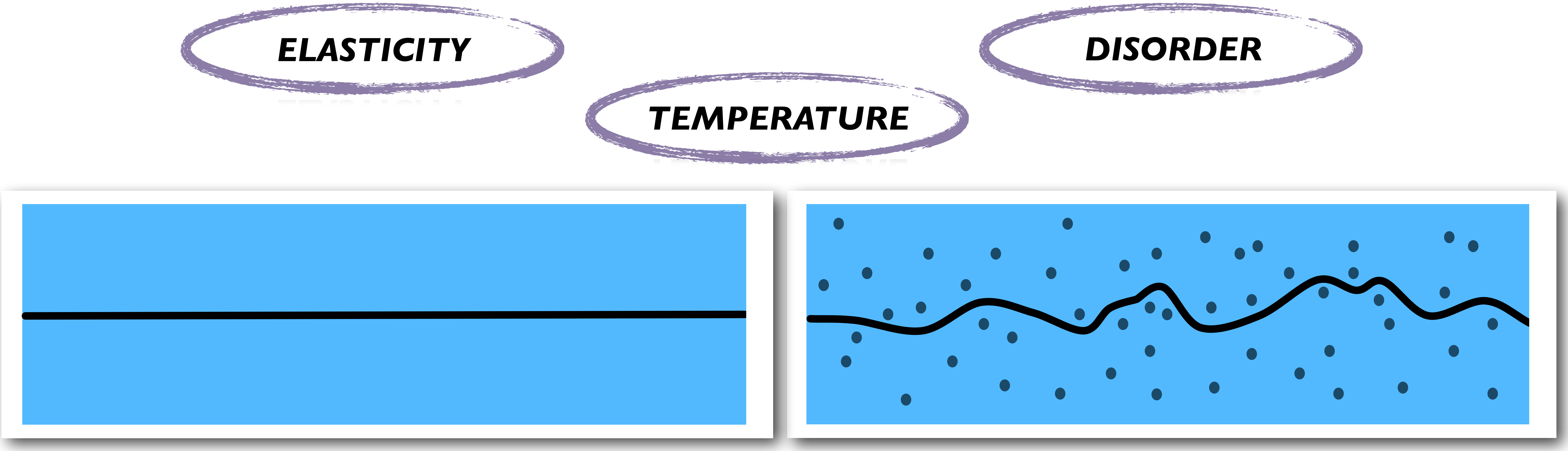
How do they respond when one pulls at them?

How does disorder modify a pure system?

Probe of disorder-conditioned features in statics/dynamics

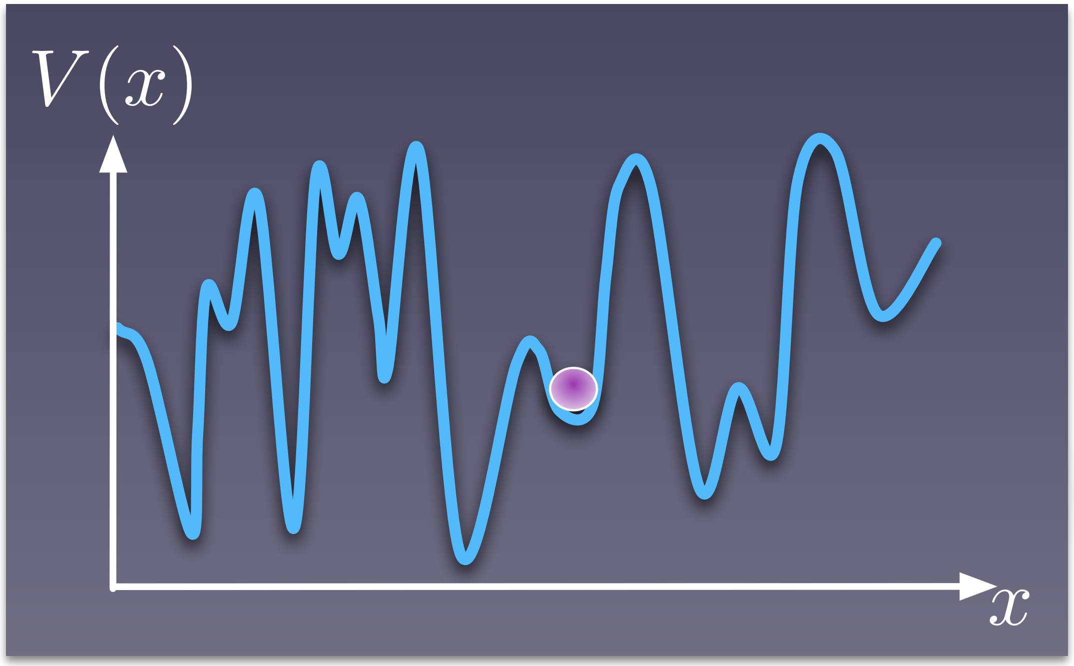
Disordered Elastic Systems (DES) — Generic theoretical framework

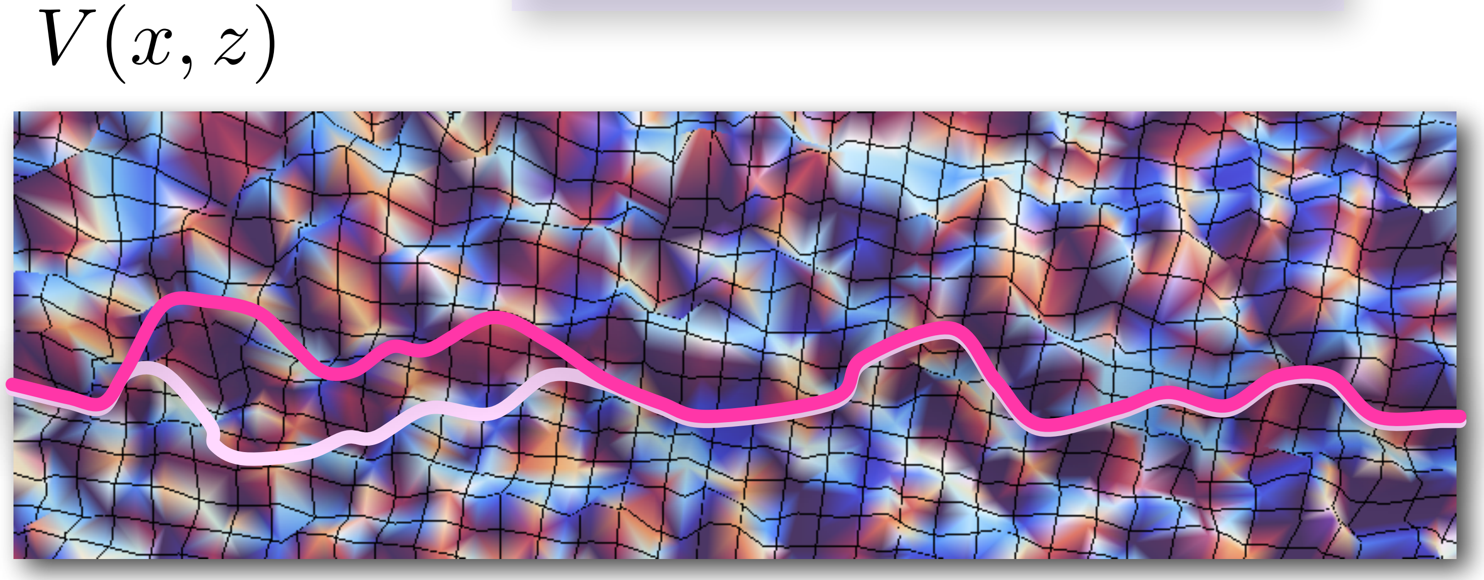
- Competition of three physical ingredients:



- Exploration of disordered energy landscapes

Simplest extended objects in a quenched disordered landscape





Mini-review on DES: E. Agoritsas, V. Lecomte, & T. Giamarchi, *Physica B* 407, 1725 (2012).

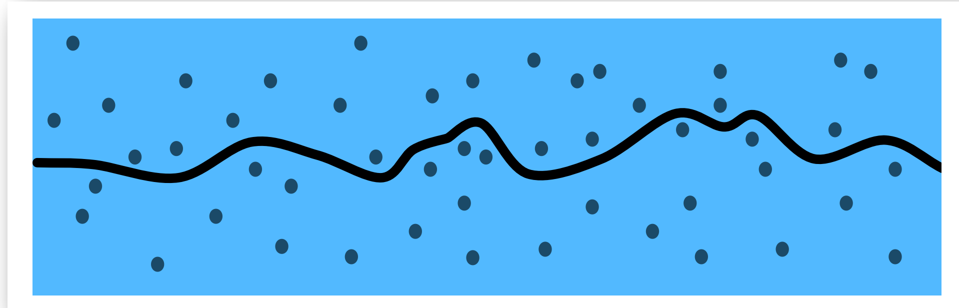
Disordered Elastic Systems (DES) — Physical ingredients

- **Dimensionality** Interfaces or periodic systems

Internal dimension: d

Transverse dimension: m

Physical space dim. $D = d + m$

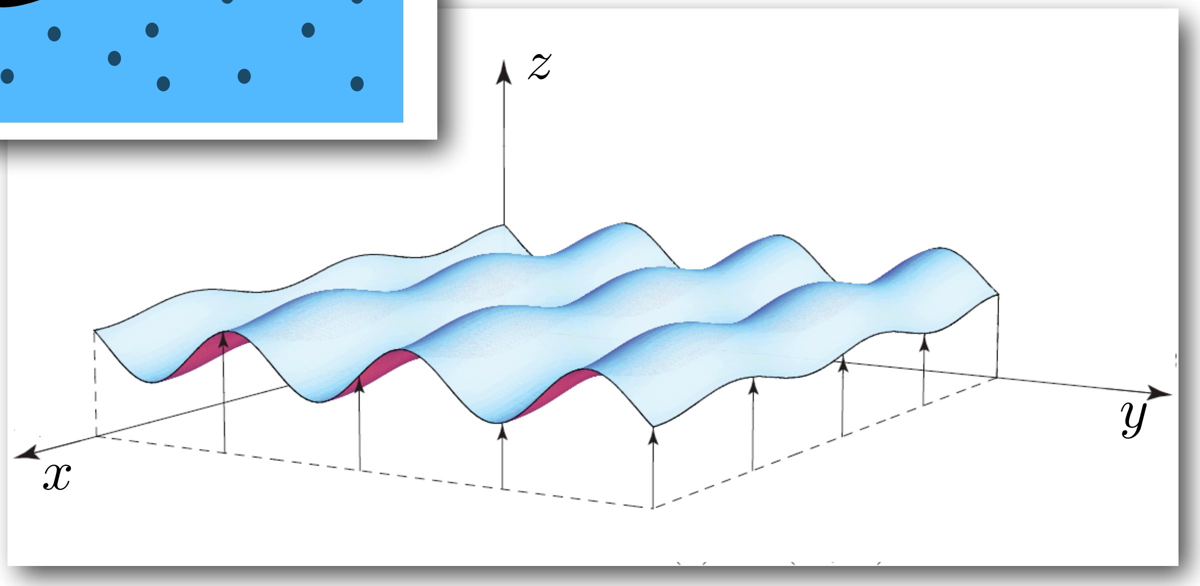


$d = m = 1$

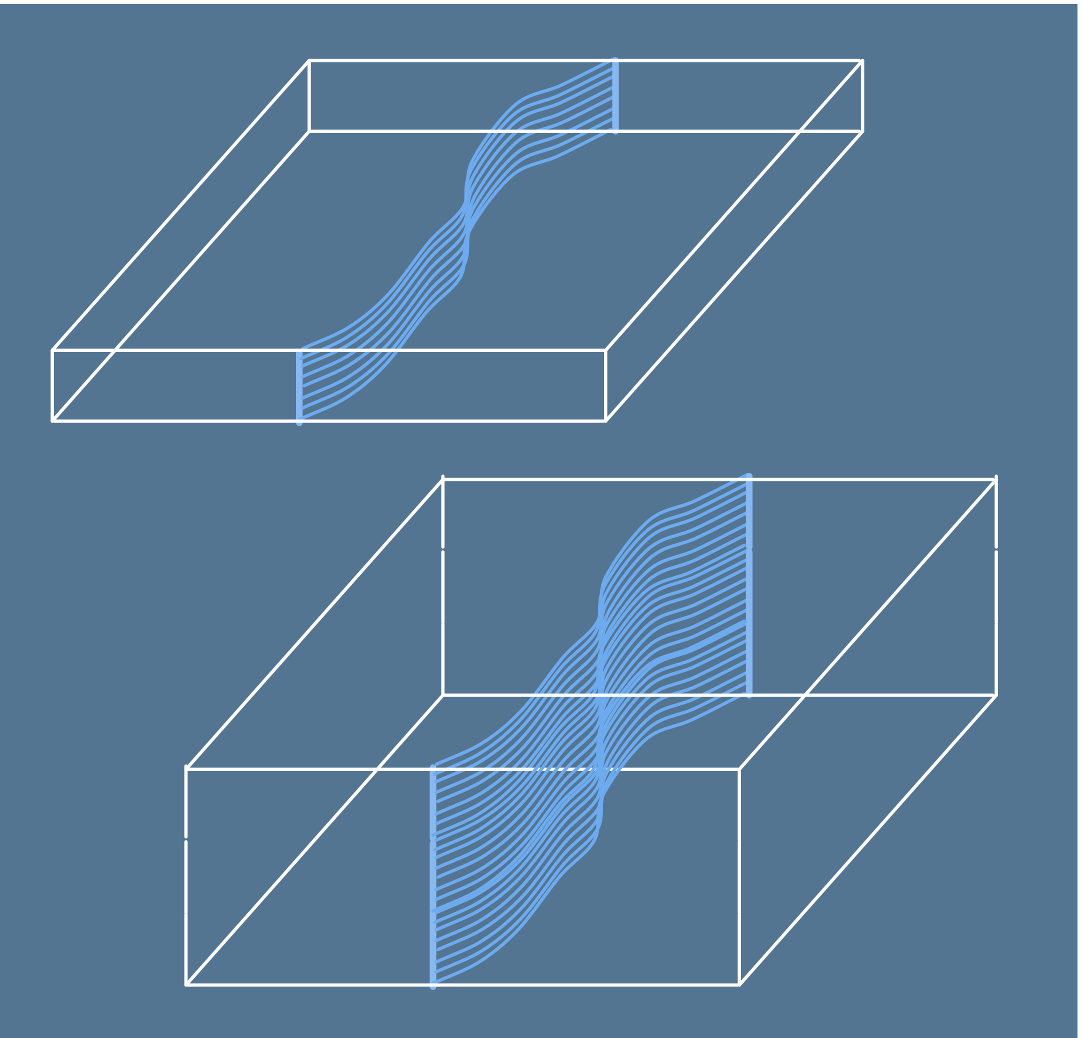
← 1D interface

2D interface ➡

$d = 2, m = 1$

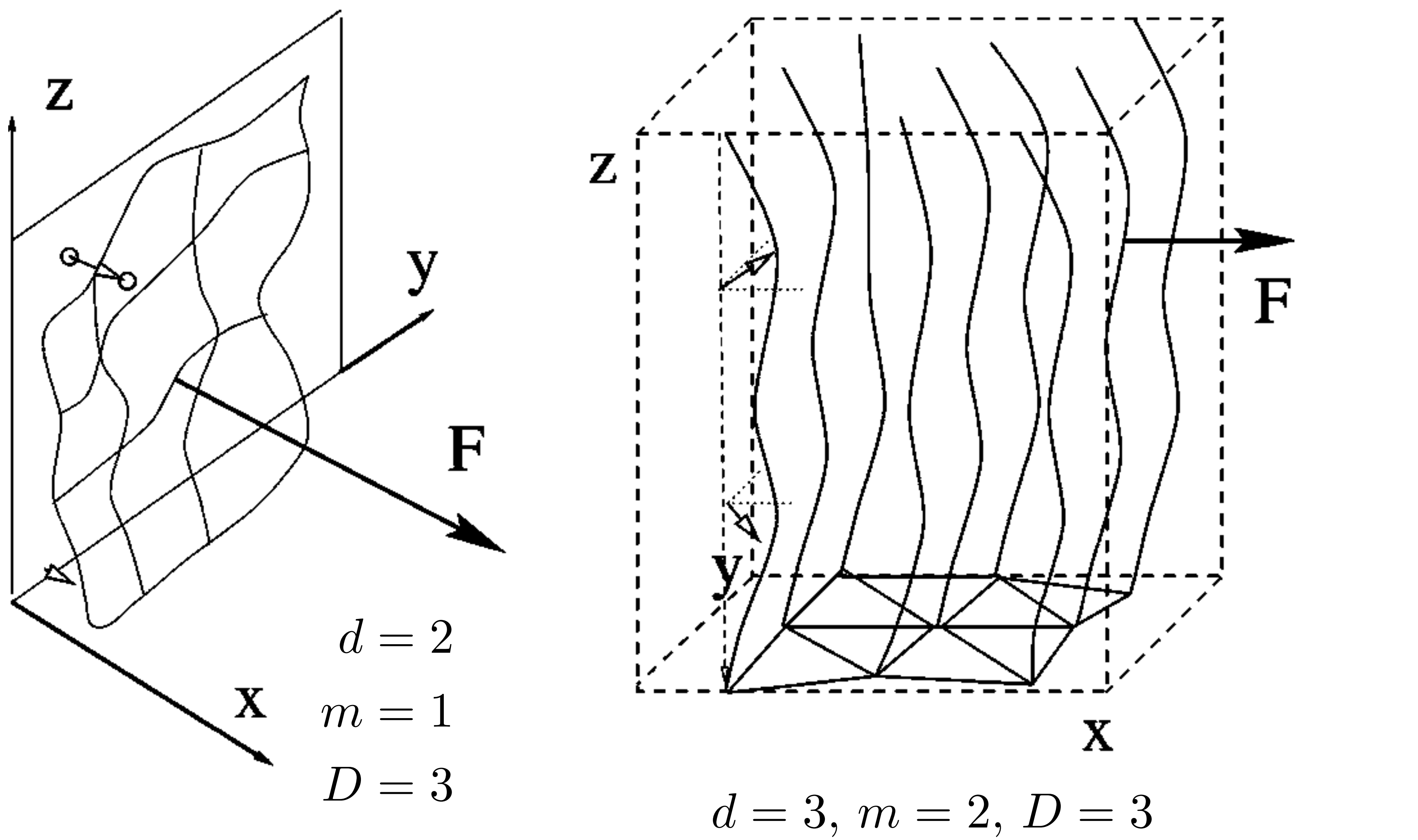


Dimensional crossover?



Effective 1D interface? NO!

- **Dimensionality** Interfaces or periodic systems

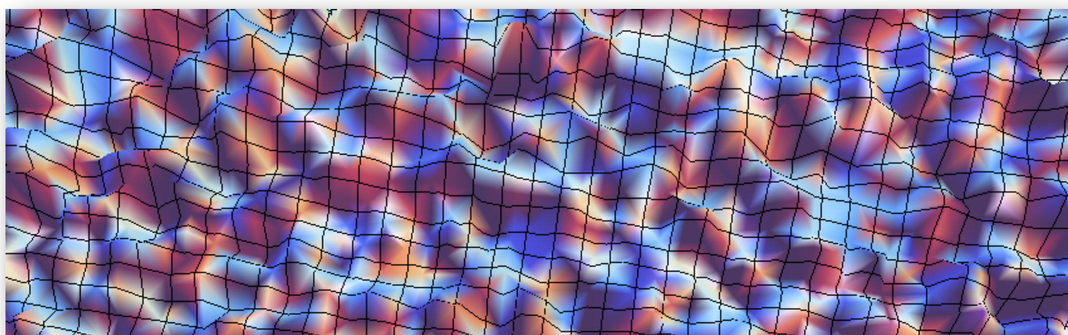


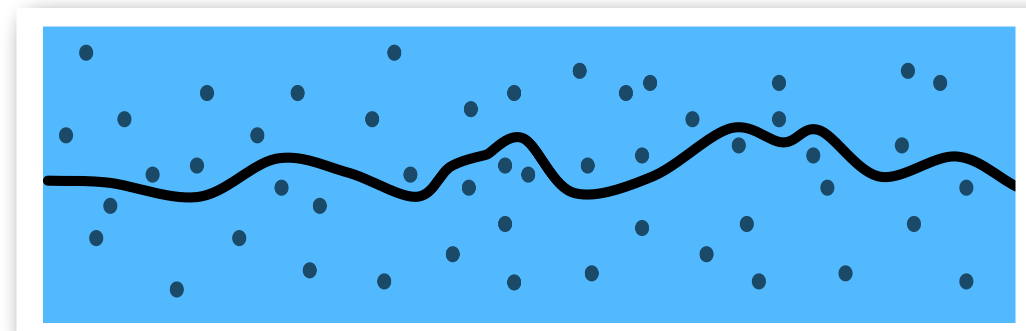
P. Le Doussal and T. Giamarchi, , "Moving glass theory of driven lattices with disorder", *Phys.Rev. B* 57, 11356 (1998).

Disordered Elastic Systems (DES) — Physical ingredients

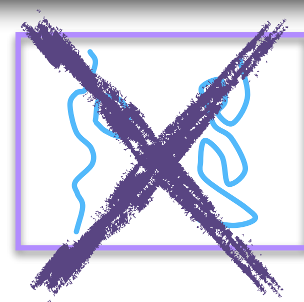
- **Dimensionality** Interfaces or periodic systems
- **Elasticity**: Long-range versus short-range, e.g. $\mathcal{H}_{\mathrm{el}} \propto$ system size
- **Disorder**:
  - Quenched *versus* annealed disorder
  - 'Random-bond' *versus* 'random field'
  - Collective weak pinning *versus* strong individual pinning centers

$$\mathcal{H}_{\mathrm{DES}} = \mathcal{H}_{\mathrm{el}} + \mathcal{H}_{\mathrm{dis}}$$

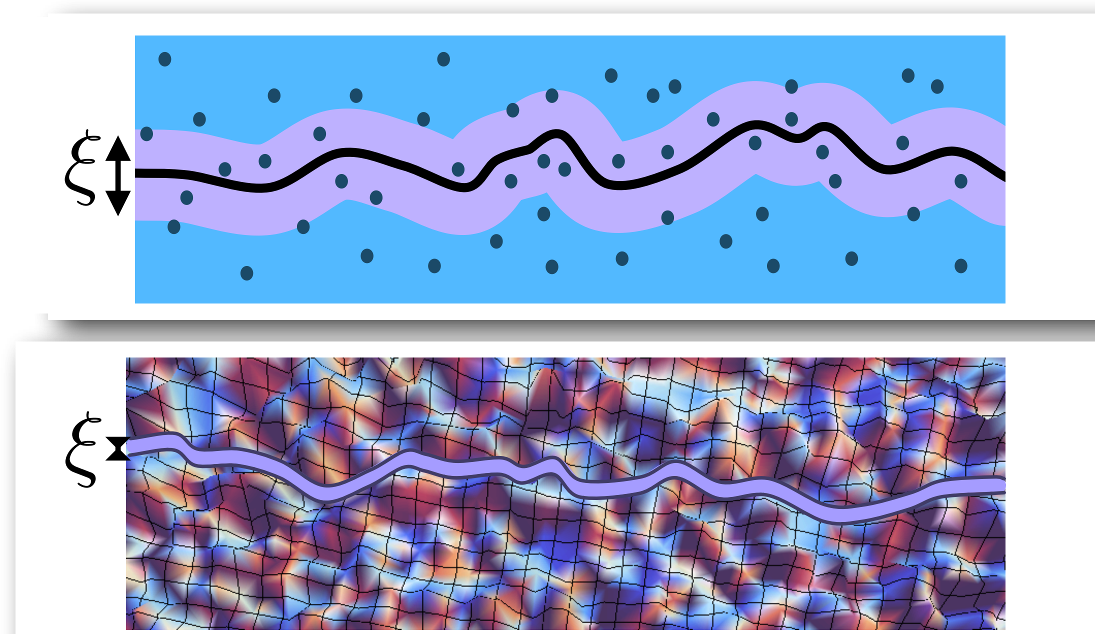




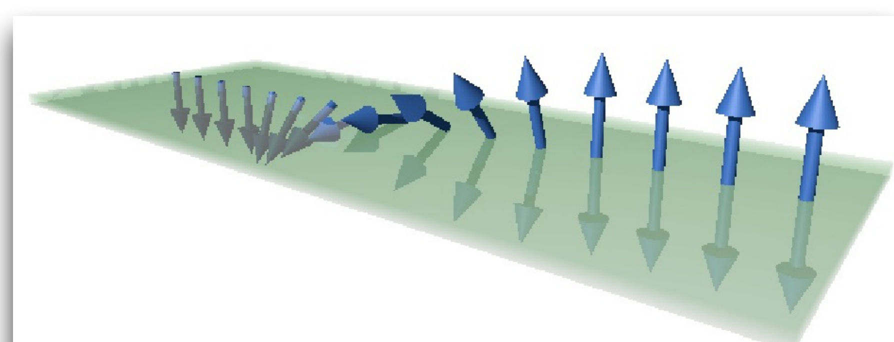
- **No bubbles nor overhangs**



- **Finite width / Disorder correlation**



- **Internal degree of freedom?**

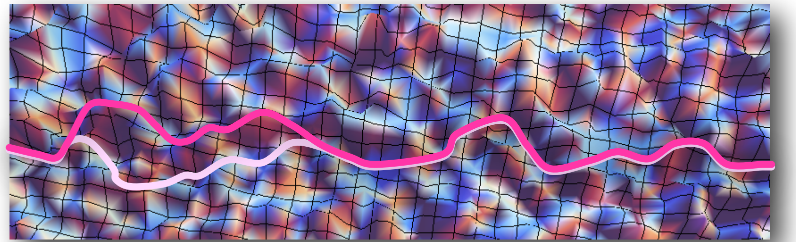


- **Equilibrium / Out-of-equilibrium?**

<u>Mini-review on DES</u>: E. Agoritsas, V. Lecomte, & T. Giamarchi, *Physica B* <u>407</u>, 1725 (2012).

Disordered Elastic Systems (DES) — Observables for probing disorder

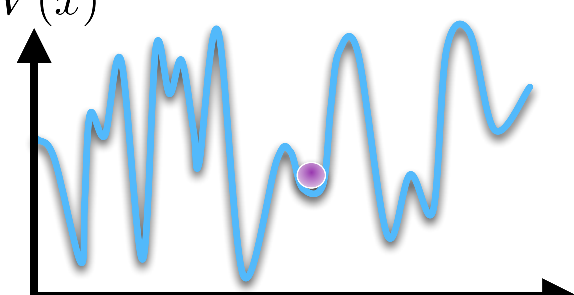
$V(x,z)$



How do they look like?
How do they respond when one pulls at them?
How does disorder modify a pure system?

DES recipe
- Dimensionality
- Elasticity
- Disorder
- Temperature

- No bubbles/overhangs
- Internal structure (width, internal DoF, ...)
- At (non-)equilibrium / external drive



Probe of disorder-conditioned features
in STATICS/DYNAMICS

Geometrical fluctuations
& roughness
as function of the lengthscale

Center-of-mass dynamics:
Steady-state velocity
under an external force

Mini-review on DES: E. Agoritsas, V. Lecomte, & T. Giamarchi, *Physica B* 407, 1725 (2012).

Disordered Elastic Systems (DES) — Selective bibliography

About ~50 years of literature in theory/experiments/numerics! ⇒ See reviews with refs therein

- T. Giamarchi, Encyclopedia of Complexity and Systems Science (2009), "Disordered Elastic Media" — Introduction to DES
- E. Agoritsas, V. Lecomte, T. Giamarchi, *Physica B* 407, 1725 (2012), "Disordered elastic systems and one-dimensional interfaces" — Statics
- T. Giamarchi, A. B. Kolton, A. Rosso, *Lecture Notes in Physics* 688, 91 (2006) [arXiv:0503437], "Dynamics of disordered elastic systems" — Dynamics
- K. J. Wiese, *Reports on Progress in Physics* 85, 086502 (2022), "Theory and experiments for disordered elastic manifolds, depinning, avalanches, and sandpiles" — Very recent extended review
- D. S. Fisher, *Physics Reports* 301, 113 (1998), "Collective transport in random media: from superconductors to earthquakes" — Lectures (1998)
- G. Blatter, M.V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, V. M. Vinokur, *Reviews of Modern Physics* 66, 1125 (1994), "Vortices in high-temperature superconductors" — "Canonical review"

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Interfaces in disordered systems and directed polymer

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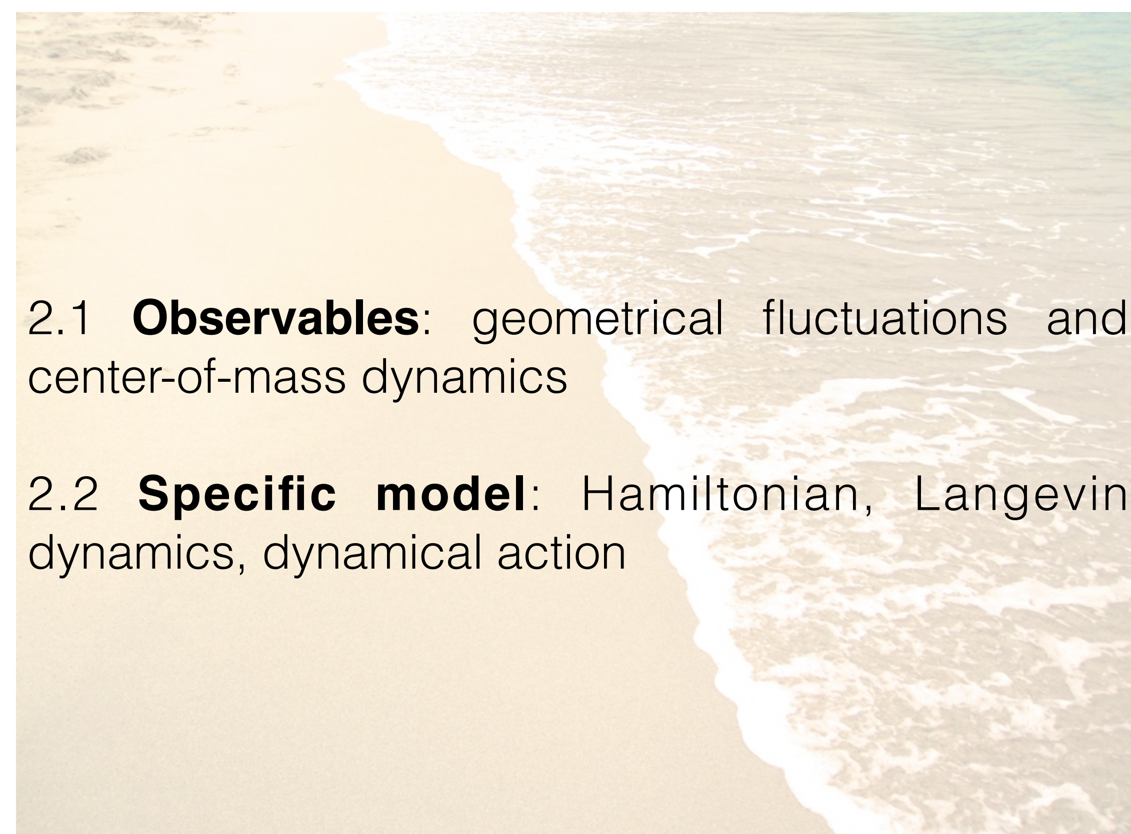


2.1 **Observables**: geometrical fluctuations and center-of-mass dynamics

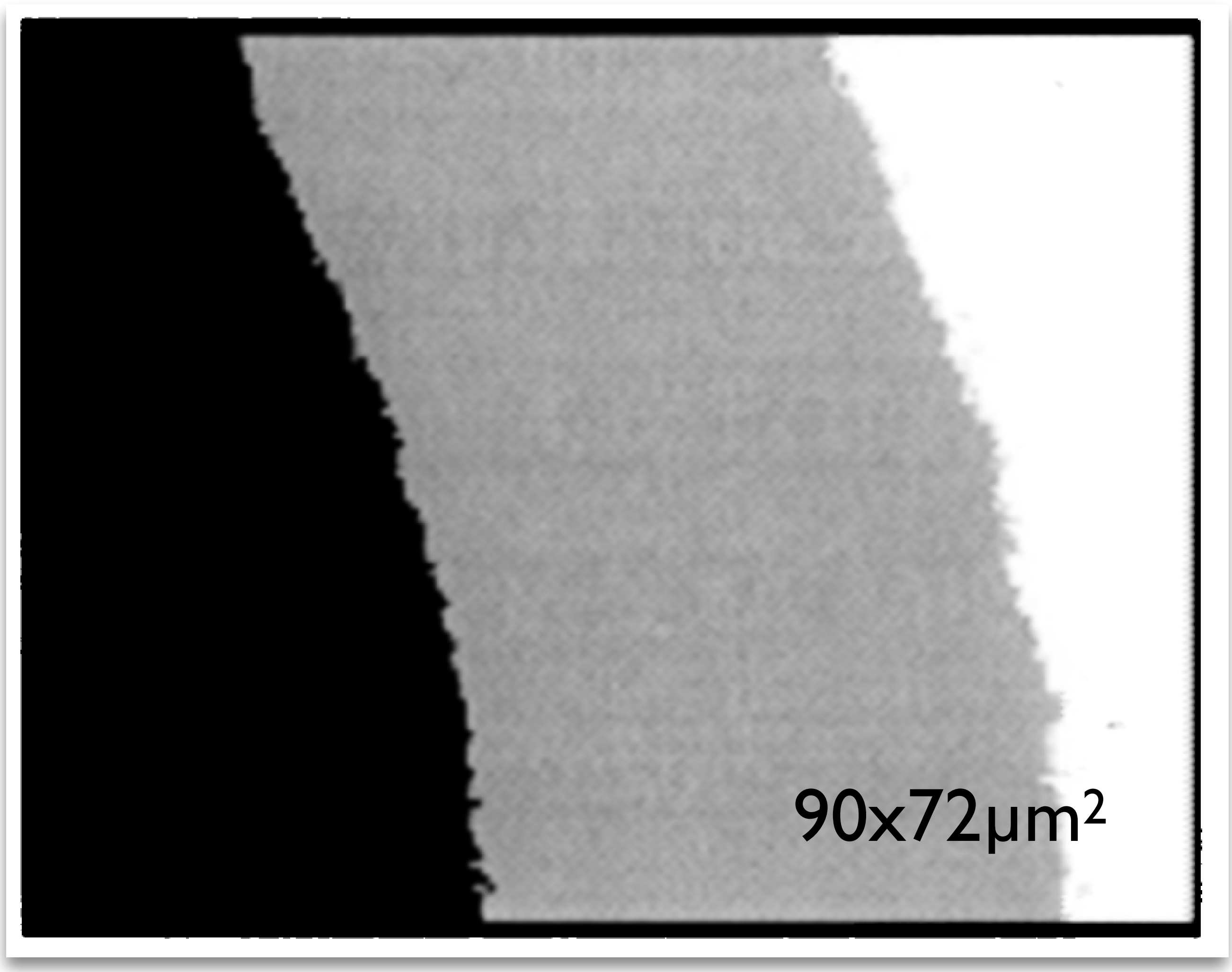
2.2 **Specific model**: Hamiltonian, Langevin dynamics, dynamical action

Observable 1: Geometrical fluctuations & roughness

Ferromagnetic domain wall $(\xi \sim 50\text{nm})$

RESOLUTION: 1µm

Ultrathin film of Pt/Co/Pt (a few atomic layers)

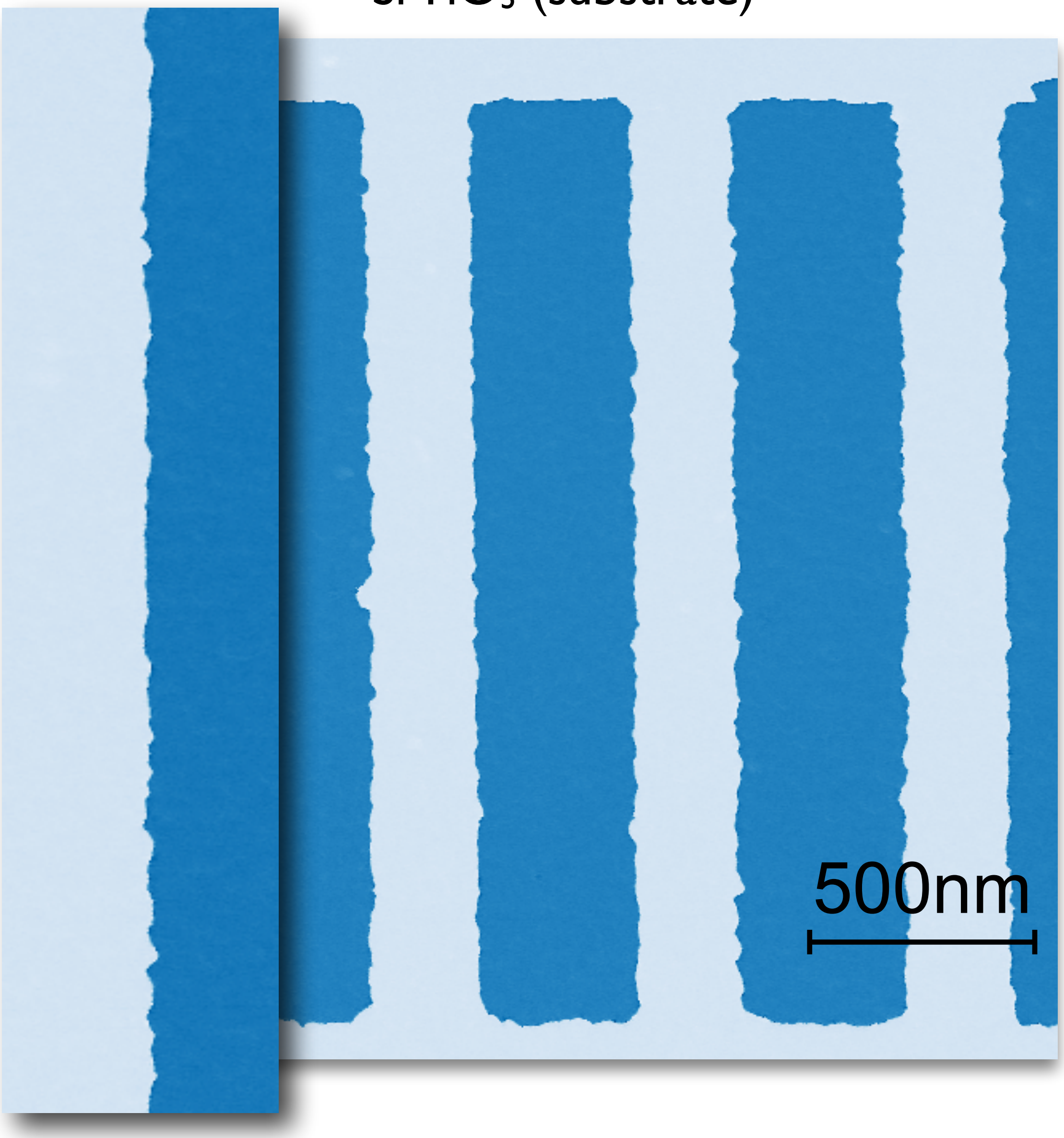


S. Lemerle, J. Ferré, C. Chappert, V. Mathet, T. Giamarchi, & P. Le Doussal, *Phys. Rev. Lett.* 80, 849 (1998).

Ferroelectric domain wall $(\xi \sim 1\text{nm})$

RESOLUTION: 5nm

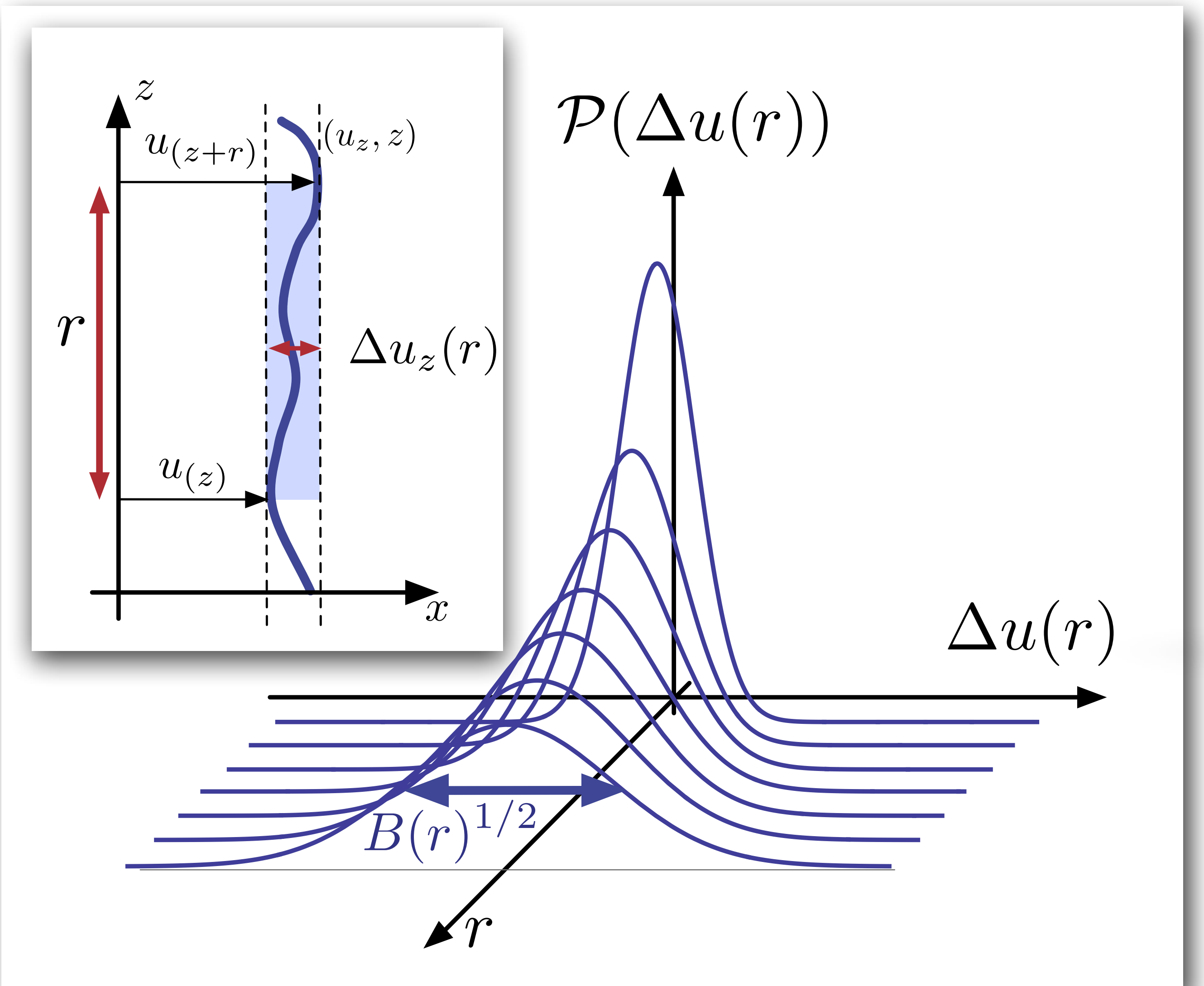
$PbZr_{0.2}Ti_{0.8}O_3$ 70nm / $SrRuO_3$ 30nm (electrode) / $SrTiO_3$ (substrate)



J. Guyonnet, E. Agoritsas, S. Bustingorry, T. Giamarchi, & P. Paruch, *Phys. Rev. Lett.* 109, 147601 (2012).

Observable 1: Geometrical fluctuations & roughness

- Statistical analysis of a snapshot of an interface configuration ⇒ access to the roughness



- Lengthscale r - Relative displacement $\Delta u(r)$
- Probability distribution function $\mathcal{P}(\Delta u(r))$
- Roughness function: $B(r) = \overline{\langle \Delta u(r)^2 \rangle} \sim A\, r^{2\zeta}$

Roughness exponent ζ:

Signature of the predominant physics
⇒ Universality classes?

Different roughness regimes?
Amplitude(s)/Crossover(s)

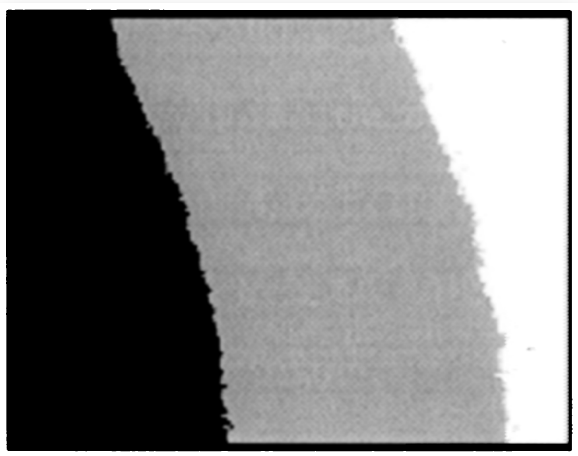
- Roughness exponent for 1D interface:
  (short-range elasticity & random-bond disorder)

$$\begin{cases} \zeta_{\text{thermal}} & = 1/2 \\ \zeta_{\text{KPZ}} & = 2/3 \end{cases}$$

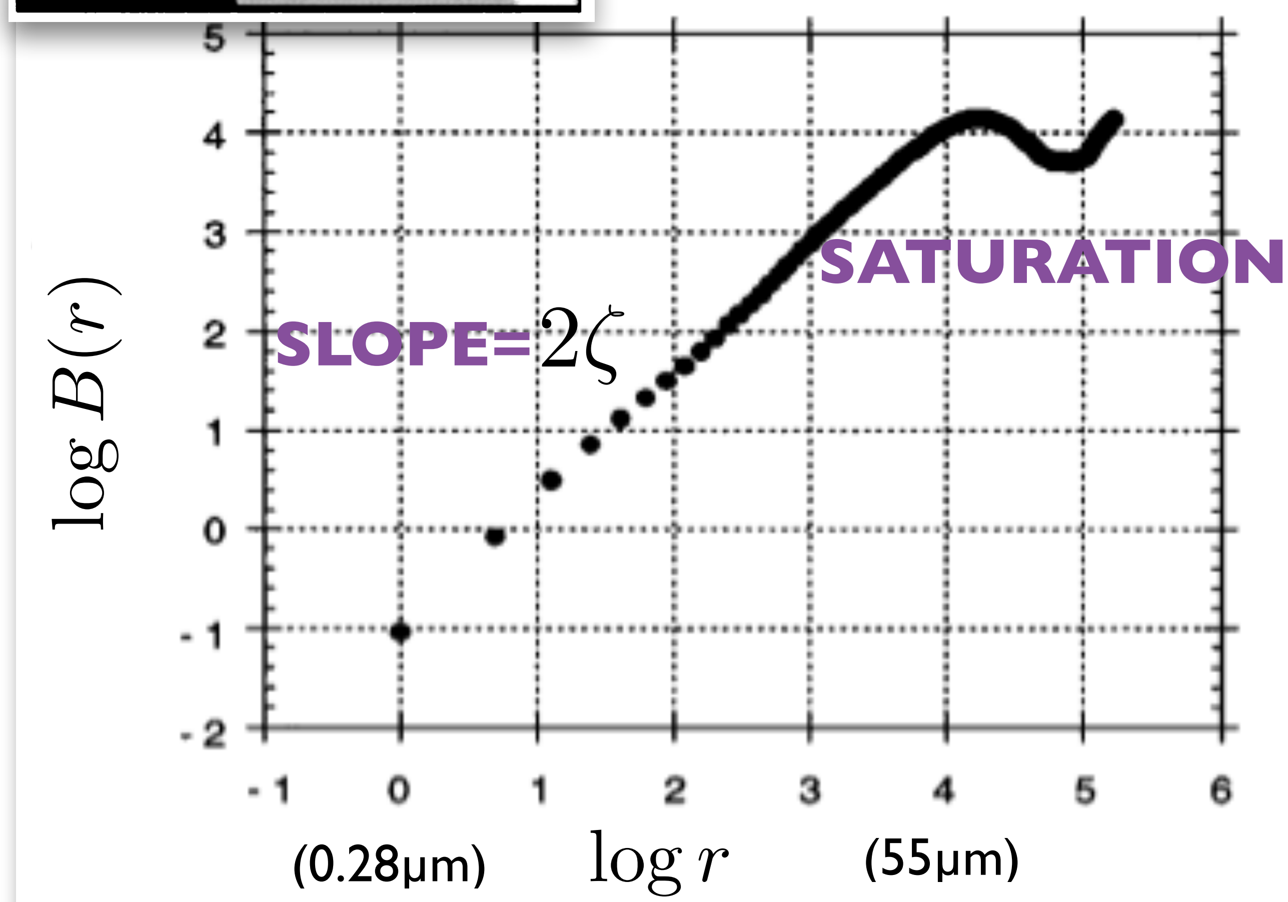
Mini-review on DES: E. Agoritsas, V. Lecomte, & T. Giamarchi, *Physica B* 407, 1725 (2012).

Observable 1: Geometrical fluctuations & roughness

Domain walls in ultrathin Pt/Co/Pt ferromagnetic films

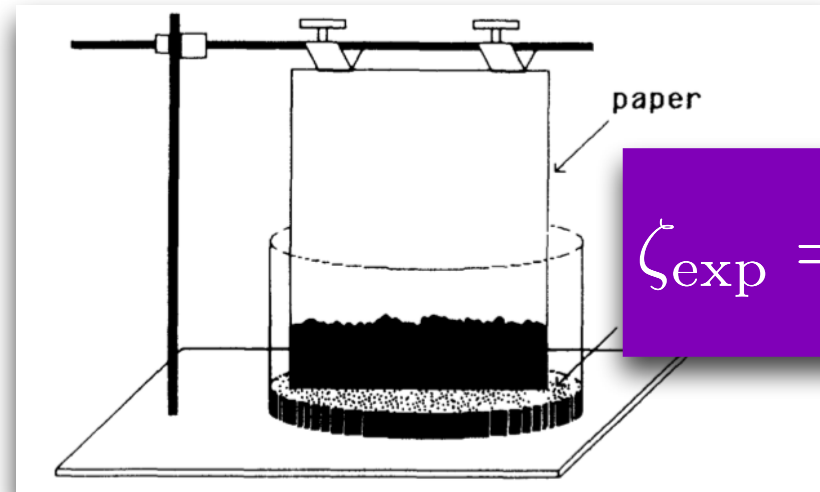


$$\zeta_{\rm exp} = 0.69 \pm 0.07$$

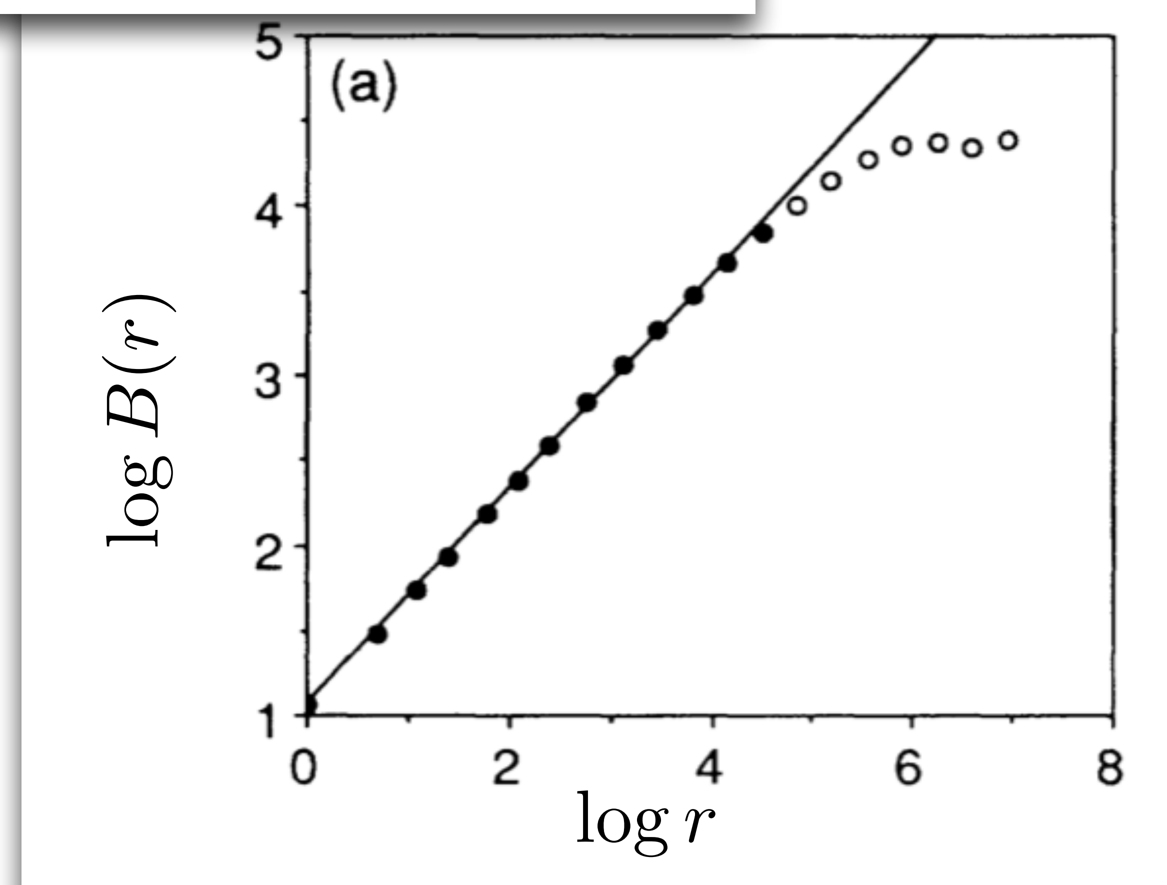


S. Lemerle et al., *Phys. Rev. Lett.* 80, 849 (1998).

Fluid invasion in a porous medium

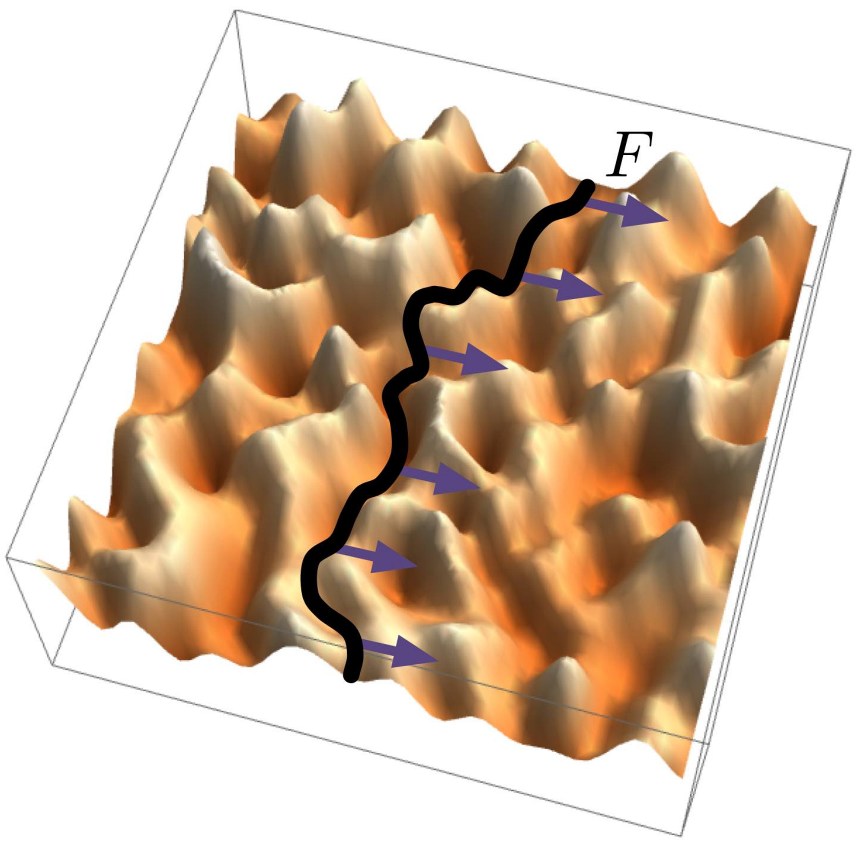


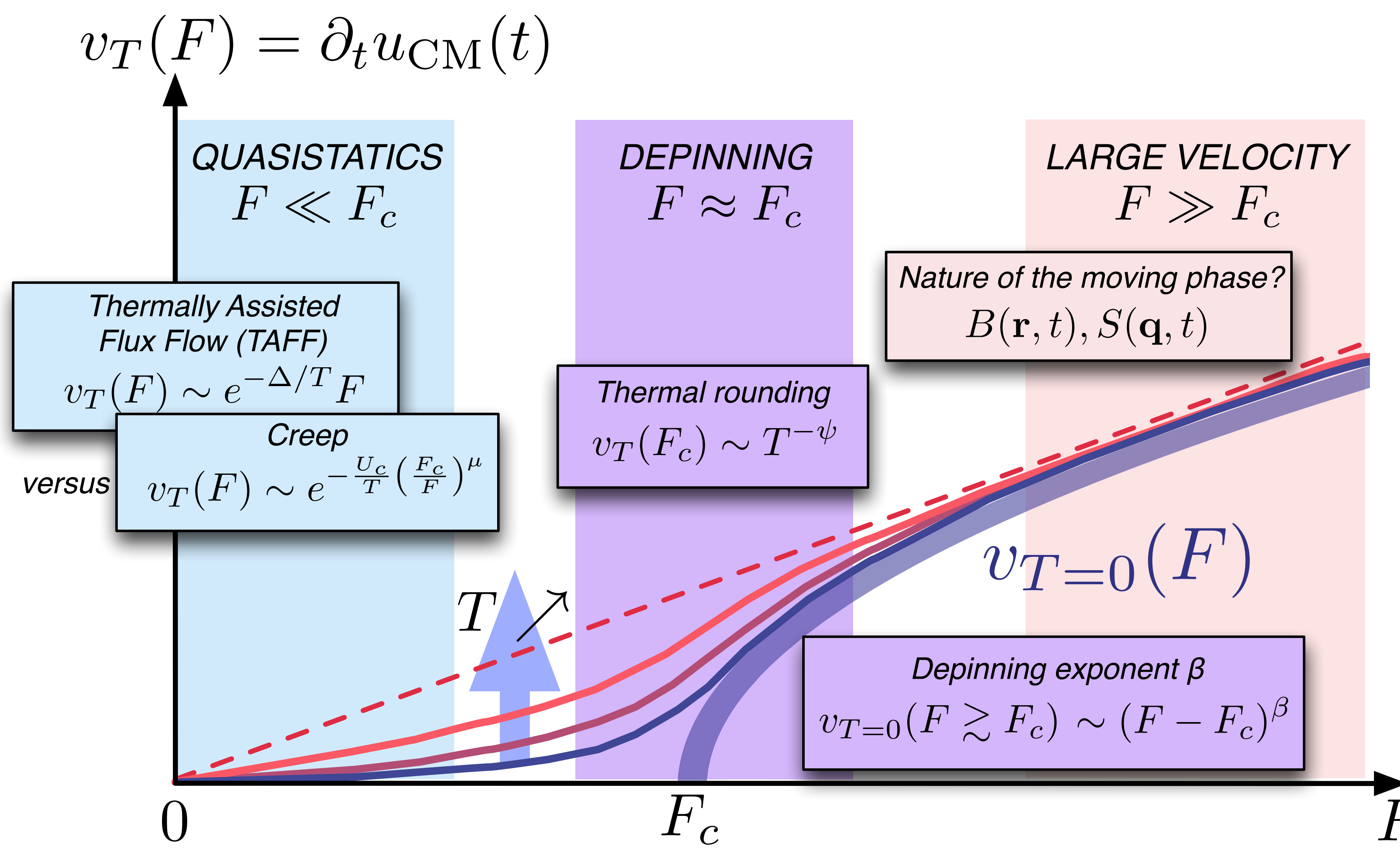
$$\zeta_{\rm exp} = 0.63 \pm 0.04$$



Buldyrev et al., *Phys. Rev. A* 45, 8313 (1992).

Observable 2 — Steady-state velocity-force characteristics





'Creep' regime:
quasistatics at low temperature
& small force

E. Agoritsas et al., *Physica B* 407, 1725 (2012).
T. Giamarchi et al., *Lecture Notes in Physics* 688, 91 (2006).
E. Ferrero et al., *Comptes Rendus Physique* 14, 641 (2013).
E. Ferrero et al., *Annu. Rev. Con. Math. Phys.* 12, 111(2021).

- Focus on low temperature T / small force f / large system size L: creep prediction?

$$v_T(F) \sim e^{-\frac{U_c}{T}\left(\frac{F_c}{F}\right)^{\mu}}$$

1D interface, short-range elasticity (elastic limit), random-bond disorder

$$\mu = \frac{d-2+2\zeta}{2-\zeta} \underset{(d=1)}{\overset{(\zeta=2/3)}{=}} \frac{1}{4}$$

In ferromagnetic domain walls: S. Lemerle *et al.*, *Phys. Rev. Lett.* 80, 849 (1998).

Observable 2 — Steady-state velocity-force characteristics

- Vortices in high-T_c supraconductors

Elastic line, short-range elasticity (elastic limit), random-bond disorder

A. I. Larkin, *Sov. Phys. JETP* 31, 784 (1970),

"Effect of inhomogeneities on the structure of the mixed state of superconductors"

VOLUME 63, NUMBER 20 PHYSICAL REVIEW LETTERS 13 NOVEMBER 1989

Theory of Collective Flux Creep

M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin,[a] and V. M. Vinokur[b]

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PHYSICAL REVIEW B VOLUME 41, NUMBER 13 1 MAY 1990

Thermal fluctuations of vortex lines, pinning, and creep in high-T_c superconductors

M. V. Feigel'man
*Institut für Theoretische Physik, Eidgenossische Technische Hochschule, Hönggerberg, 8093 Zürich, Switzerland and Landau Institute for Theoretical Physics, 117940 Moscow, U.S.S.R.**

V. M. Vinokur

value of the
in the pres-
single-vortex

VOLUME 64, NUMBER 20 PHYSICAL REVIEW LETTERS 14 MAY 1990

Scaling Approach to Pinning: Charge-Density Waves and Giant Flux Creep in Superconductors

Thomas Nattermann[a]

Institut

A
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pone
wave

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Elastic theory of flux lattices in the presence of weak disorder

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(Received 16 January 1995)

Observable 2 — Steady-state velocity-force characteristics

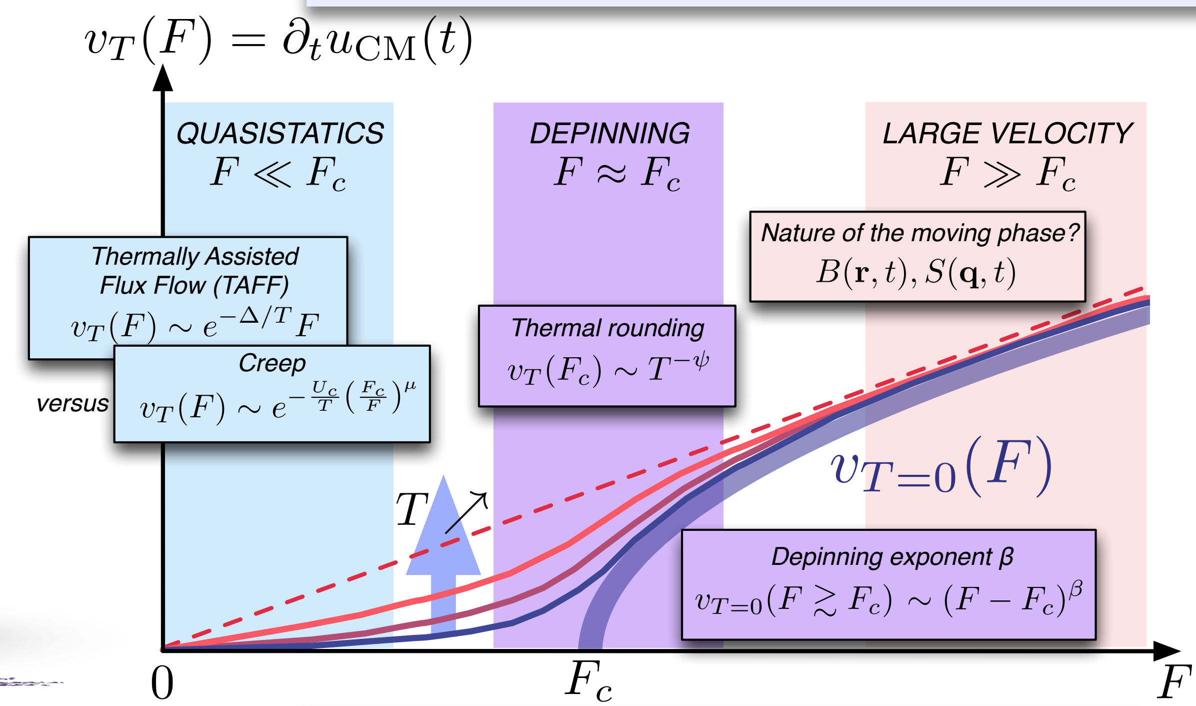
Elastic line, short-range elasticity (elastic limit), random-bond disorder

- Vortices in high-T_c supraconductors

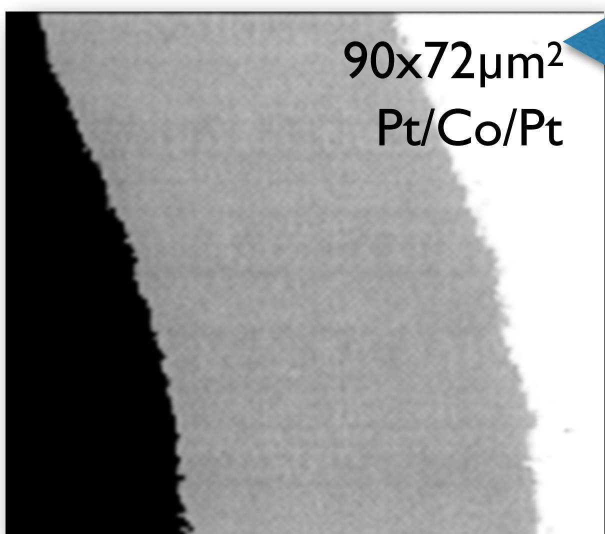
A. I. Larkin, *Sov. Phys. JETP* 31, 784 (1970).
M.V. Feigel'man et al., *Phys. Rev. Lett.* 63, 2303 (1989).
M.V. Feigel'man & V. M. Vinokur., *Phys. Rev. B* 41, 8986 (1990).
T. Nattermann, *Phys. Rev. Lett.* 64, 2454 (1990).
T. Giamarchi & P. Le Doussal, *Phys. Rev. B* 52, 1242 (1995).
Etc.

Individual vortex *v.s.* collective pinning

Scaling argument for the creep



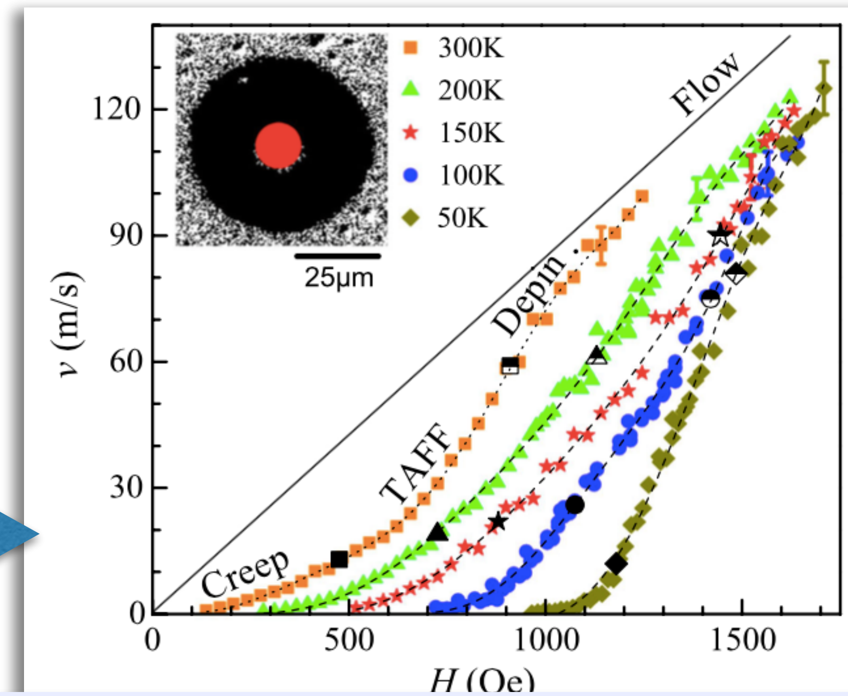
- Ferromagnetic domain walls in ultra-thin films



S. Lemerle *et al.*, *Phys. Rev. Lett.* 80, 849 (1998).

J. Ferré, P. T. Metaxas, A. Mougin, J.-P. Jamet, J. Gorchon, & V. Jeudy, *Comptes Rendus Physique* 14, 651 (2013).

S. Bustingorry *et al.*, *Phys. Rev. B* 85, 214416 (2012).
J. Gorchon *et al.*, *Phys. Rev. Lett.* 113, 027205 (2014).
V. Jeudy *et al.*, *Phys. Rev. Lett.* 117, 057201 (2016).
etc.



Other dimensionality/elasticity/disorder

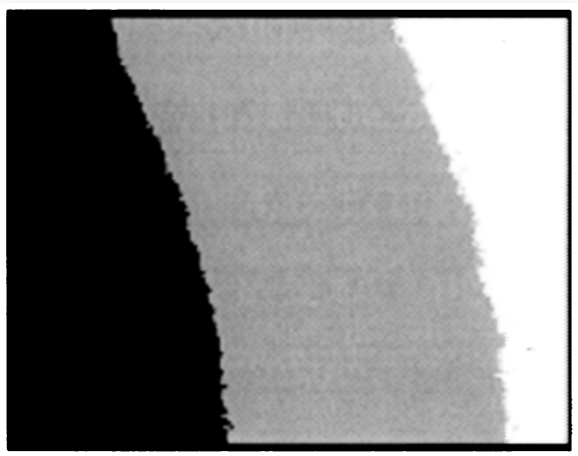
- Ferroelectric domain walls in thin film (e.g. $Pb(Zr_{0.2}Ti_{0.8})O_3$)

P. Paruch & J. Guyonnet, *Comptes Rendus Physique* 14, 667 (2013).

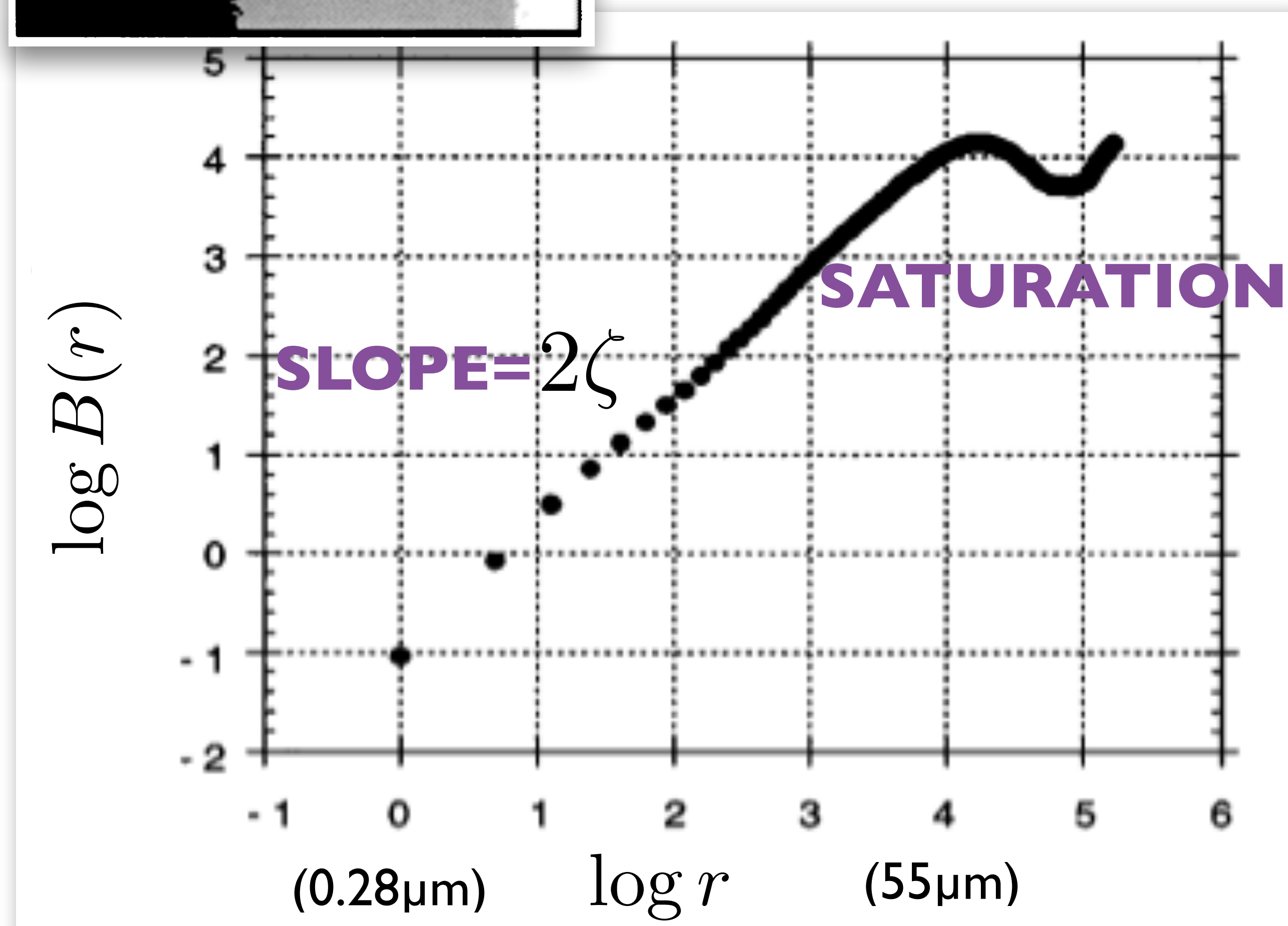
- Imbibition fronts, fractures, fluid invasion in porous medium, etc. *via* DES modelling

Roughness and creep exponents

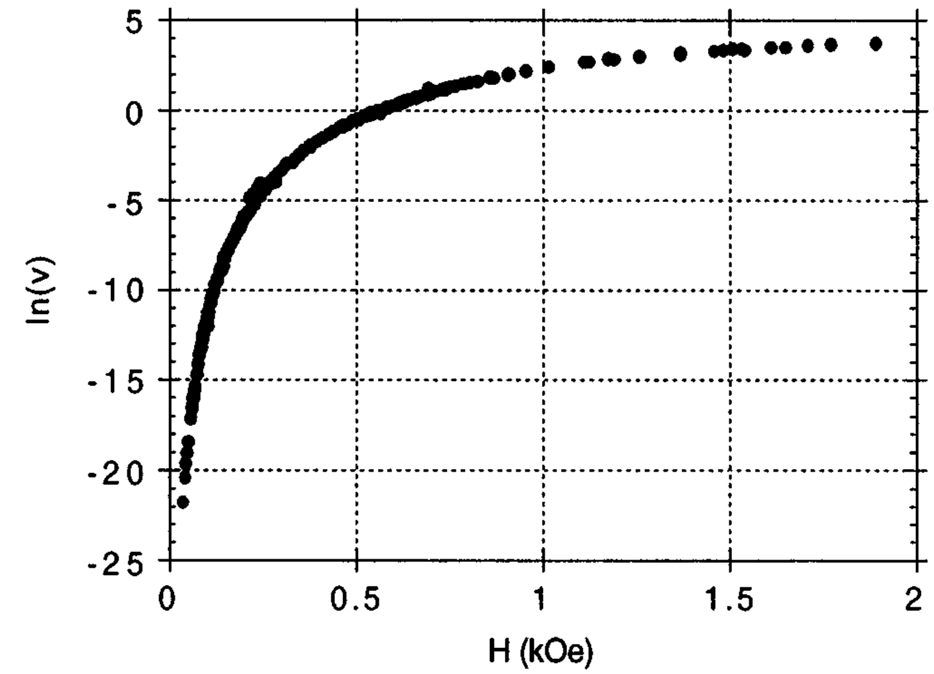
Domain walls in ultrathin Pt/Co/Pt ferromagnetic films



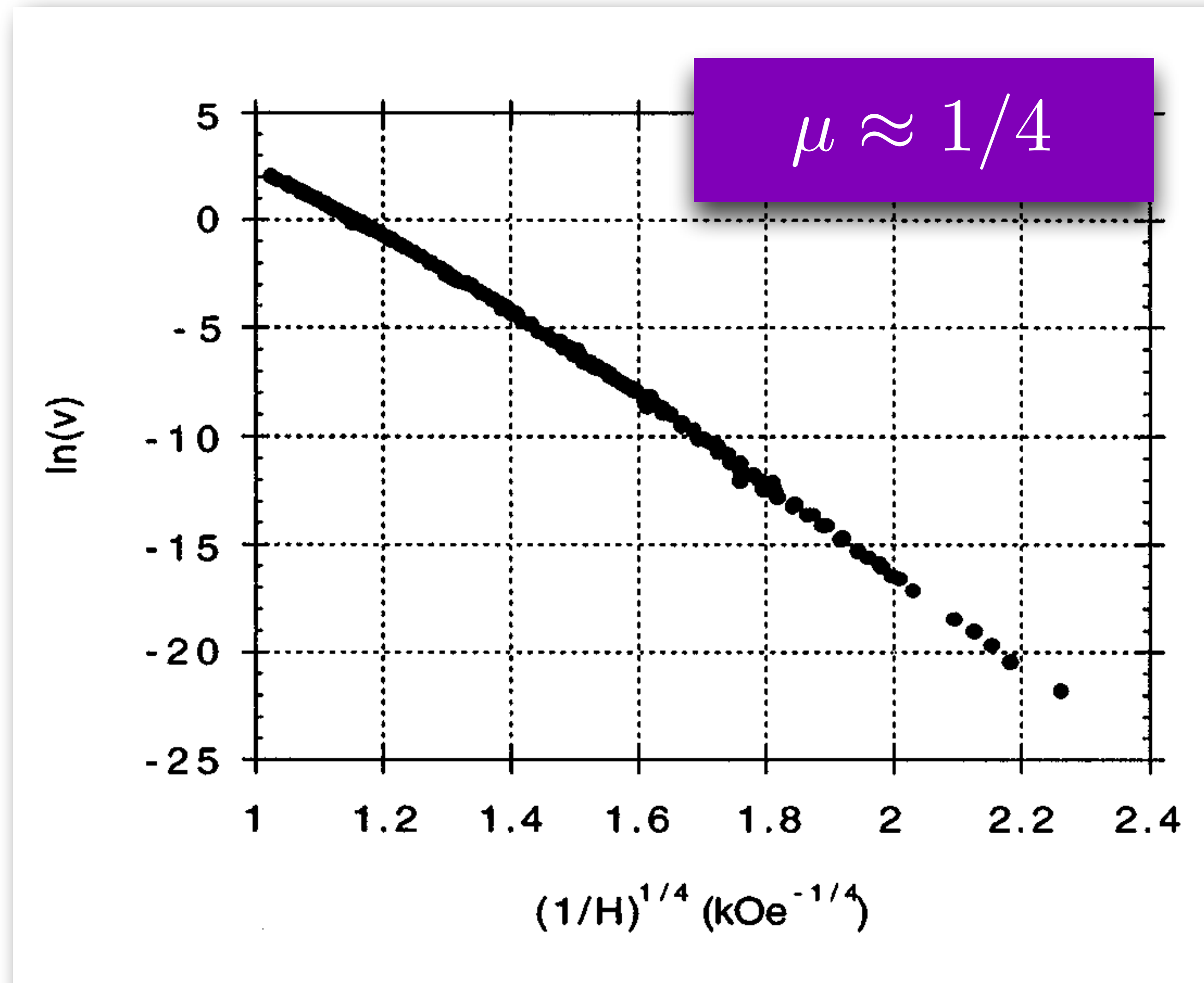
$\zeta_{\text{exp}} = 0.69 \pm 0.07$



S. Lemerle et al., *Phys. Rev. Lett.* 80, 849 (1998).



$\mu \approx 1/4$



Model: 1D interface with finite width / short-range correlated disorder

- Short-range elasticity & Elastic limit / Quenched random-bond weak disorder

$$\mathcal{H}_{\text{DES}} = \mathcal{H}_{\text{el}} + \mathcal{H}_{\text{dis}}$$

Hamiltonian:

$$\mathcal{H}[u,\widetilde{V}] = \int_{\mathbb{R}} dz \cdot \left[\frac{c}{2} (\nabla_z u(z))^2 + \int_{\mathbb{R}} dx \cdot \rho_\xi(x - u(z))\, \widetilde{V}(z,x) \right]$$

Elasticity **Effective random potential** $V(z, u(z))$

- Density $\rho_\xi(x-u(z))$ & random potential $\widetilde{V}(z,x)$

$$\overline{\widetilde{V}(z,x)} = 0$$

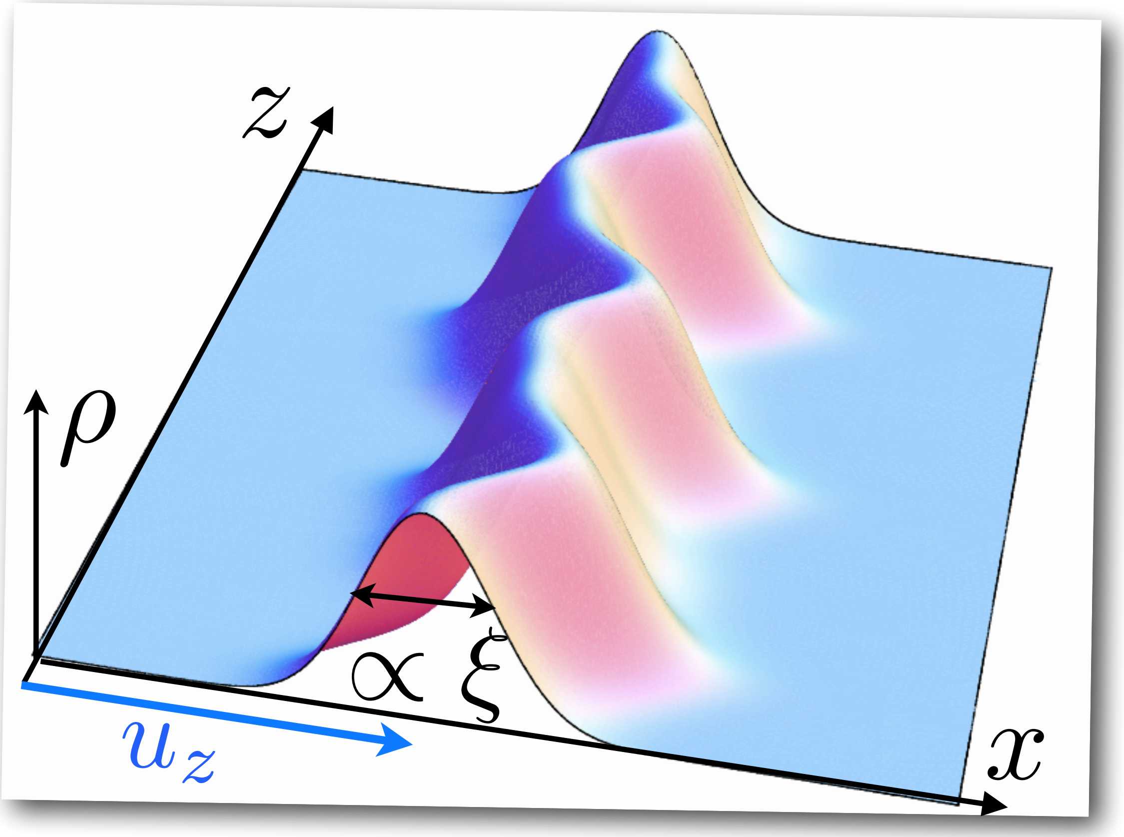
$$\overline{\widetilde{V}(z,x)\widetilde{V}(z',x')} = D\,\delta(z-z')\delta(x-x')$$

- Alternative: ***correlated*** effective potential $V(z,u(z))$

$$\overline{V(z,x)V(z',x')} = D\,\delta(z-z')R_\xi(x-x')$$

- Exponentially decaying with following scaling:

$$R_\xi(x) = \xi^{-1} R_1(x/\xi) \quad \text{e.g.} \quad R_\xi^{\text{G}}(x) = \frac{e^{-x^2/(2\xi^2)}}{\sqrt{2\pi}\xi}$$



Elastic constant c / Width ξ / Disorder strength D / Temperature T

- Overdamped dynamics: 'quenched Edwards-Wilkinson':

$$\gamma\,\partial_t u(z,t) = c\partial_z^2 u(z,t) + F_{\text{dis}}(z,u(z)) + f_{\text{ext}} + \eta_{\text{thermal}}(z,t)$$

$$\langle \eta_{\text{th}}(z,t)\eta_{\text{th}}(z',t')\rangle = 2\gamma T\delta(z-z')\delta(t-t')$$

$$F_{\text{dis}}(z,x) = -\partial_x V(z,x)$$

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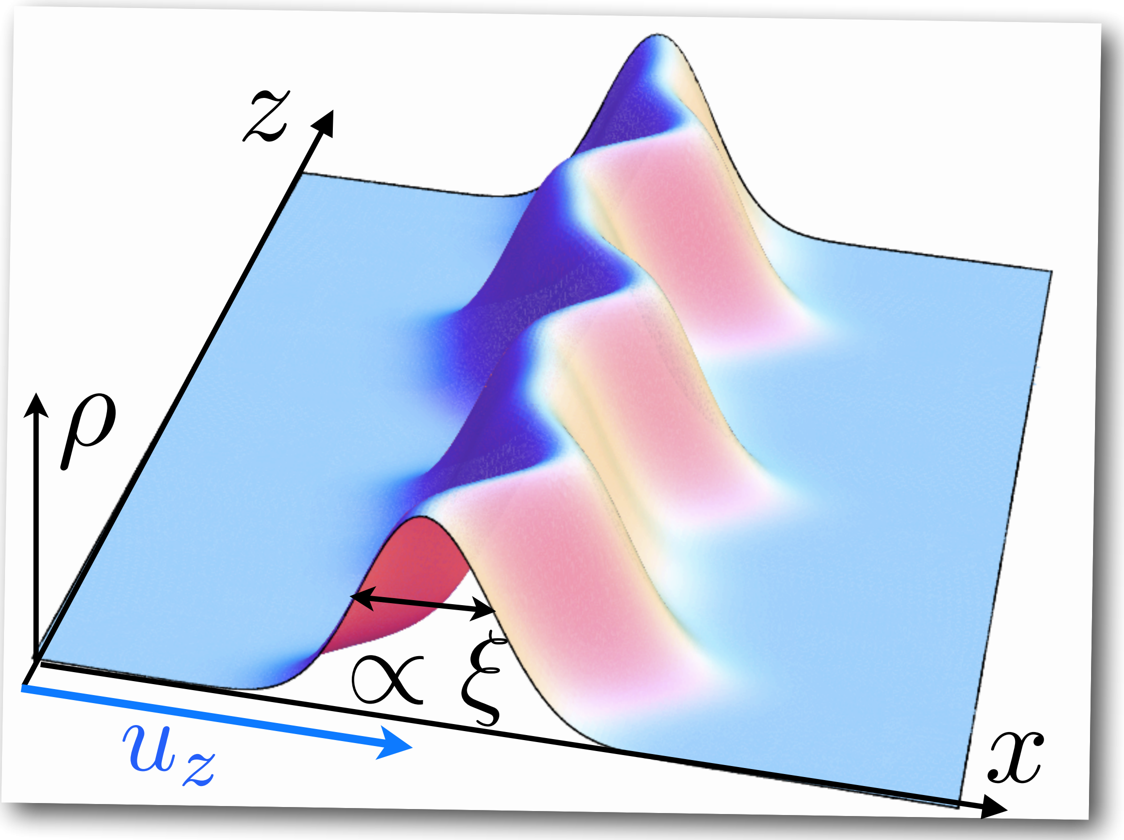

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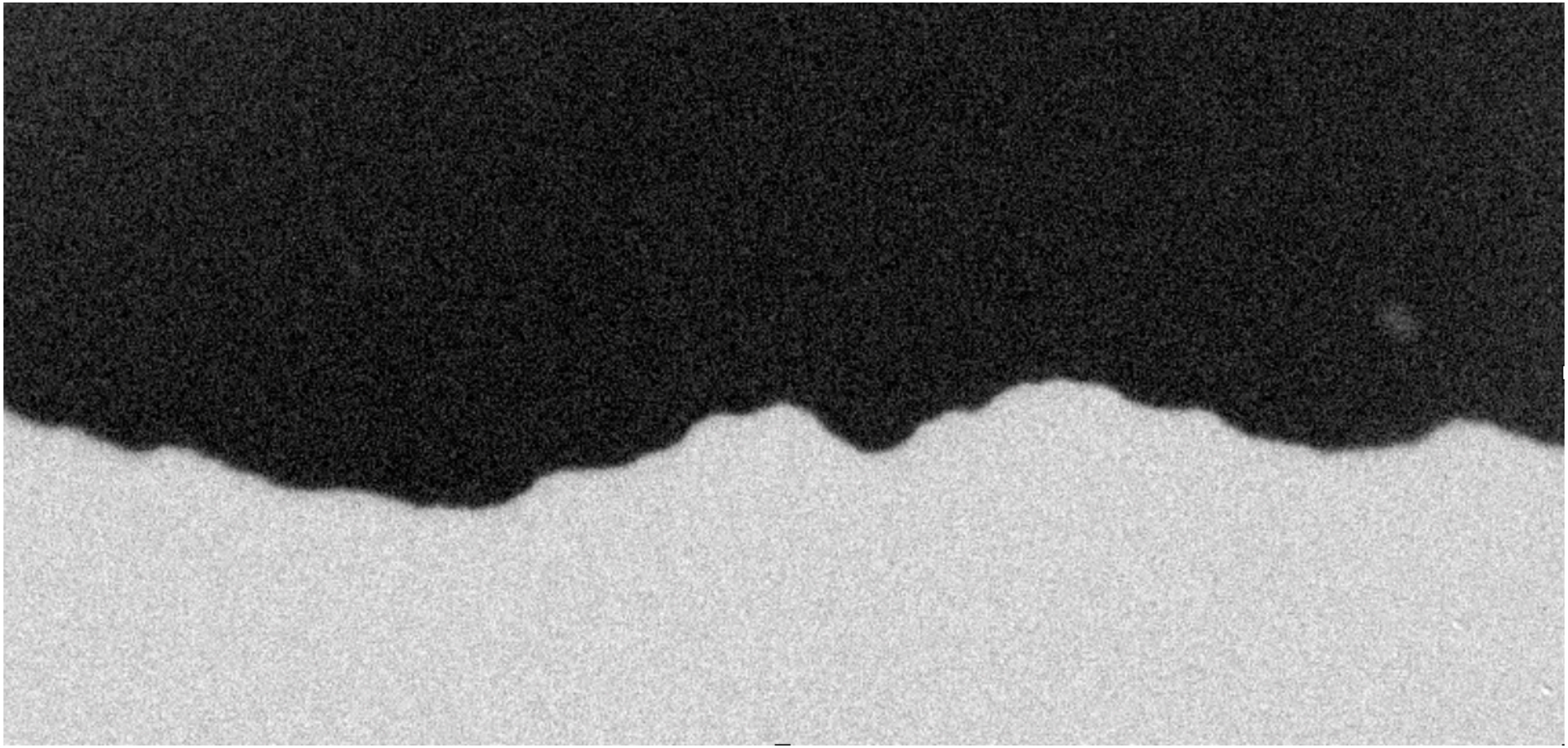
Experimental realisation: moving ferromagnetic domain wall

- Here focus on 'quenched Edwards-Wilkinson' (qEW):

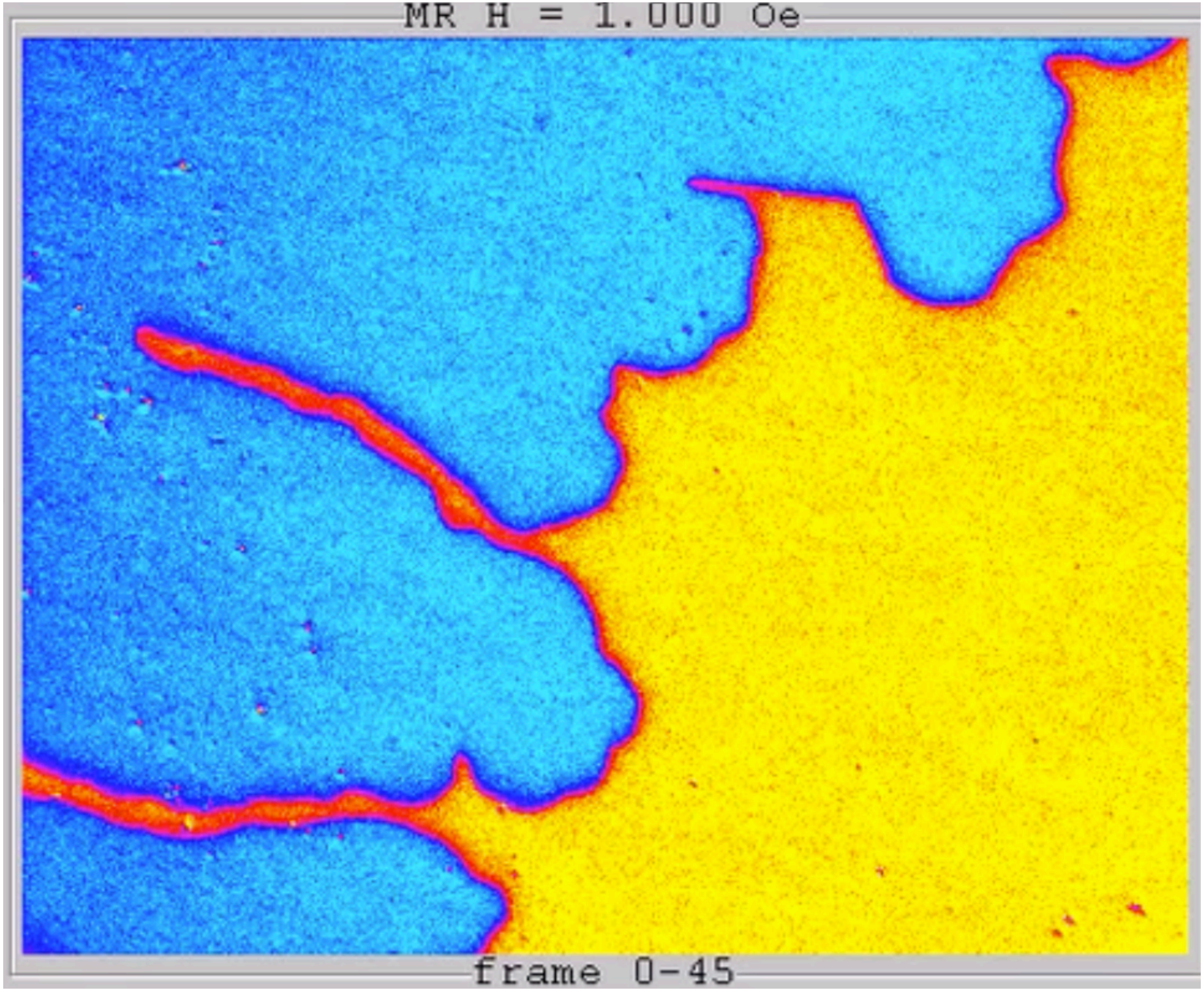
$$\gamma\,\partial_t u(z,t) = c\partial_z^2 u(z,t) + F_{\mathrm{dis}}(z,u(z)) + f_{\mathrm{ext}} + \eta_{\mathrm{thermal}}(z,t)$$

- V. Repain et al. @ Orsay (Pt/Co/Pt)



Experimental realisation: moving ferromagnetic domain wall

- V. Repain et al. @ Orsay: **Pt/Co(0,5 nm)/Pt/SiO_2**



Overdamped dynamics / RB vs RF disorder

- **Overdamped dynamics for numerics / field-theory:**

$$\mathcal{H}_{\rm DES} = \mathcal{H}_{\rm el} + \mathcal{H}_{\rm dis}$$

$$\underbrace{\cancel{m\partial_t^2 u(z,t)}}_{\text{Acceleration}} + \gamma\partial_t u(z,t) = \underbrace{-\frac{\delta\mathcal{H}_{\rm el}[u]}{\delta u(z,t)}}_{F_{\rm el}(z,t)} \underbrace{-\frac{\delta\mathcal{H}_{\rm dis}[u,V]}{\delta u(z,t)}}_{F_{\rm dis}(z,t)} + f_{\rm ext} + \eta_{\rm thermal}(z,t)$$

  - Gaussian noise of zero mean and 2-pt correlator: $\langle \eta_{\rm th}(z,t)\eta_{\rm th}(z',t')\rangle = 2\gamma T\delta(z-z')\delta(t-t')$

- Here focus on 'quenched Edwards-Wilkinson' (qEW):

$$\gamma\,\partial_t u(z,t) = c\partial_z^2 u(z,t) + F_{\rm dis}(z,u(z)) + f_{\rm ext} + \eta_{\rm thermal}(z,t)$$

  - Random force: $F_{\rm dis}(z,x) = -\partial_x V(z,x)$

$$\Delta(x) = -D\,R''(x)$$

$$\overline{V(z,x)V(z',x')} = D\delta(z-z')R(x-x')$$

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$\int dx\,\Delta(x) = 0$ random-bond (RB)

$\int dx\,\Delta(x) > 0$ random-field (RF)

  - Typically with periodic boundary condition: $u(z+L_z) = u(z)$, and also $L_x \sim L_z^\zeta$

Numerics: E. E. Ferrero, S. Bustingorry, A. B. Kolton, A. Rosso, *Comptes Rendus de Physique* 14, 641 (2013)

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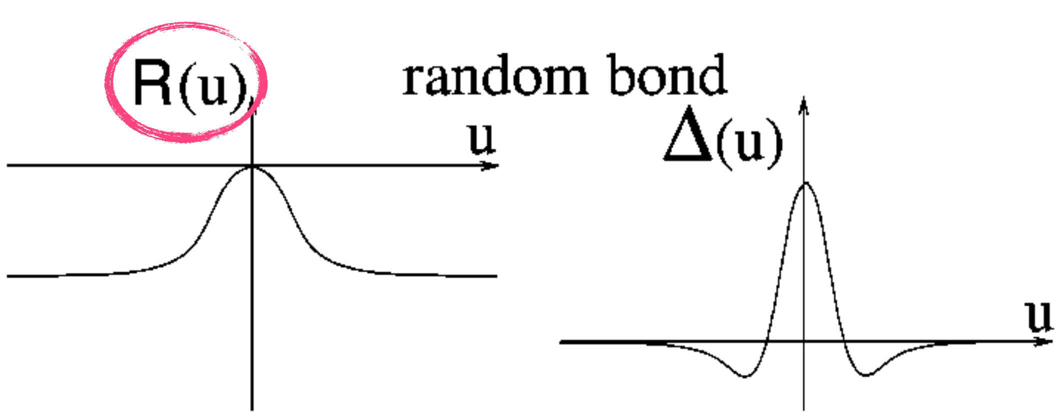
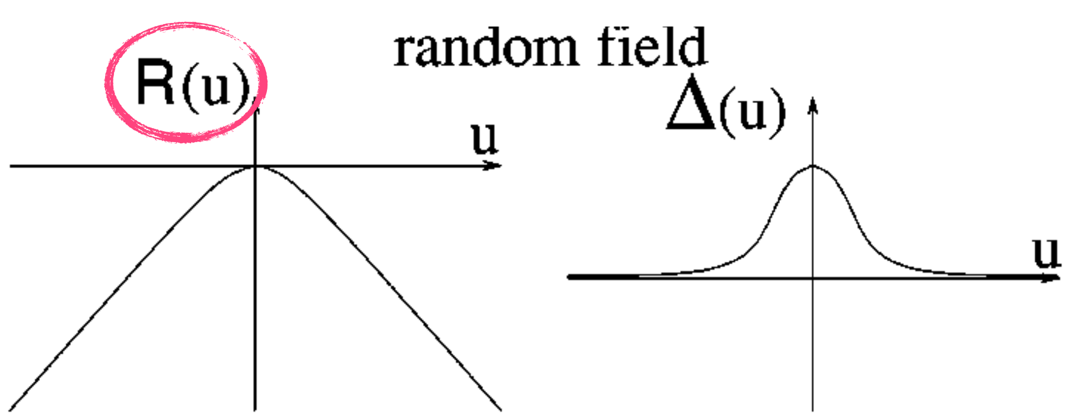
- Be careful about slightly alternative definitions:

$$\overline{[V(z,x) - V(z',x')]^2} = 2\delta(z-z')D\,[R(0) - R(x-x')] \equiv -2\delta(z-z')\mathrm{R}(x-x')$$





P. Chauve, T. Giamarchi, P. Le Doussal, "Creep and depinning in disordered media", *Phys.Rev. B* 62, 6241 (2000)

Overdamped dynamics & MSR dynamical action

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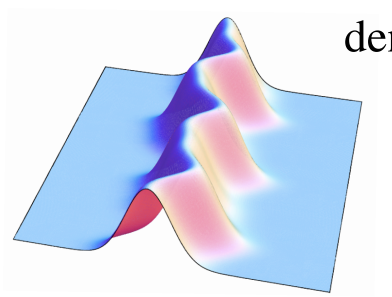
Force-force correlator = central object for FRG

Interlude: random-bond versus random-field

- Two equivalent formulations of the disorder Hamiltonian:

$$-\nabla_x V(x,z) = h(x,z)$$

$$\mathcal{H}_{\text{dis}} = \int d^d z\, d^m x \cdot \underbrace{\rho_u\left(x,z\right)}_{\text{density}} \cdot \underbrace{V\left(x,z\right)}_{\text{random potential}} = \int d^d z\, d^m x \cdot \underbrace{P_u\left(x,z\right)}_{\text{profile}} \cdot \underbrace{h\left(x,z\right)}_{\text{random field}}$$



$$\nabla_x P_u(x,z) = \rho_u(x,z)$$

- Random-bond ↔ delta-correlated random potential:

$$\mathcal{H}_{\text{dis}}^{\text{RB}}\left[u,V\right] = \int dz \underbrace{\int dx \cdot \rho_u\left(x,z\right) \cdot V\left(x,z\right)}_{\text{effective random potential}} = \int dz \cdot \widetilde{V}\left(u_z,z\right)$$

$$\begin{aligned} \overline{V(x,z)V(x',z')} &= D \cdot \delta\left(z-z'\right) \cdot \delta\left(x-x'\right) \\ \overline{\widetilde{V}\left(x,z\right)\widetilde{V}\left(x',z'\right)} &= \delta\left(z-z'\right) \cdot R_\rho^{\text{RB}}\left(u_z,u'_z\right) \end{aligned} \quad \Longrightarrow \quad \boxed{R_\rho^{\text{RB}}\left(u_z,u'_z\right) \equiv D \int dx \cdot \rho_u\left(x,z\right)\, \rho_{u'}\left(x,z\right)}$$

- Random-field ↔ delta-correlated random force:

$$\mathcal{H}_{\text{dis}}^{\text{RF}}\left[u,h\right] = \int dz\, dx \cdot P_u\left(x,z\right) \cdot h\left(x,z\right) = \int dz\, \frac{1}{2}\left(\int_{-\infty}^{u_z} - \int_{u_z}^{\infty}\right) dx \cdot \tilde{h}\left(x,z\right)$$

$$\begin{aligned} \overline{h(x,z)h(x',z')} &= D \cdot \delta\left(z-z'\right) \cdot \delta\left(x-x'\right) \\ \overline{\tilde{h}\left(x,z\right)\tilde{h}\left(x',z'\right)} &= \delta\left(z-z'\right) \cdot \Delta_\rho^{\text{RF}}\left(x,x'\right) \end{aligned} \quad \Longrightarrow \quad \boxed{\Delta_\rho^{\text{RF}}(u_z,u'_z) \equiv D \int dx \cdot \rho_u\left(x,z\right)\, \rho_{u'}\left(x,z\right)}$$

Interlude: random-bond versus random-field

- Quenched Gaussian disorder ⇒ Focus on the mean & two-point correlator

  - Random force: $F_{\rm dis}(z,x) = -\partial_x V(z,x)$

$$\Delta(x) = -D\,R''(x)$$

$$\overline{V(z,x)V(z',x')} = D\delta(z-z')R(x-x')$$

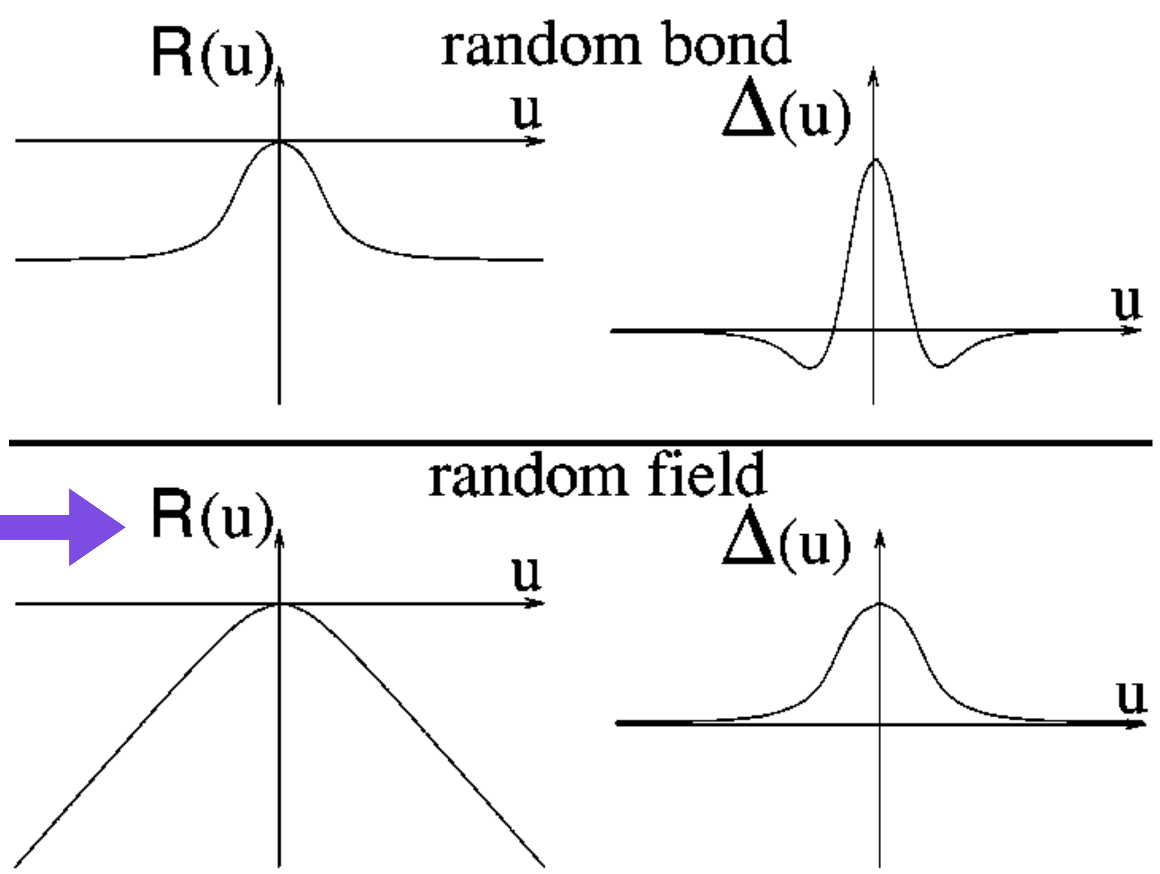
$$\overline{F_{\rm dis}(z,x)F_{\rm dis}(z',x')} = \delta(z-z')\,\Delta(x-x')$$

$$\int dx\,\Delta(x) = 0 \quad \text{random-bond (RB)}$$

$$\int dx\,\Delta(x) > 0 \quad \text{random-field (RF)}$$

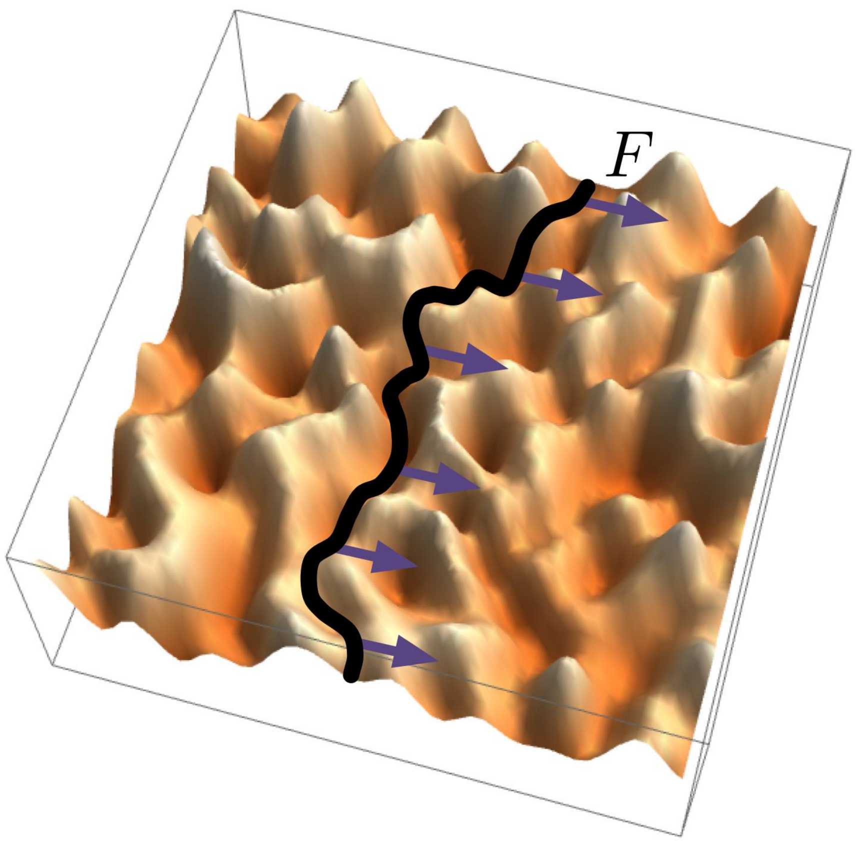
  - Be careful about slightly alternative definitions:

$$\overline{[V(z,x)-V(z',x')]^2}$$
$$= 2\delta(z-z')D\,[R(0)-R(x-x')]$$
$$\equiv -2\delta(z-z')\mathrm{R}(x-x')$$



P. Chauve, T. Giamarchi, P. Le Doussal, "Creep and depinning in disordered media", *Phys.Rev. B* 62, 6241 (2000)

Case study of the 1D interface in short-range correlated disorder $(\xi > 0)$



- **Experimental motivation:**
  relevant e.g. for ultrathin ferromagnetic domain walls Pt/Co/Pt looking for characteristic length- and energy-scales

- **Statistical-physics motivation:**
  simplest extended object in a *quenched* disorder
  $\Longrightarrow$ Model reduction? Center-of-mass dynamics, effective disorder/noise? Mean-field descriptions?

- **Technical motivation:**
  comparison of analytical approximation schemes on a well-defined system, while predictions possibly relevant more broadly for the 1D KPZ universality class

Universal scalings:
roughness exponent ζ=2/3 at asymptotically large scales

Non-universal features:
characteristic crossover scales & roughness amplitude

- Accessible assuming uncorrelated disorder (because of ad hoc exact solutions & symmetries, renormalization fixed points, …)
- How does universality emerge from specific microphysics, e.g. from correlated disorder?

- If correlated disorder, no exact solutions anymore but possible pathologies/divergences are regularized
- Allows for existence of physically relevant temperature-induced crossover

cnrs GDR Groupement de recherche

IDE Interactions, Désordre, Elasticité

Vivien Lecomte, Elisabeth Agoritsas, Damien Vandembroucq

Laura Foini, Kirsten Martens, Alberto Rosso, Jérôme Weiss

https://groupes.renater.fr/sympa/info/gdr-ide

Next annual meeting: **June 17-21, 2024 in Grenoble**

Registration deadline: April 30, 2024 ⇒ ***https://gdr-ide-2024.sciencesconf.org***