Doctoral Training School *Kardar-Parisi-Zhang equation: new trends in theories and experiments*
April 15-26, 2024 — Ecole de Physique des Houches (France)

Interfaces in disordered systems and directed polymer

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Interfaces in disordered systems and directed polymer

Elisabeth Agoritsas

- 1. Introduction
- 2. Disordered elastic systems: Recipe
- 3. Disordered elastic systems: Statics
- ➔ 4. Disordered elastic systems: Dynamics
- 5. Concluding remarks

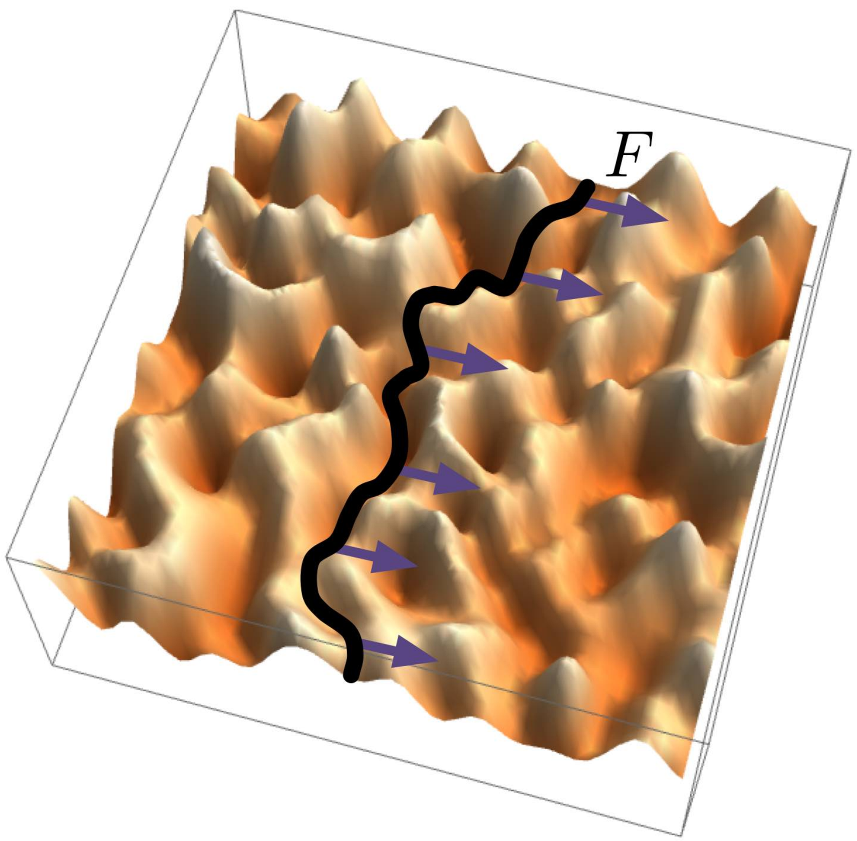
4.1 Velocity-force characteristics

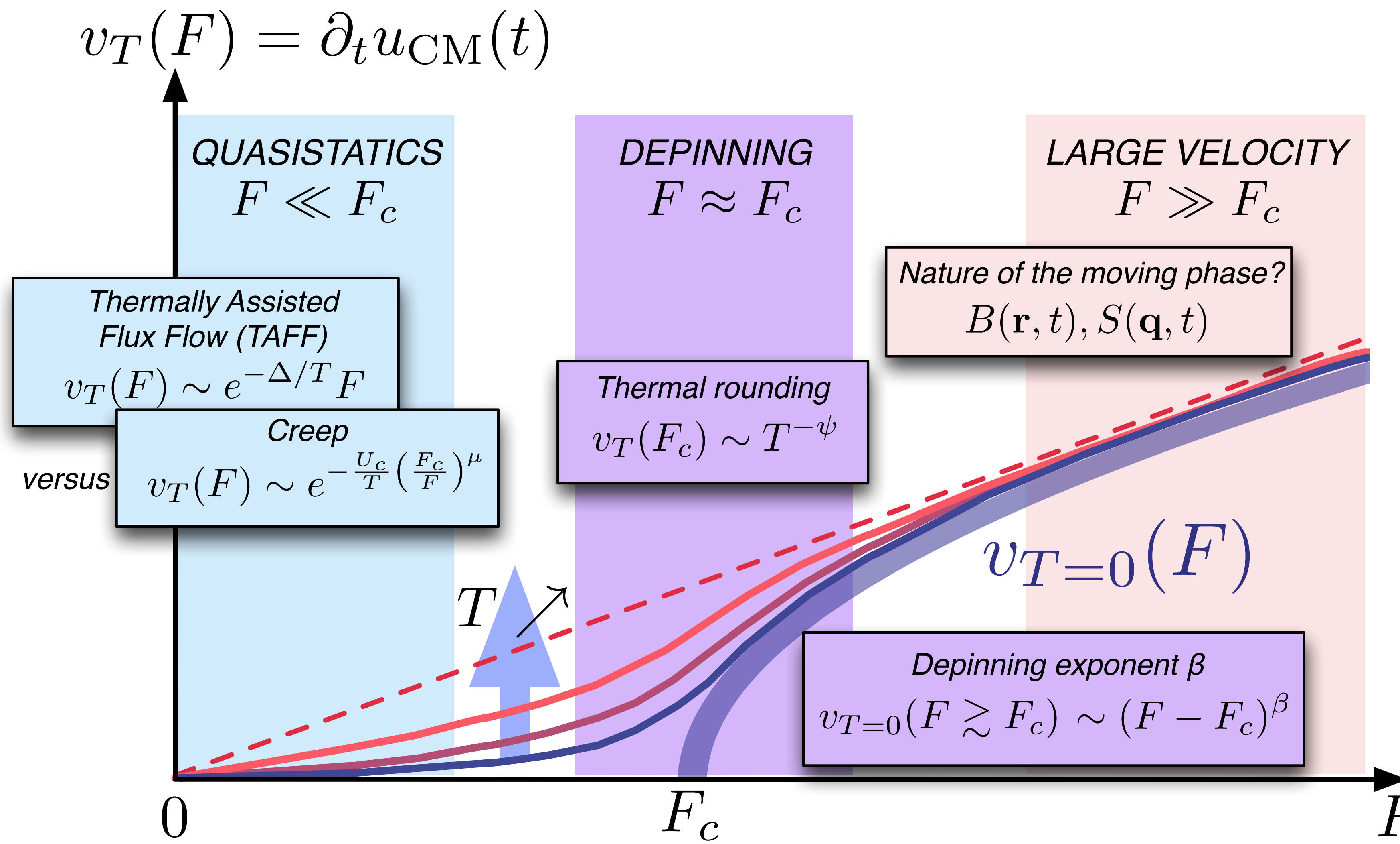
4.2 Example of **model reduction**: effective 1D interface starting from a 2D Ginzburg-Laudau description

4.3 **Creep regime**: how to recover 1/4 creep exponent from 2/3 KPZ exponent

4.4 **Fast-flow regime**

Observable 2 — Steady-state velocity-force characteristics





'Creep' regime:
quasistatics at low temperature & small force

E. Agoritsas et al., *Physica B* 407, 1725 (2012).
T. Giamarchi et al., *Lecture Notes in Physics* 688, 91 (2006).
E. Ferrero et al., *Comptes Rendus Physique* 14, 641 (2013).
E. Ferrero et al., *Annu. Rev. Con. Math. Phys.* 12, 111(2021). **[Review!]**

- Focus on low temperature T / small force f / large system size L: creep prediction?

$$v_T(F) \sim e^{-\frac{U_c}{T}\left(\frac{F_c}{F}\right)^\mu}$$

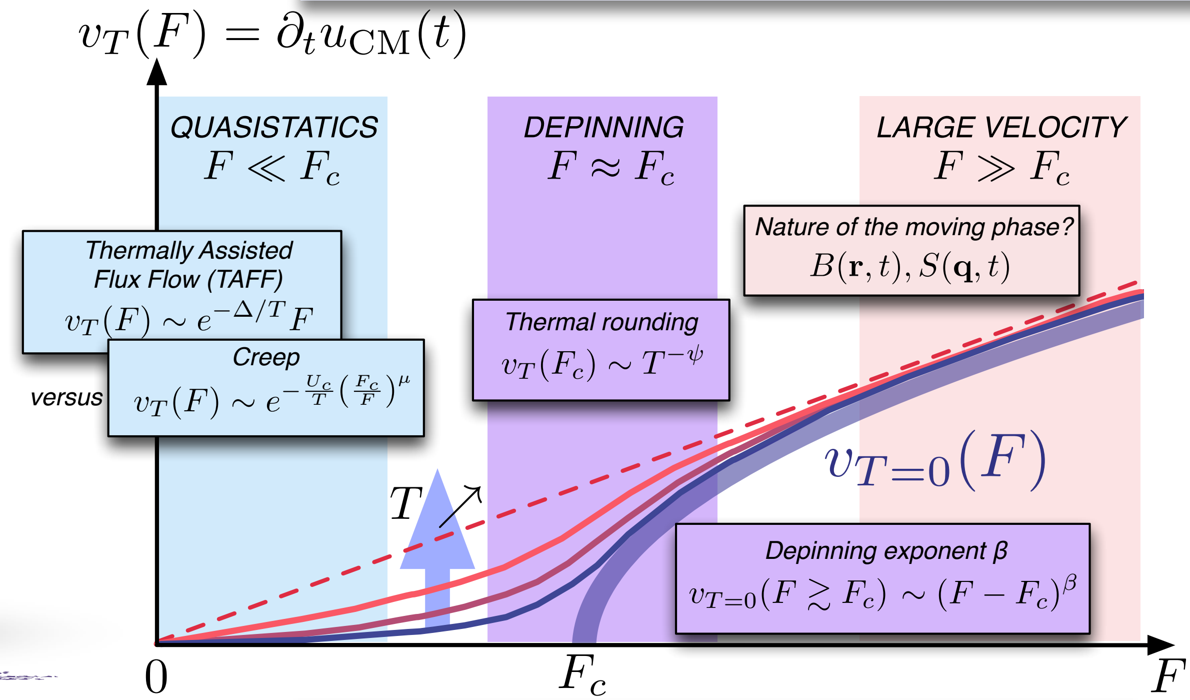
1D interface, short-range elasticity (elastic limit), random-bond disorder

$$\mu = \frac{d-2+2\zeta}{2-\zeta} \underset{(d=1)}{\overset{(\zeta=2/3)}{=}} \frac{1}{4}$$

In ferromagnetic domain walls: S. Lemerle *et al.*, *Phys. Rev. Lett.* 80, 849 (1998).

Observable 2 — Steady-state velocity-force characteristics

Elastic line, short-range elasticity (elastic limit), random-bond disorder



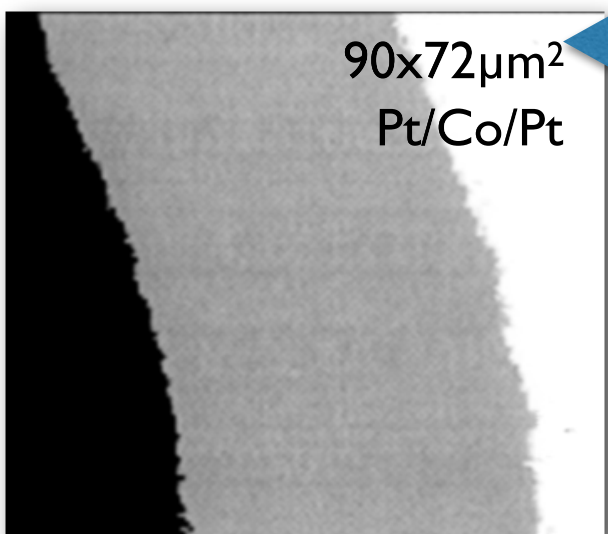
- Vortices in high-T_c supraconductors

A. I. Larkin, *Sov. Phys. JETP* 31, 784 (1970).
M.V. Feigel'man et al., *Phys. Rev. Lett.* 63, 2303 (1989).
M.V. Feigel'man & V. M. Vinokur., *Phys. Rev. B* 41, 8986 (1990).
T. Nattermann, *Phys. Rev. Lett.* 64, 2454 (1990).
T. Giamarchi & P. Le Doussal, *Phys. Rev. B* 52, 1242 (1995).
Etc.

Individual vortex *v.s.* collective pinning

Scaling argument for the creep

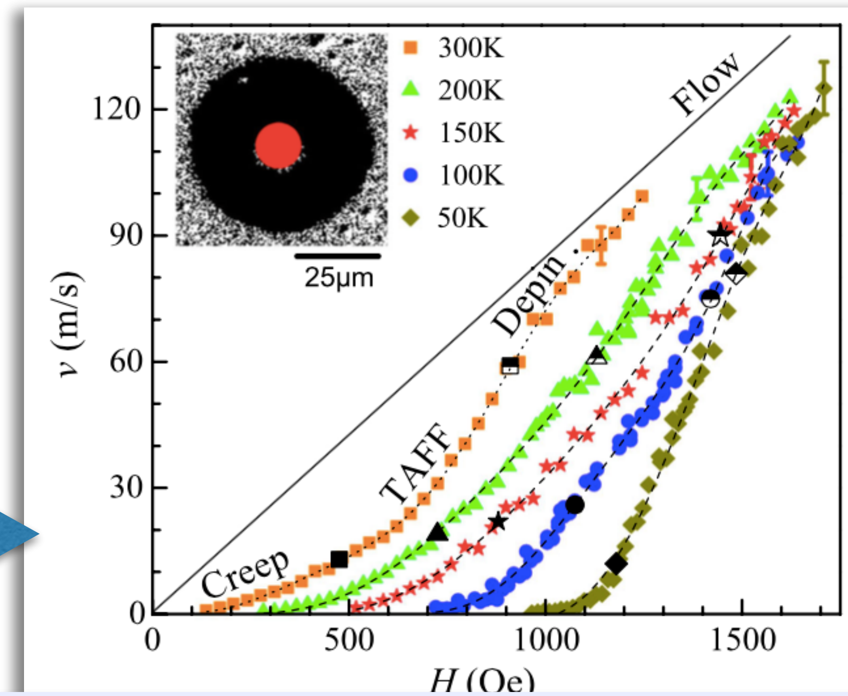
- Ferromagnetic domain walls in ultra-thin films



S. Lemerle *et al.*, *Phys. Rev. Lett.* 80, 849 (1998).

J. Ferré, P. T. Metaxas, A. Mougin, J.-P. Jamet, J. Gorchon, & V. Jeudy, *Comptes Rendus Physique* 14, 651 (2013).

S. Bustingorry *et al.*, *Phys. Rev. B* 85, 214416 (2012).
J. Gorchon *et al.*, *Phys. Rev. Lett.* 113, 027205 (2014).
V. Jeudy *et al.*, *Phys. Rev. Lett.* 117, 057201 (2016).
etc.



Other dimensionality/elasticity/disorder

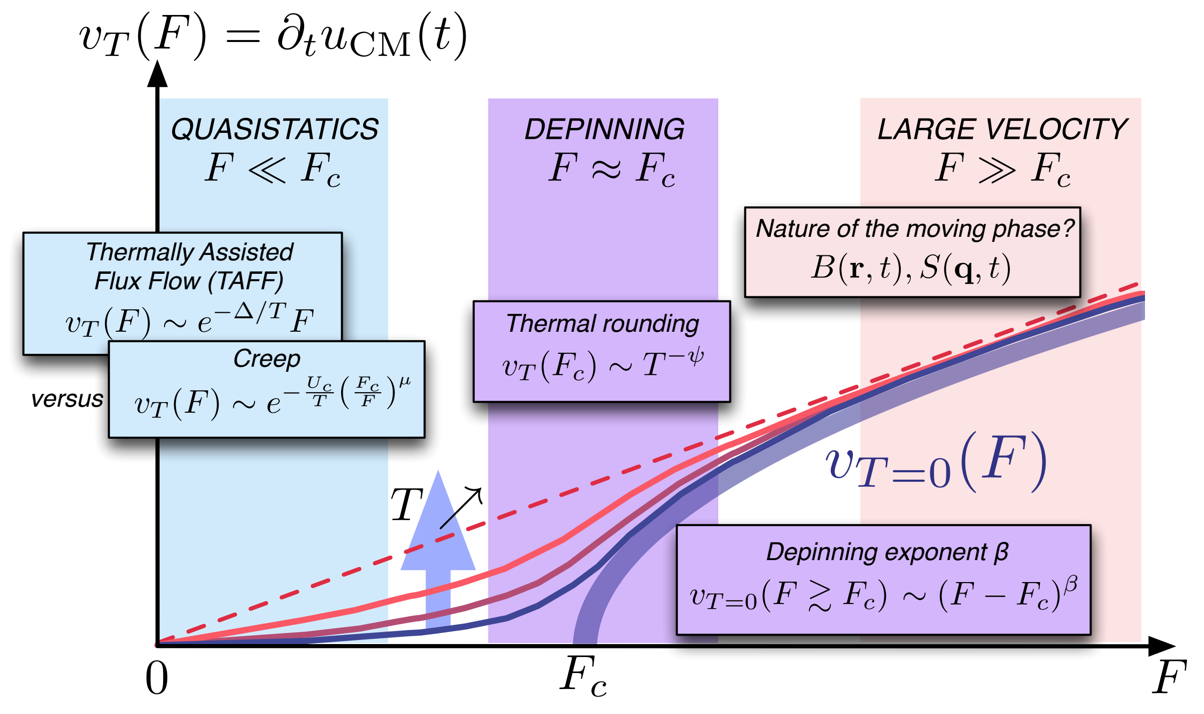
- Ferroelectric domain walls in thin film (e.g. $Pb(Zr_{0.2}Ti_{0.8})O_3$)

P. Paruch & J. Guyonnet, *Comptes Rendus Physique* 14, 667 (2013).

- Imbibition fronts, fractures, fluid invasion in porous medium, etc. *via* DES modelling

Non-exhaustive biblio. — Numerical studies of DES

- Numerical studies of the quenched Edwards-Wilkinson elastic line [non-exhaustive!]
  - A. B. Kolton, A. Rosso, & T. Giamarchi, *Phys. Rev. Lett.* 94, 047002 (2005): "Creep Motion of an Elastic String in a Random Potential"
  - A. B. Kolton, A. Rosso, & T. Giamarchi, *Phys. Rev. Lett.* 95, 180604 (2005): "Nonequilibrium Relaxation of an Elastic String in a Random Potential"
  - A. B. Kolton, A. Rosso, T. Giamarchi, & W. Krauth, *Phys. Rev. Lett.* 97, 057001 (2006): "Dynamics below the Depinning Threshold in Disordered Elastic Systems"
  - A. B. Kolton, A. Rosso, T. Giamarchi, & W. Krauth, *Phys. Rev. B* 79, 184207 (2009): "Creep dynamics of elastic manifolds via exact transition pathways"



- **Review:** E. Ferrero, S. Bustingorry, A. B. Kolton, & A. Rosso, *Comptes Rendus Physique* 14, 641 (2013): "Numerical approaches on driven elastic interfaces in random media"

- **Distribution of critical force (highest barrier):** C. Bolech & A. Rosso, *Phys. Rev. Lett.* 93, 125701 (2004): "Universal Statistics of the Critical Depinning Force of Elastic Systems in Random Media"

- **Finite-size fluctuations of the velocity:** A. B. Kolton, S. Bustingorry, E. Ferrero, & A. Rosso, *J. Stat. Mech.* 2013, P12004 (2013): "Uniqueness of the thermodynamic limit for driven disordered elastic interfaces"

- **Avalanches organisation & statistics from creep to depinning:**
  E. Ferrero, L. Foini, T. Giamarchi, A. B. Kolton, & A. Rosso, *Phys. Rev. Lett.* 118, 147208 (2017): "Spatio-temporal patterns in ultra-slow domain wall creep dynamics"

Overdamped dynamics & MSR dynamical action

$$\mathcal{H}_{\rm DES} = \mathcal{H}_{\rm el} + \mathcal{H}_{\rm dis}$$

- Here focus on 'quenched Edwards-Wilkinson' (qEW):

$$\gamma\,\partial_t u(z,t) = c\partial_z^2 u(z,t) + F_{\rm dis}(z,u(z)) + f_{\rm ext} + \eta_{\rm thermal}(z,t)$$

  - Gaussian noise of zero mean and 2-pt correlator: $\langle \eta_{\rm th}(z,t)\eta_{\rm th}(z',t')\rangle = 2\gamma T\delta(z-z')\delta(t-t')$
  - Random force: $F_{\rm dis}(z,x) = -\partial_x V(z,x)$

$$\Delta(x) = -D\,R''(x)$$

$$\overline{V(z,x)V(z',x')} = D\delta(z-z')R(x-x')$$

$$\overline{F_{\rm dis}(z,x)F_{\rm dis}(z',x')} = \delta(z-z')\,\Delta(x-x')$$

$\int dx\,\Delta(x) = 0$ random-bond (RB)

$\int dx\,\Delta(x) > 0$ random-field (RF)

- Martin-Siggia-Rose (MSR) dynamical action: $\overline{\langle \mathcal{O}(t)\rangle} = \int \mathcal{D}u_{zt}\int \mathcal{D}\hat{u}_{zt}\,\mathcal{O}[u,\hat{u}]\,e^{-S[u,\hat{u}]}$

$$S[u,\hat{u}] = \int_{zt} i\hat{u}_{zt}(\gamma\partial_t - c\partial_z^2)u_{zt} - \frac{1}{2}\int_{ztt'} i\hat{u}_{zt}\,i\hat{u}_{zt'}\,\Delta(u_{zt}-u_{zt'}) - f_{\rm ext}\int_{zt} i\hat{u}_{zt} + \gamma T\int_{zt} i\hat{u}_{zt}\,i\hat{u}_{zt}$$

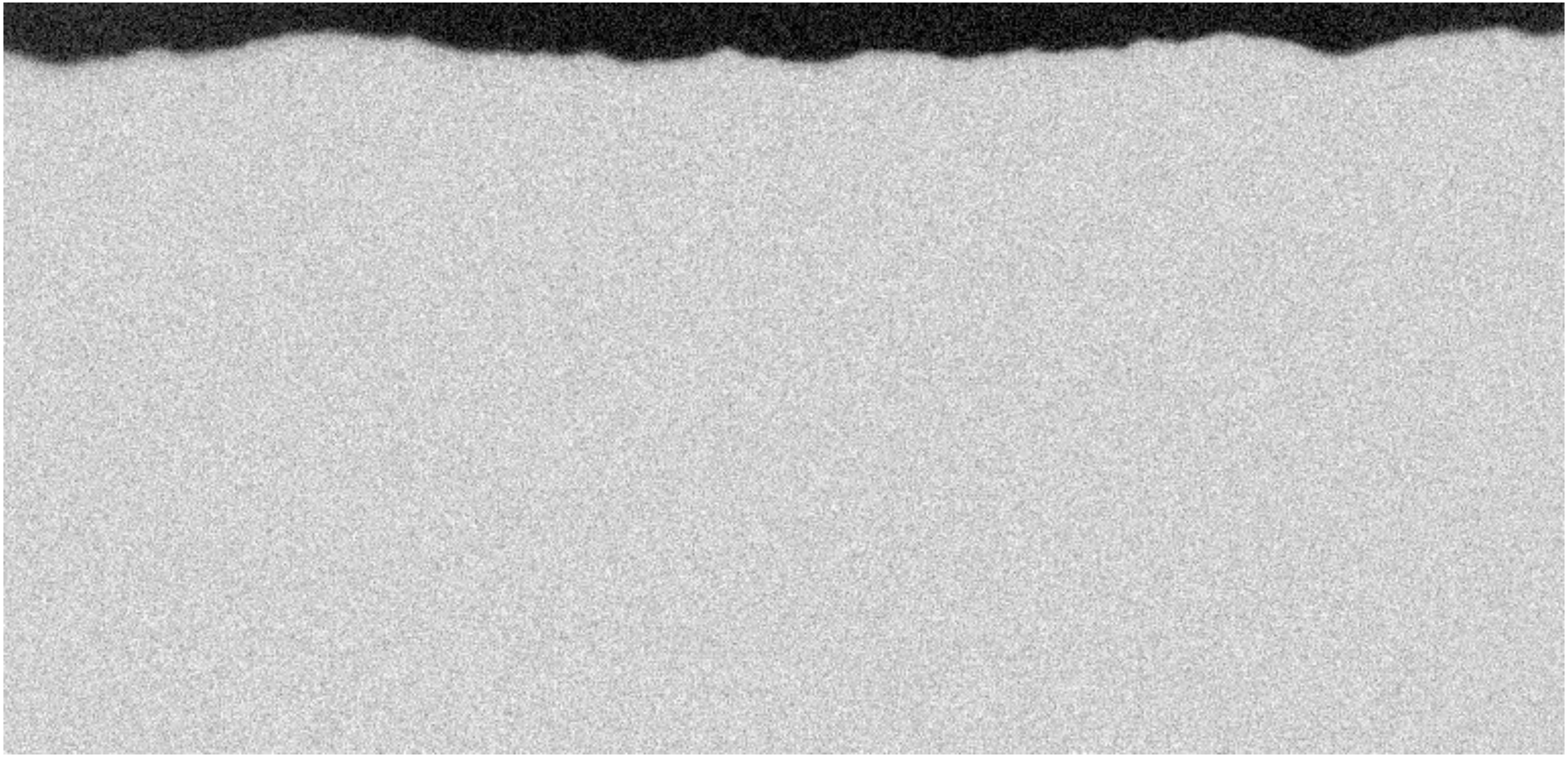
Force-force correlator = central object for FRG

Experimental realisation: moving ferromagnetic domain wall

- Here focus on 'quenched Edwards-Wilkinson' (qEW):

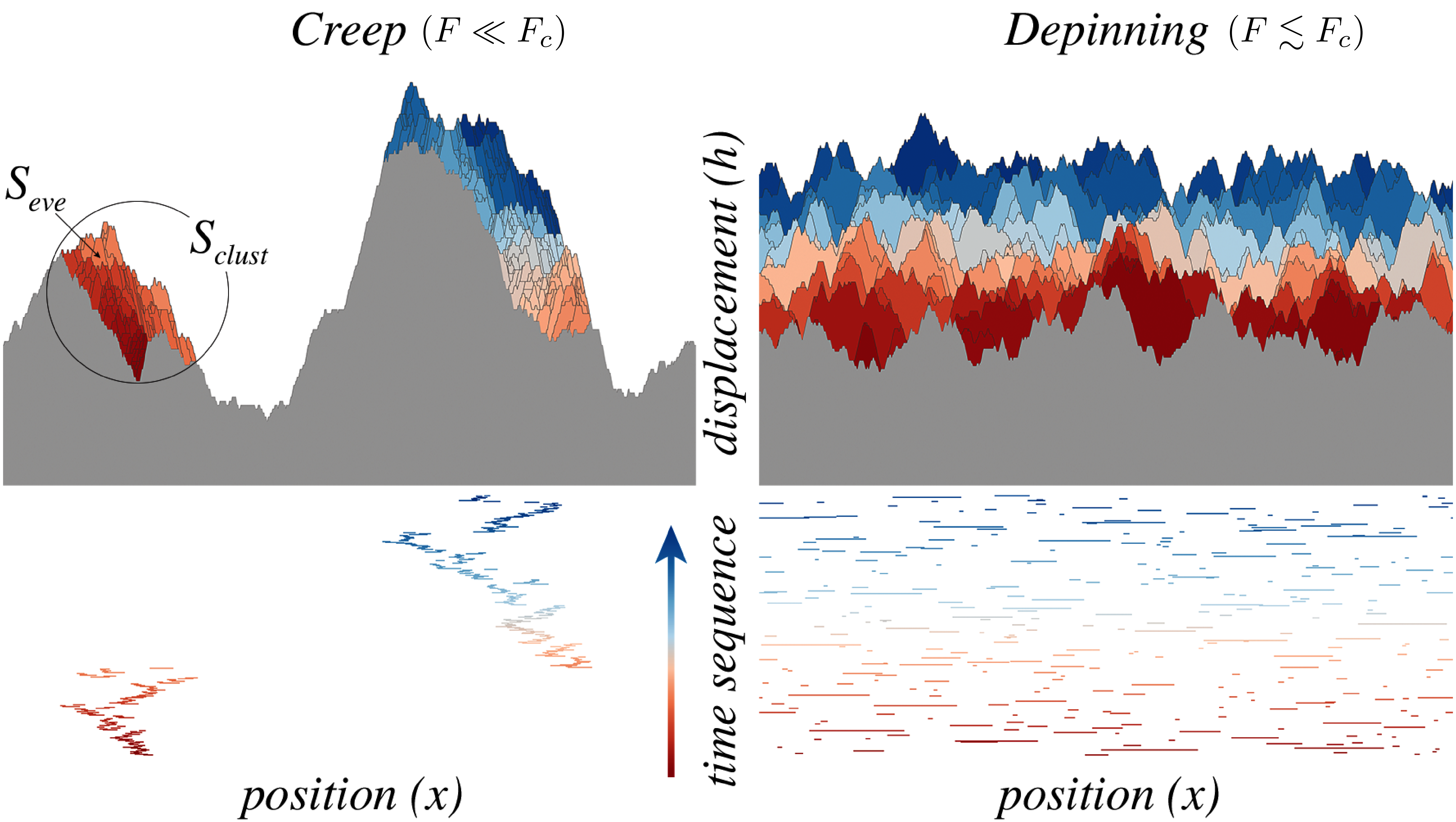
$$\gamma\,\partial_t u(z,t) = c\partial_z^2 u(z,t) + F_{\rm dis}(z,u(z)) + f_{\rm ext} + \eta_{\rm thermal}(z,t)$$

- V. Repain et al. @ Orsay (Pt/Co/Pt)



Avalanches organisation & statistics from creep to depinning

E. Ferrero, L. Foini, T. Giamarchi, A. B. Kolton, & A. Rosso, *Phys. Rev. Lett.* 118, 147208 (2017): "Spatio-temporal patterns in ultra-slow domain wall creep dynamics", *cf. Figs. 1&3.*



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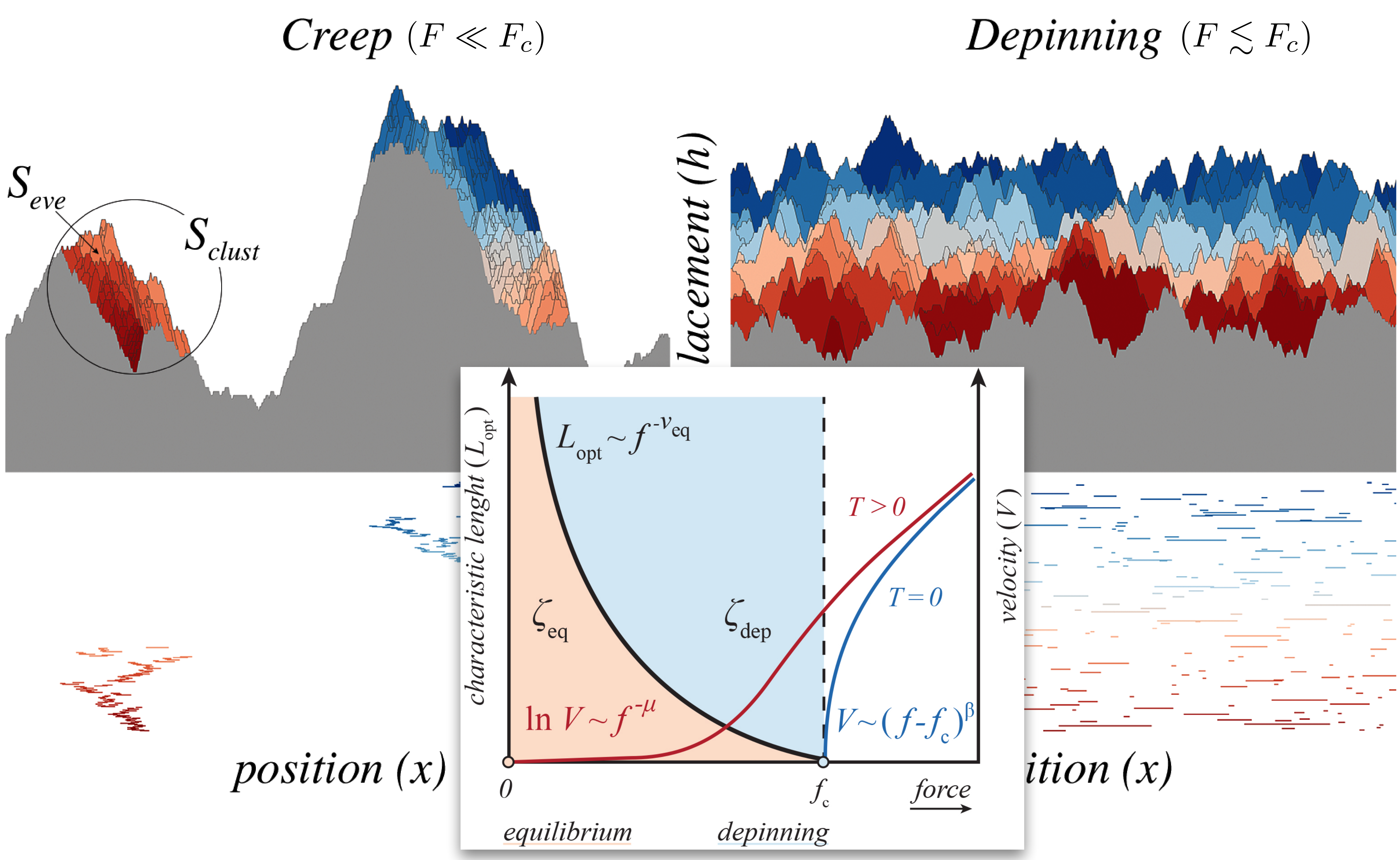
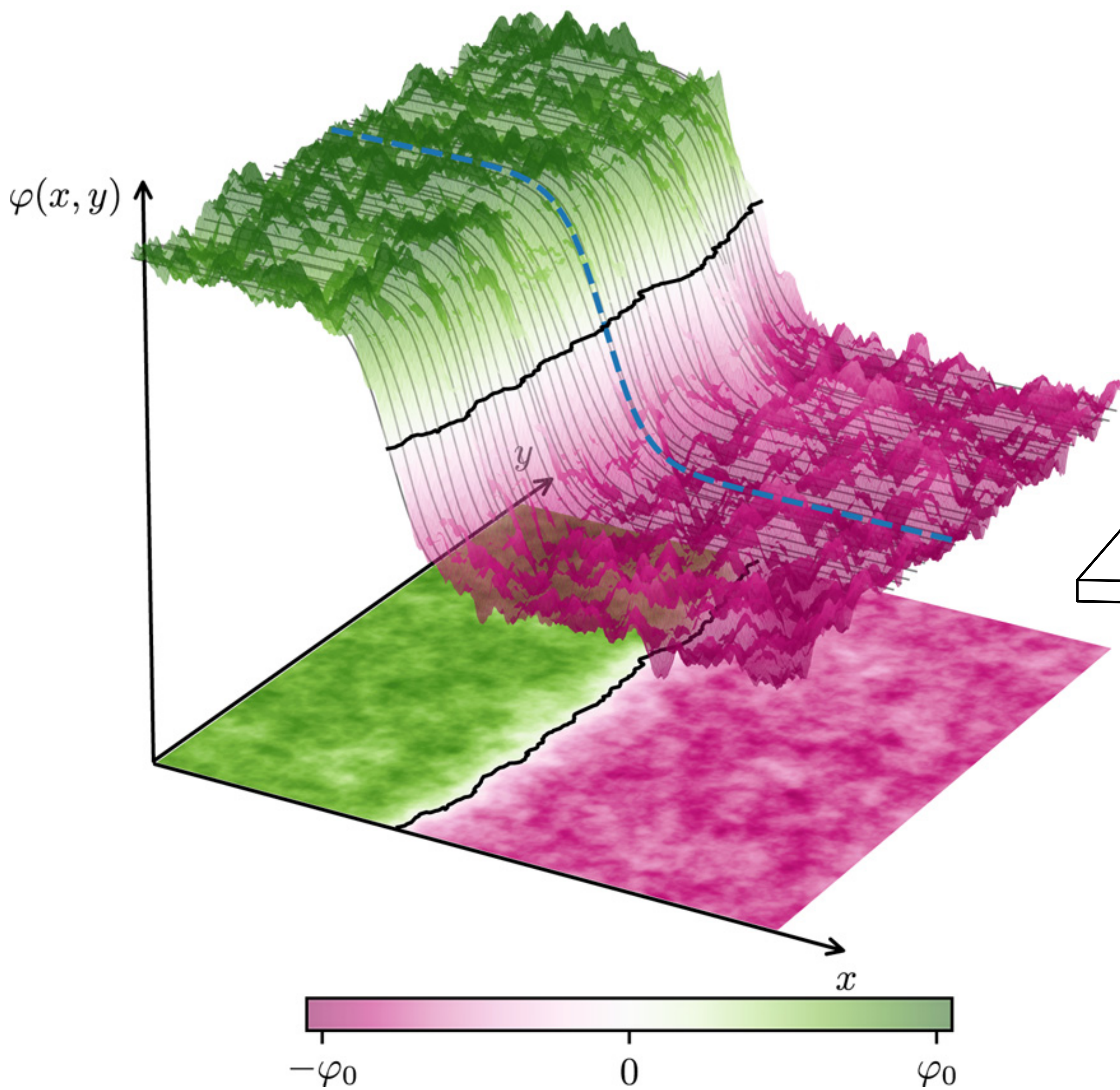
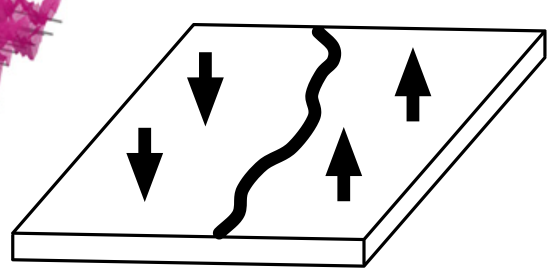


Model reduction at equilibrium from 2D Ginzburg-Landau to 1D qEW

- Let's start from a 2D Ginzburg-Landau description of the magnetisation in thin films **[no disorder]**





N. Caballero, E. Agoritsas, V. Lecomte, T. Giamarchi, *Phys. Rev. B* 102, 104204 (2020).

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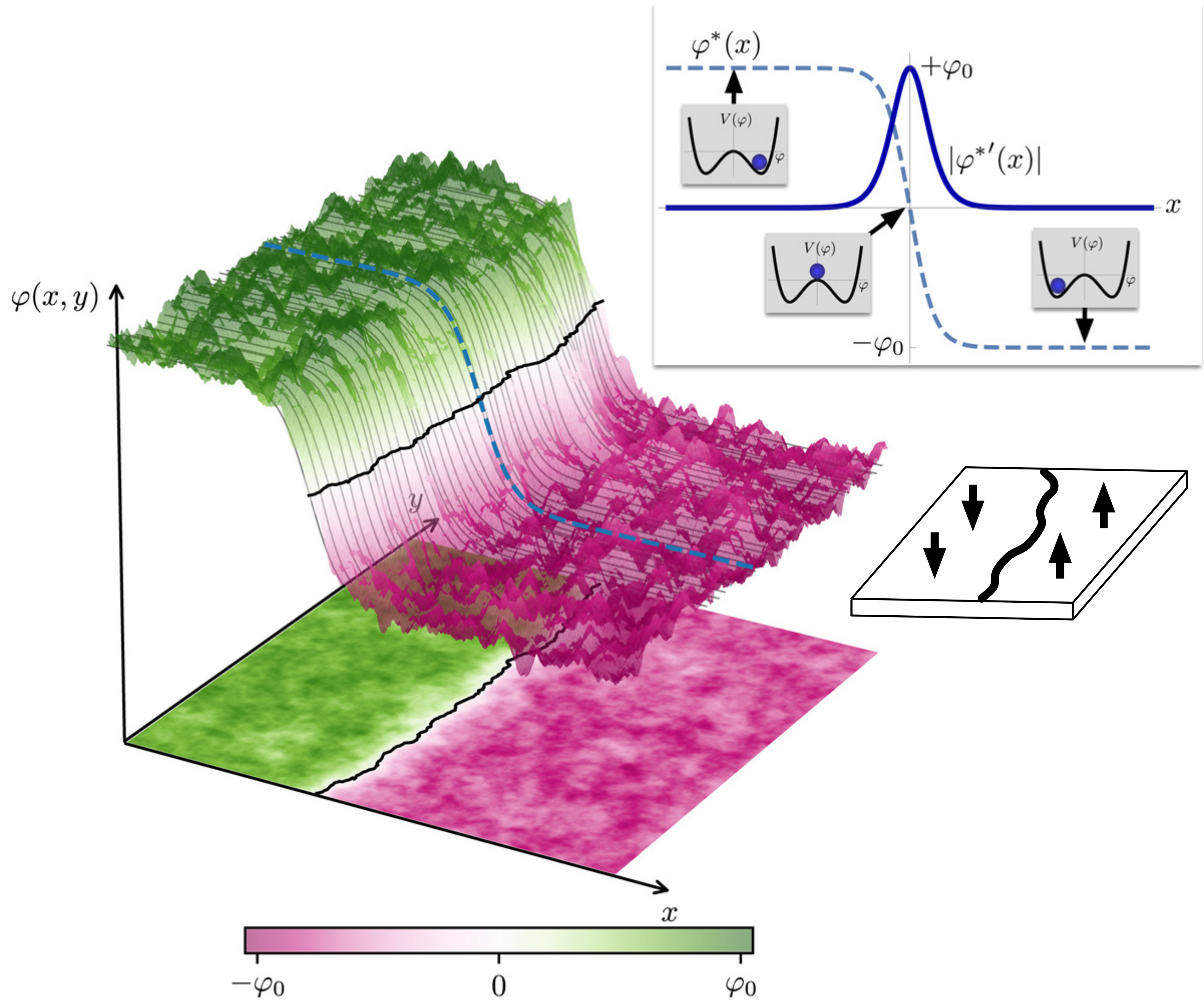

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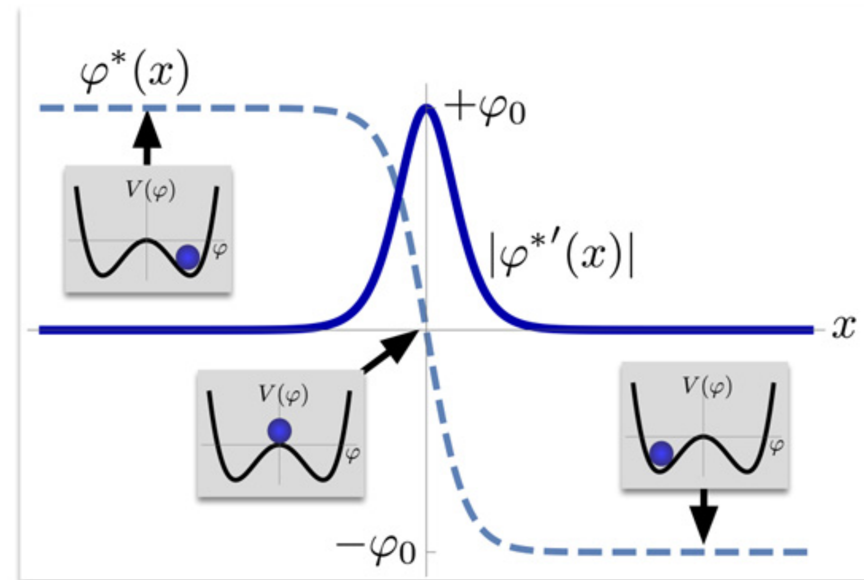
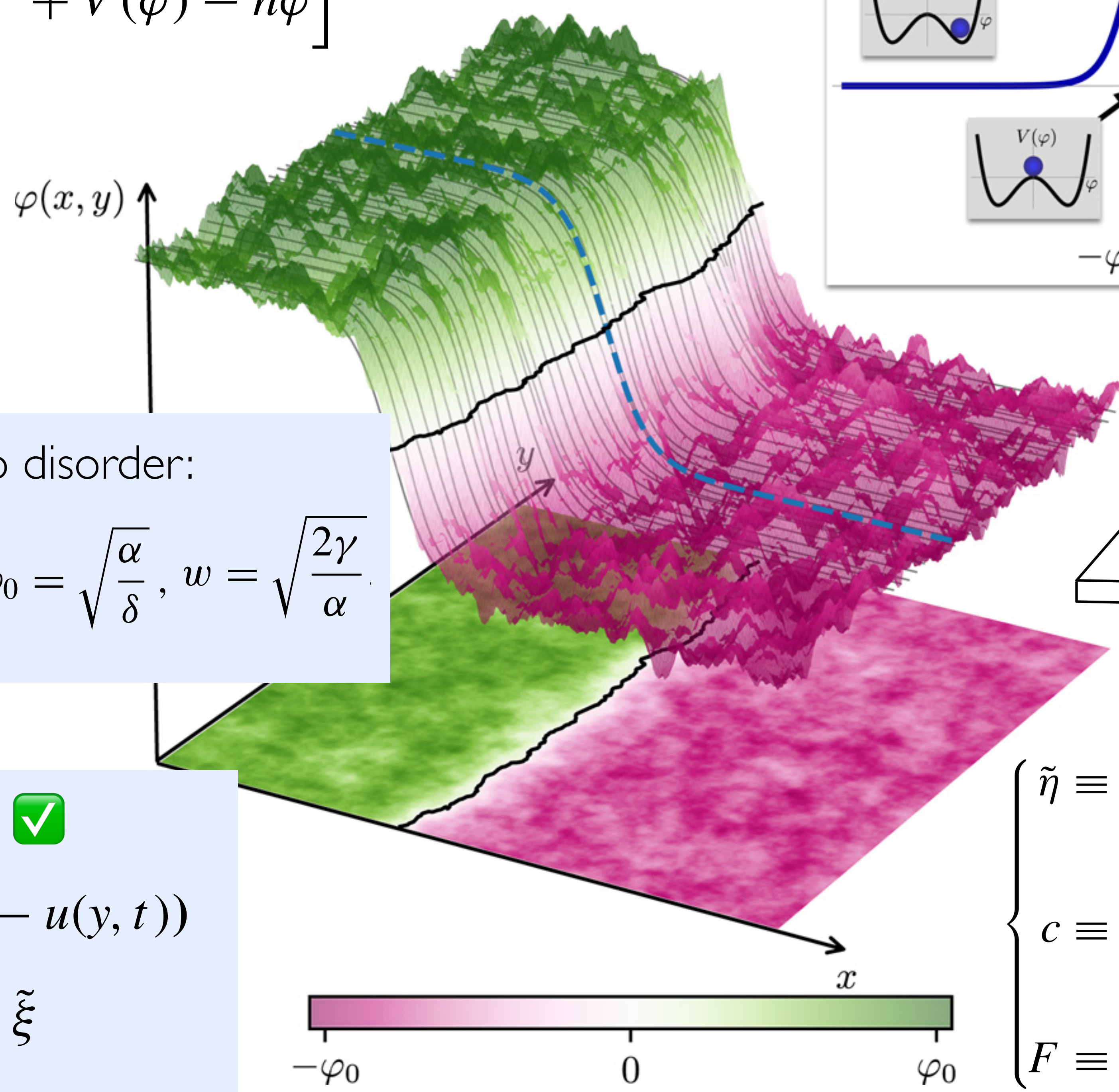
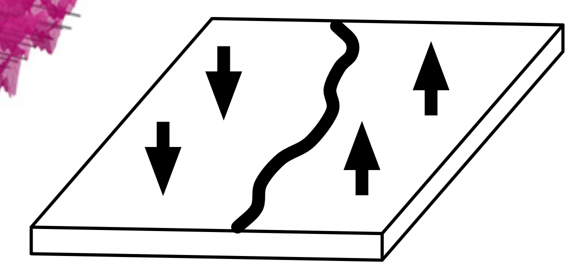
Model reduction at equilibrium from 2D Ginzburg-Landau to 1D qEW

- Let's start from a 2D Ginzburg-Landau description of the magnetisation in thin films **[no disorder]**

$$\begin{cases} \eta \partial_t \varphi = -\dfrac{\delta \mathcal{H}_{\mathrm{GL}}[\varphi]}{\delta \varphi} + \xi \qquad \text{“}\langle \xi \xi \rangle = 2\eta T \, \delta \, \delta\text{”} \\ \mathcal{H}_{\mathrm{GL}}[\varphi] = \displaystyle\int d\mathbf{r} \left[\frac{\gamma}{2} |\nabla_{\mathbf{r}} \varphi|^2 + V(\varphi) - h\varphi \right] \\ V(\varphi) = -\dfrac{\alpha}{2}\varphi^2 + \dfrac{\delta}{4}\varphi^4 \end{cases}$$







- Ideal profile at T=0/h=0/no disorder:

$$\boxed{\varphi^*(x) = -\varphi_0 \tanh\left(\frac{x}{w}\right)} \quad \varphi_0 = \sqrt{\frac{\alpha}{\delta}}, \; w = \sqrt{\frac{2\gamma}{\alpha}}$$

- Model reduction onto EW ✅

<u>Ansatz</u>: $\varphi(x, y, t) = \varphi^*(x - u(y, t))$

$$\tilde{\eta} \partial_t u = c \partial_y^2 u + F + \tilde{\xi}$$

$$\begin{cases} \tilde{\eta} \equiv \eta \mathcal{N}_1 = \eta \dfrac{2\sqrt{2}}{3} \dfrac{\alpha}{\delta} \sqrt{\dfrac{\alpha}{\gamma}} \\ c \equiv \gamma \mathcal{N}_1 = \dfrac{2\sqrt{2}}{3} \dfrac{\alpha}{\delta} \sqrt{\alpha \gamma} \\ F \equiv h \mathcal{N}_3 = -2\sqrt{\dfrac{\alpha}{\delta}} h. \end{cases}$$

N. Caballero, E. Agoritsas, V. Lecomte, T. Giamarchi, *Phys. Rev. B* <u>102</u>, 104204 (2020).

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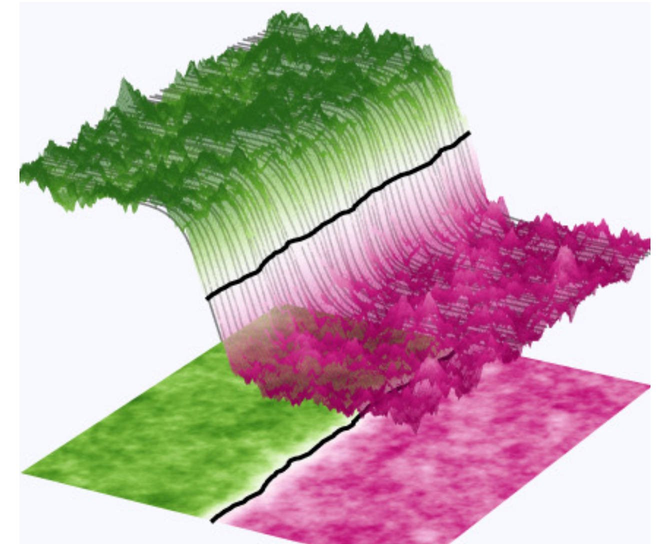
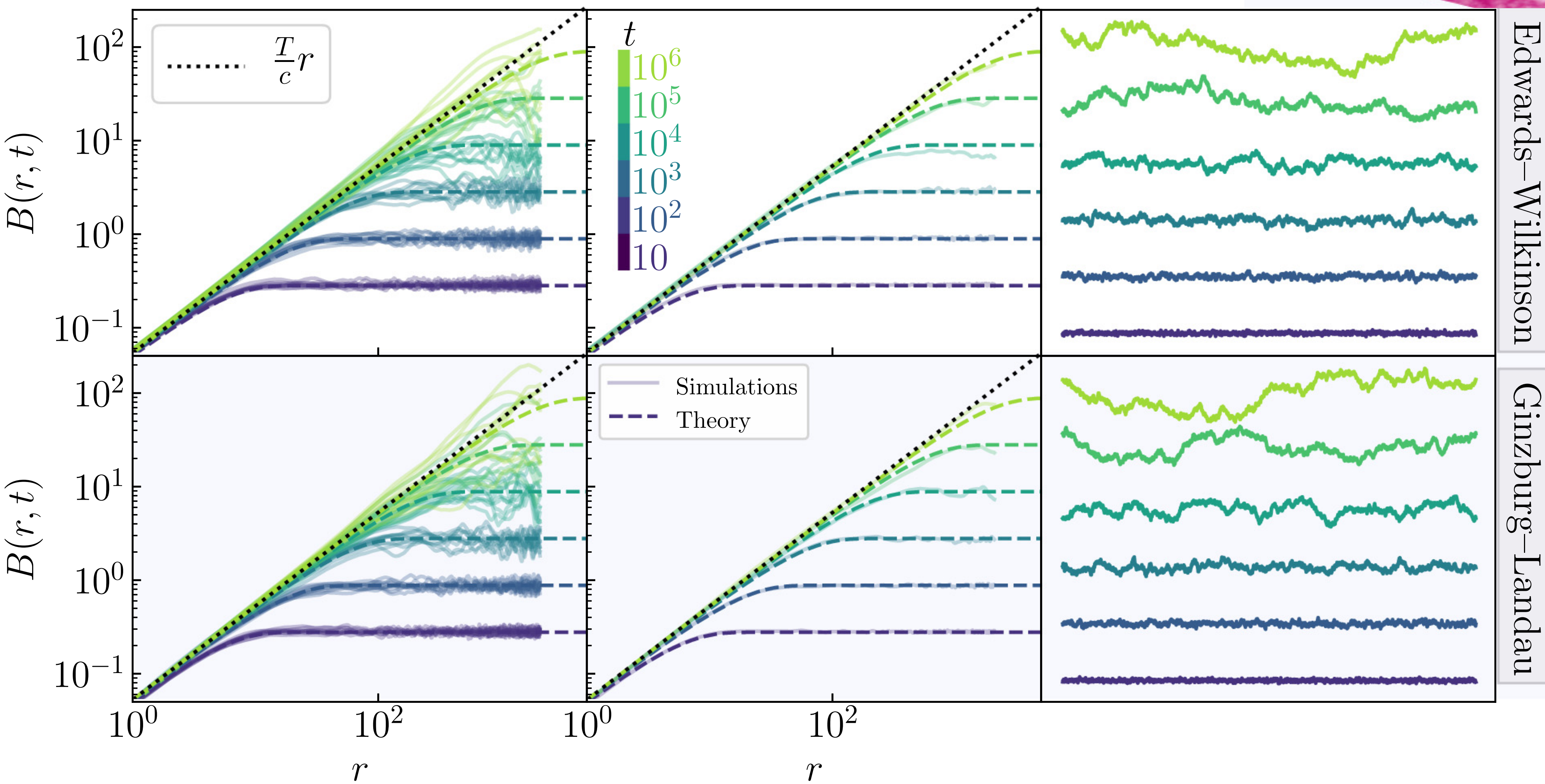


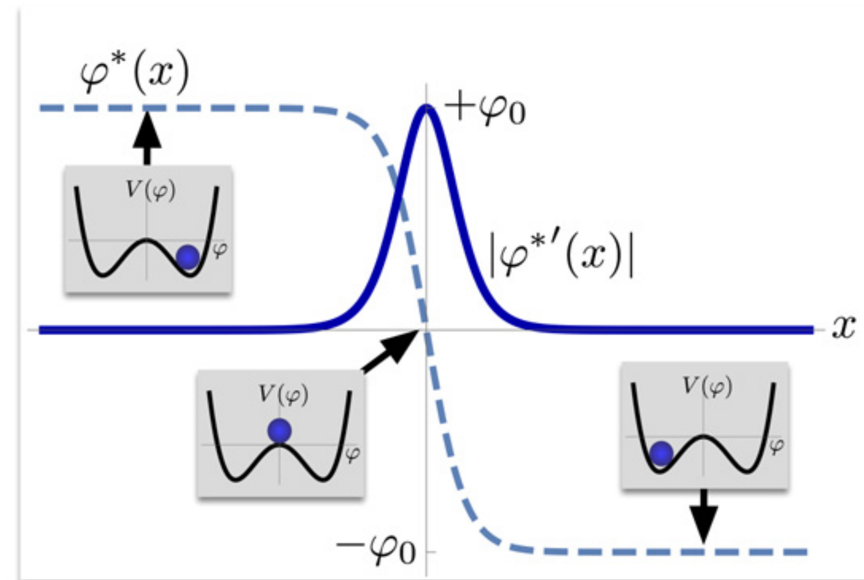


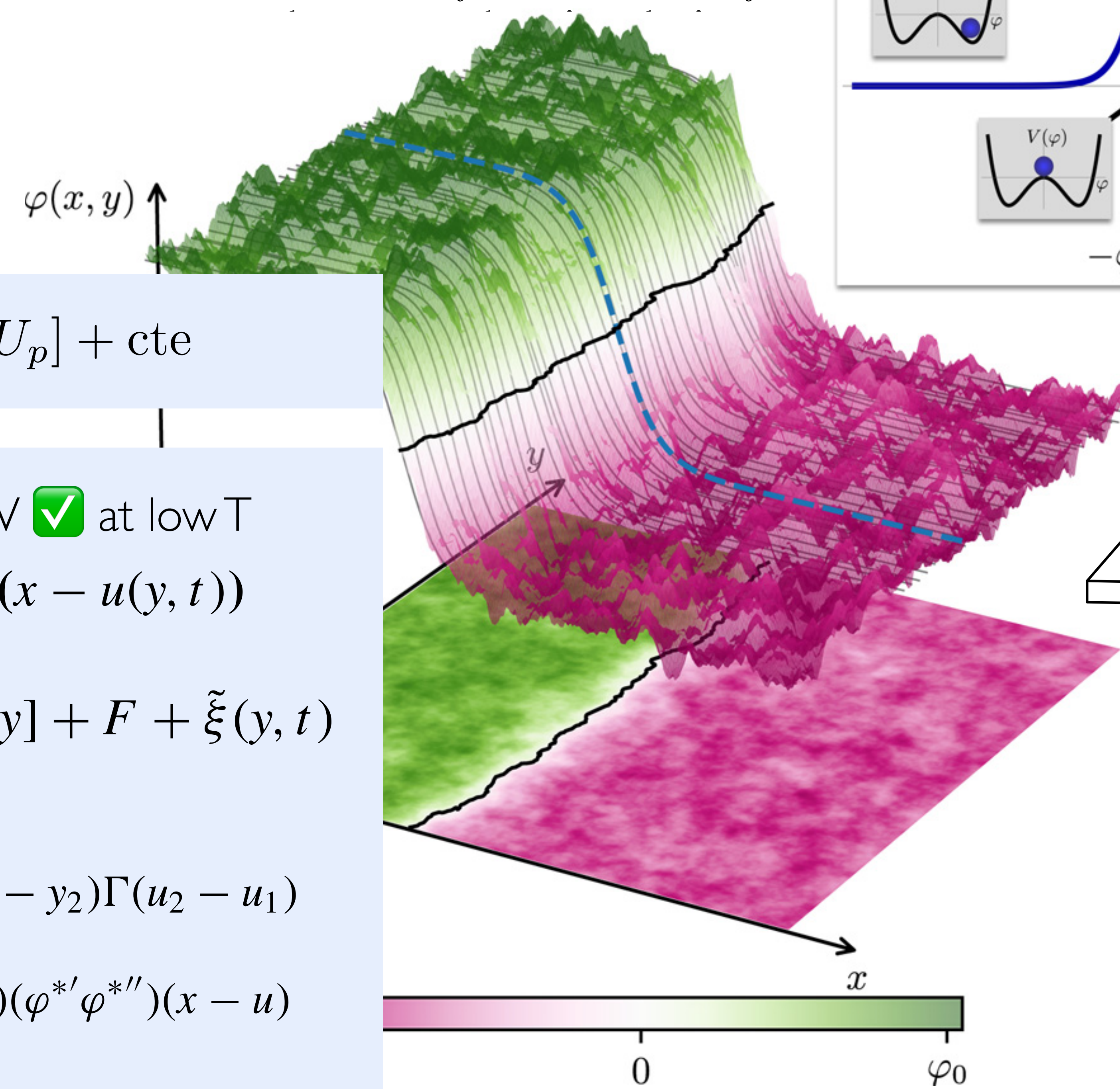
N. Caballero, E. Agoritsas, V. Lecomte, T. Giamarchi, *Phys. Rev. B* 102, 104204 (2020).

Model reduction at equilibrium from 2D Ginzburg-Landau to 1D qEW

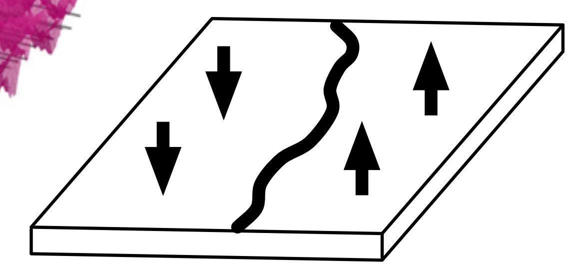
- Let's allow for fluctuations of the barrier height in the local double-well potentials **[with RB disorder]**

$$\begin{cases} V(\varphi) = -\frac{\alpha}{2}\varphi^2 + \frac{\delta}{4}\varphi^4 \\ V_\zeta(\varphi(\mathbf{r})) = V(\varphi(\mathbf{r}))[1+\epsilon\zeta(\mathbf{r})] \end{cases} \qquad \overline{\zeta(\mathbf{r}_i)\zeta(\mathbf{r}_j)} = \delta^2(\mathbf{r}_i - \mathbf{r}_j)$$





$$\mathcal{H}_{\mathrm{GL}}[\varphi_u,\zeta] = \mathcal{H}_{\mathrm{DES}}[u,U_p] + \mathrm{cte}$$



- Model reduction onto qEW ✅ at low T

<u>*Ansatz*</u>: $\varphi(x,y,t) = \varphi^*(x-u(y,t))$

$$\tilde{\eta}\partial_t u = c\partial_y^2 u + F_p[u(y,t),y] + F + \tilde{\xi}(y,t)$$

$$\begin{cases} \overline{F_p(u_1,y_1)F_p(u_2,y_2)} = \epsilon^2\delta(y_1-y_2)\Gamma(u_2-u_1) \\ \Gamma(u) = \gamma^2\int_{-\infty}^{\infty} dx\,(\varphi^{*\prime}\varphi^{*\prime\prime})(x)(\varphi^{*\prime}\varphi^{*\prime\prime})(x-u) \end{cases}$$

N. Caballero, E. Agoritsas, V. Lecomte, T. Giamarchi, *Phys. Rev. B* <u>102</u>, 104204 (2020).

Model reduction at equilibrium from 2D Ginzburg-Landau to 1D qEW

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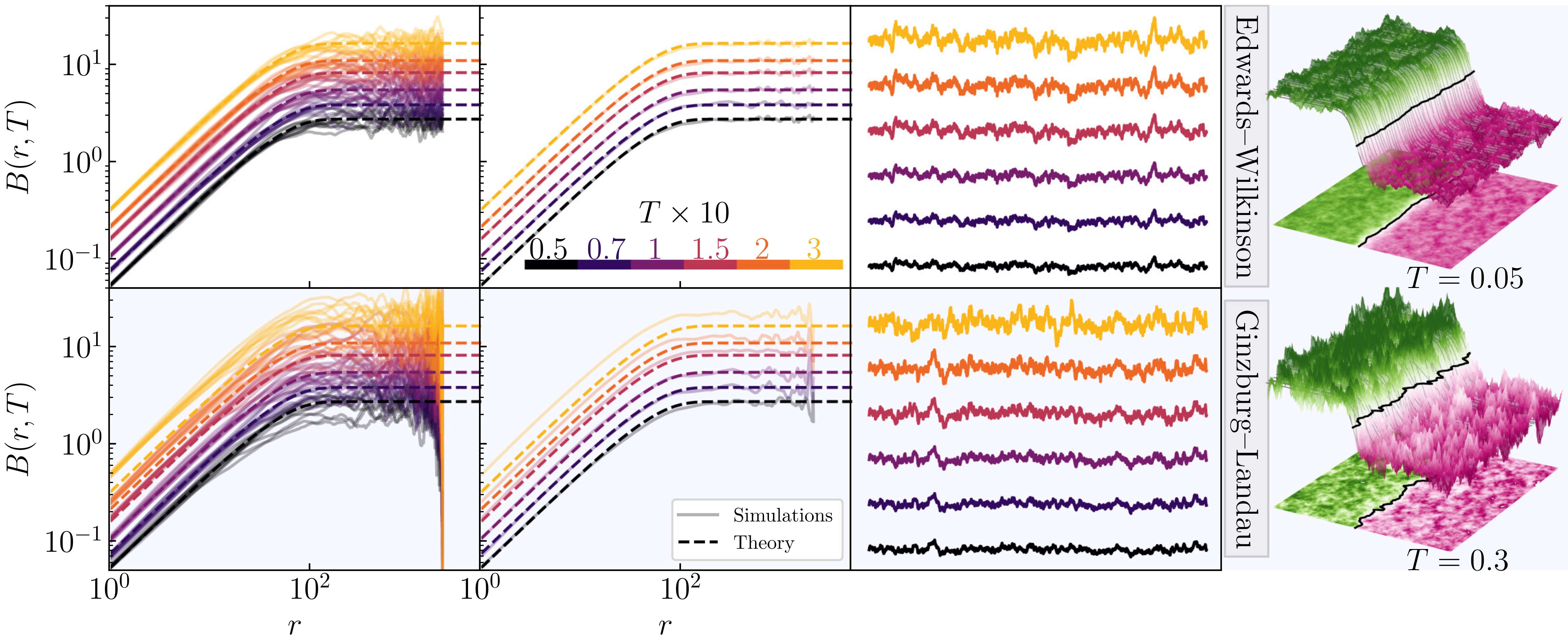


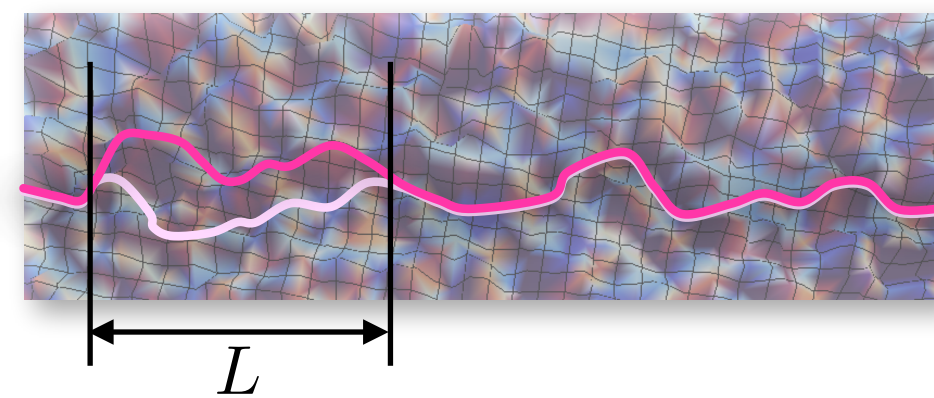
- ⚠️ **Increasing temperature leads to overhangs** ⇒ N. Caballero & T. Giamarchi, arXiv:2211.12258

N. Caballero, E. Agoritsas, V. Lecomte, T. Giamarchi, *Phys. Rev. B* 102, 104204 (2020).

Quasistatic ‘creep’ regime - Standard scaling argument

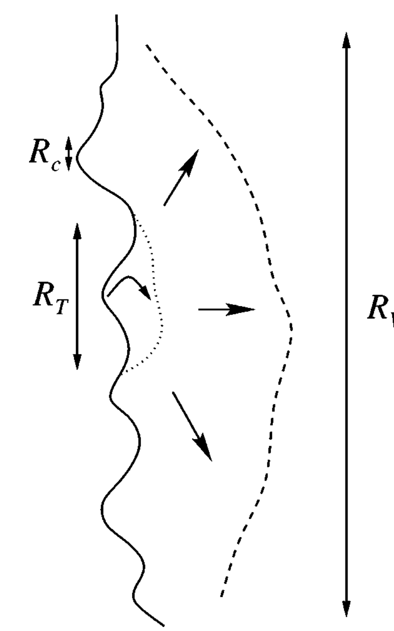
- Focus on low temperature T / small force f / large system size L: creep prediction?

- Scaling argument initially presented in L. B. Ioffe & V. M. Vinokur, *J. Phys. C* 20, 6149 (1987)
  T. Nattermann, *Phys. Rev. Lett.* 64, 2454 (1990)



- *Quasistatic assumption*: in order to move a segment of length *L* of the interface, we can estimate the energy barrier to cross from the *equilibrium* free energy.
  - Typical transverse displacement deduced from the roughness at equilibrium: $u(L) \sim L^{\zeta}$
  - Elastic (free-)energy associated to this displacement: $E_{\mathrm{el}}(L) \sim L^d \cdot c \dfrac{u(L)^2}{L^2} \sim L^{d-2+2\zeta}$
  - Under an external force, corresponding (free-)energy: $E_f(L) \sim f L^d u(L) \sim f L^{d+\zeta}$
  - Minimum size *L* for which it is worth to overcome a barrier, thus depends on the force:

$$E_{\mathrm{el}}(L_{\mathrm{opt}}) = E_f(L_{\mathrm{opt}}) \Leftrightarrow \boxed{L_{\mathrm{opt}}(f) \sim f^{-(2-\zeta)}} \Leftrightarrow \boxed{E_{\mathrm{el}}(L_{\mathrm{opt}}(f)) \sim f^{-\mu}}$$



- Mean steady-state velocity controlled by the typical ‘largest barrier’, which controls the mean first passage time (MFPT). Under an Arrhenius assumption:

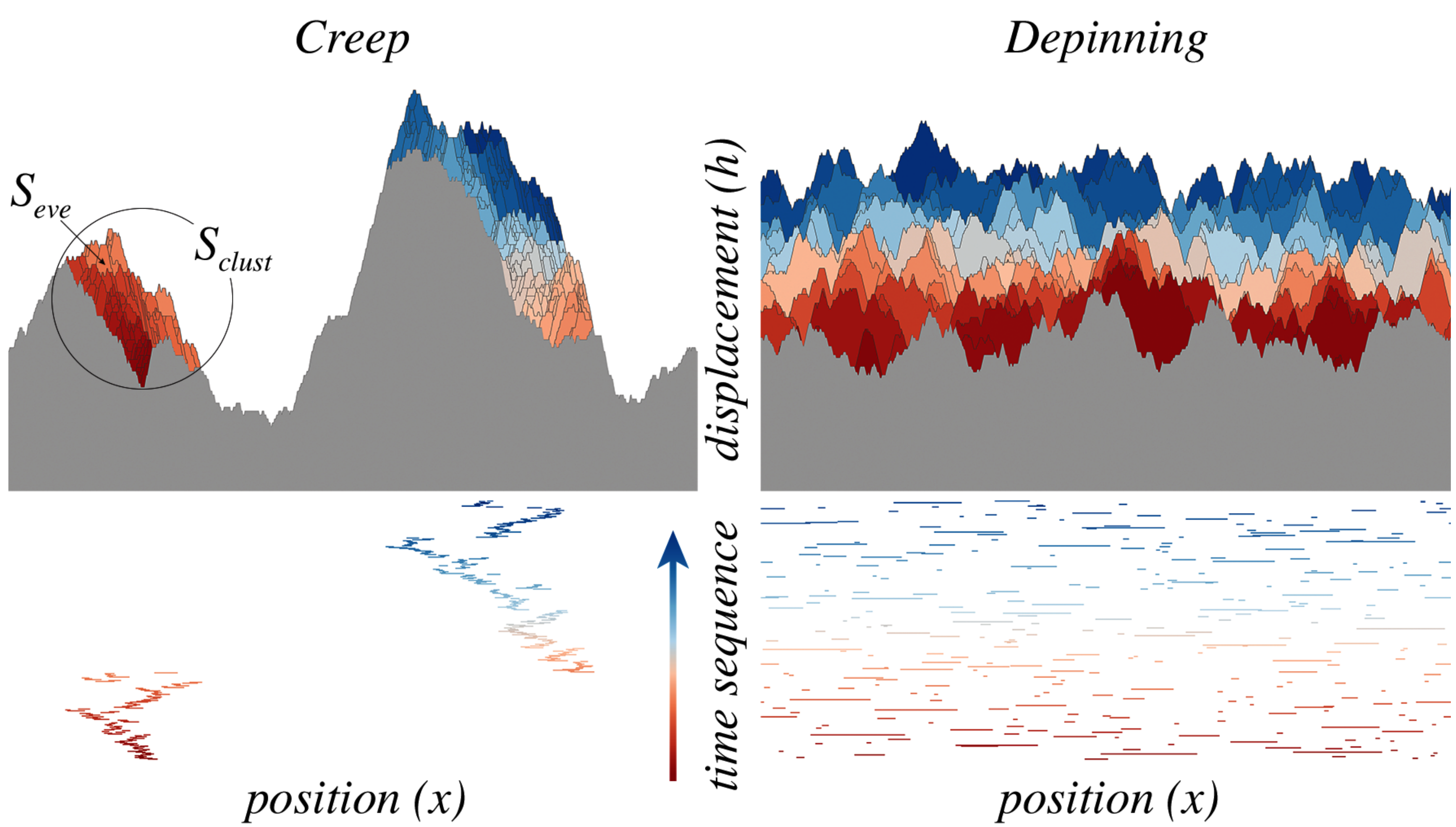
$$\boxed{v_T(F) \sim e^{-\frac{1}{T} E_{\mathrm{el}}(L_{\mathrm{opt}}(F))} \sim e^{-\frac{U_c}{T}\left(\frac{F_c}{F}\right)^{\mu}}} \qquad \mu = \frac{d-2+2\zeta}{2-\zeta} \overset{(\zeta=2/3)}{=} \frac{1}{4}$$

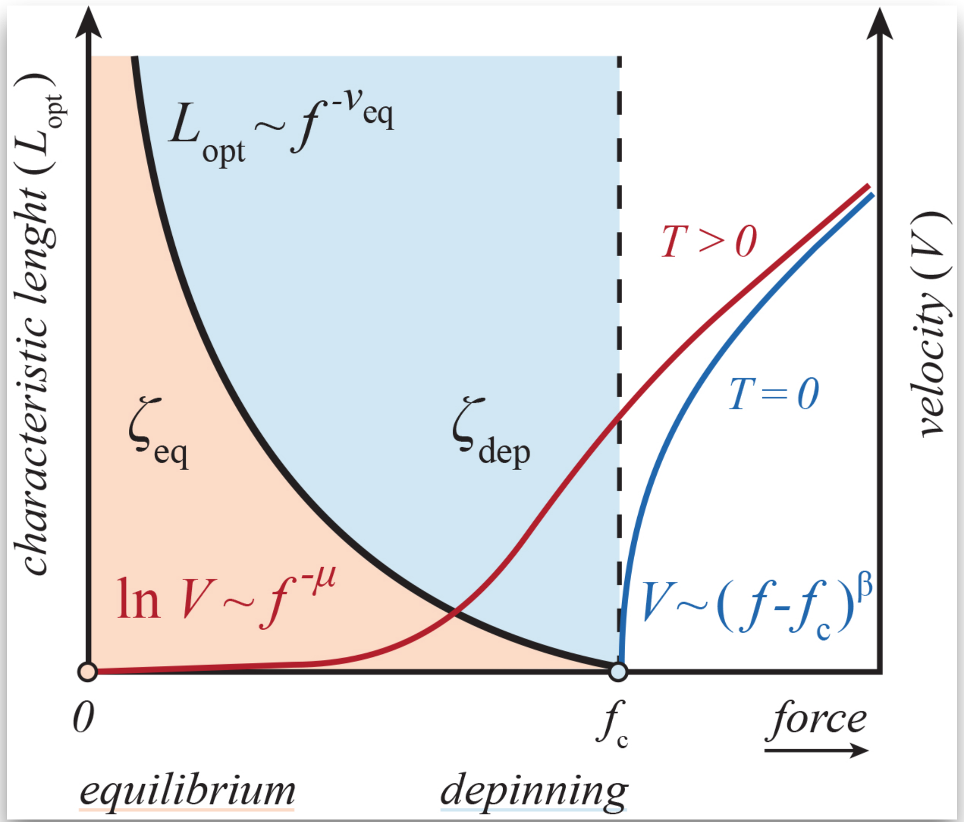
P. Chauve, T. Giamarchi, P. Le Doussal, *Phys. Rev. B* 62, 6241 (2000) **[very nice intro as well!]**

E. Ferrero, L. Foini, T. Giamarchi, A. B. Kolton, A. Rosso, *Annu. Rev. Cond. Math. Phys.* 12, 111 (2021) **[recent review]**

Quasistatic 'creep' regime — Numerical test for the assumptions for scaling

E. Ferrero, L. Foini, T. Giamarchi, A. B. Kolton, & A. Rosso, *Phys. Rev. Lett.* 118, 147208 (2017): "Spatio-temporal patterns in ultra-slow domain wall creep dynamics", *cf. Figs. 1&3.*



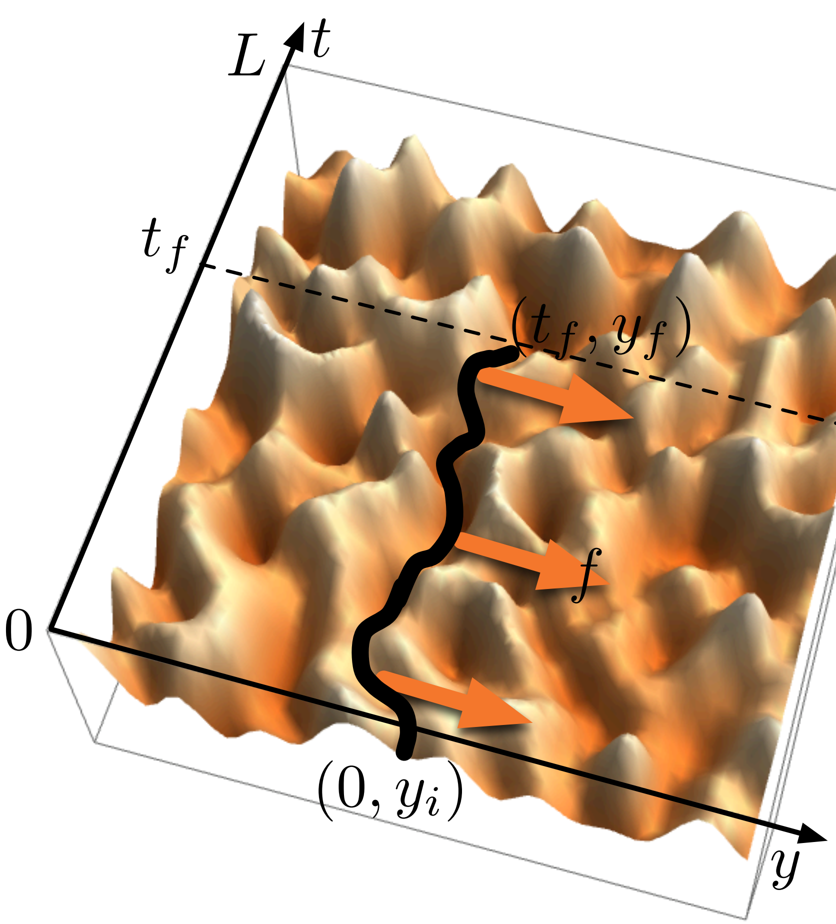


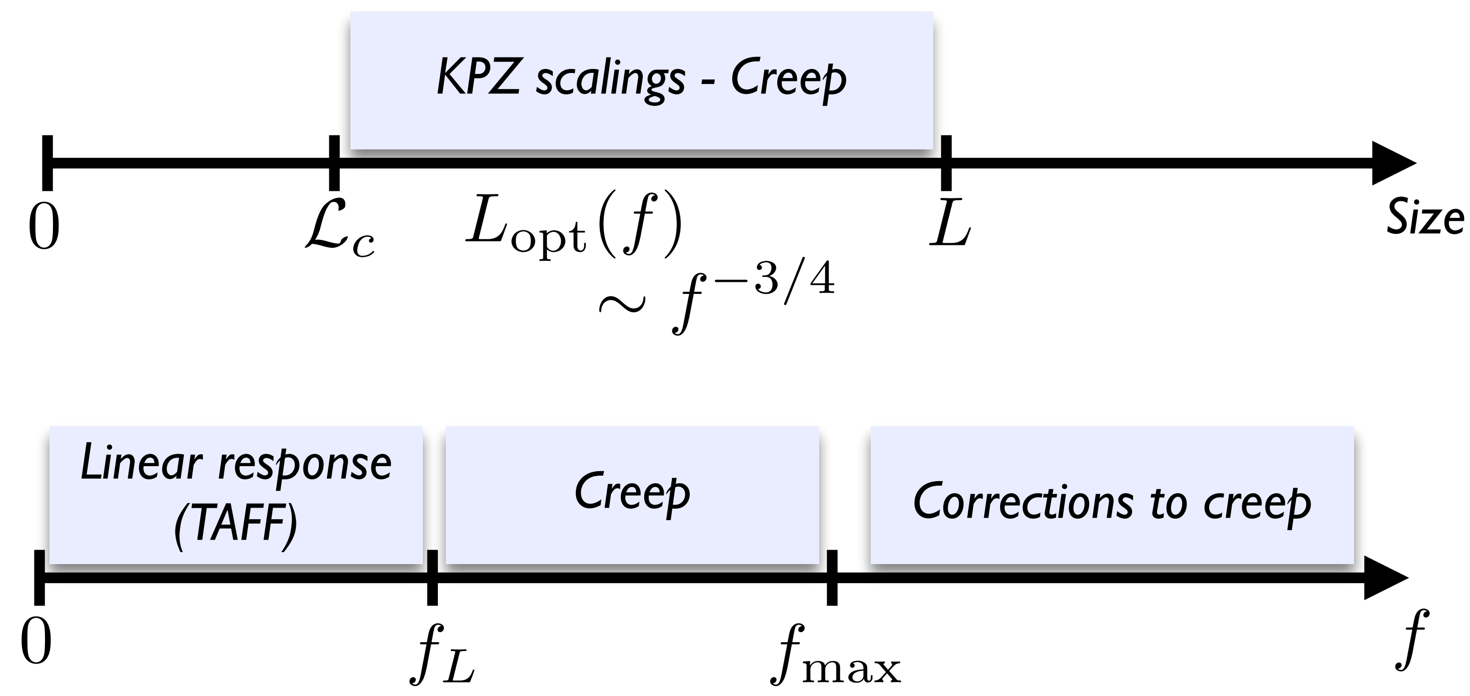
(with roughness exponent ζ)

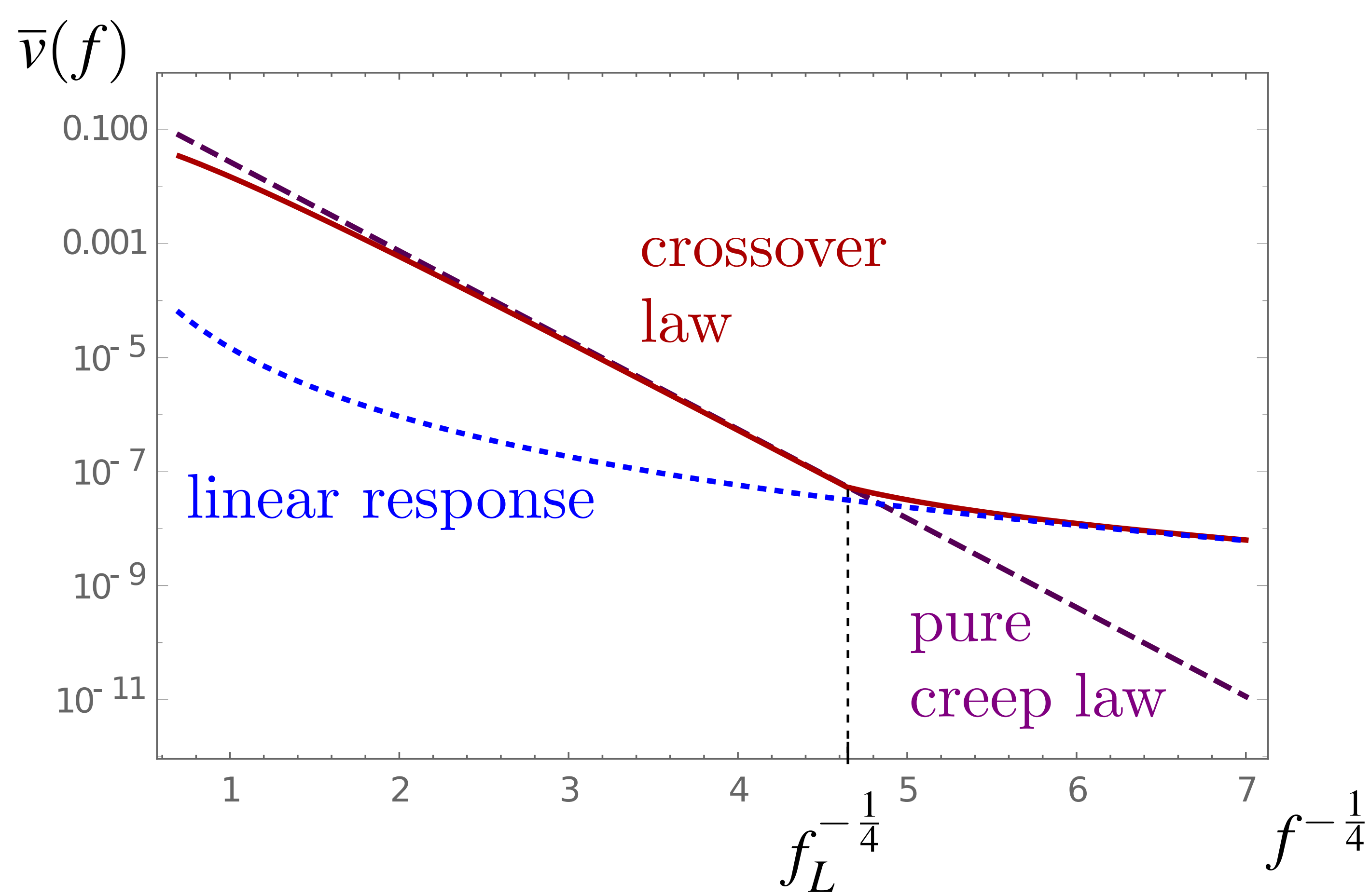
- New numerical protocol, allowing to reach much smaller forces than before (⇒ 'creep'!)
- Avalanche statistics & spatio-temporal patterns, from 'creep' to 'depinning'

- Possible to test the assumptions used in standard scaling arguments!
- In particular: broad distribution of energy barriers, with force-dependent cutoff being the 'bottleneck' governing the mean velocity in the steady state (hence the 'creep' formula)

Quasistatic 'creep' regime — Regime of validity of the creep prediction







E. Agoritsas, R. García-García, V. Lecomte, L. Truskinovsky, & D. Vandembroucq, *J. Stat. Phys.* 164, 1394 (2016).

Quasistatic 'creep' regime - Model reduction?

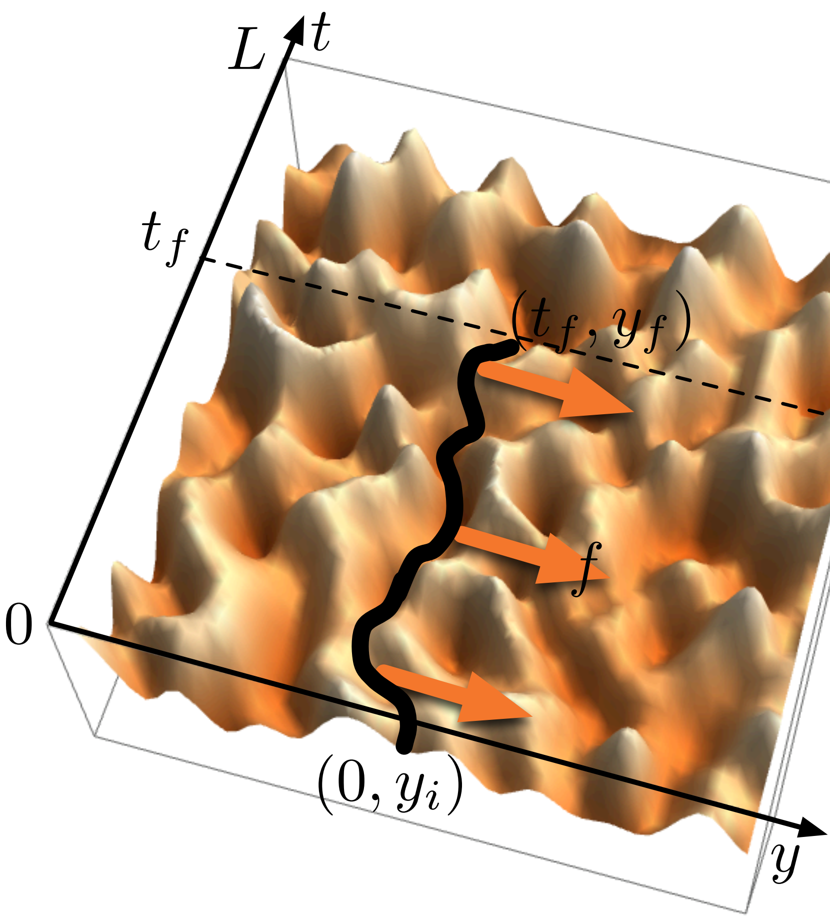
- Focus on low temperature T / small force f / large system size L: creep prediction?

$$v_T(F) \sim e^{-\frac{U_c}{T}\left(\frac{F_c}{F}\right)^{\mu}}$$

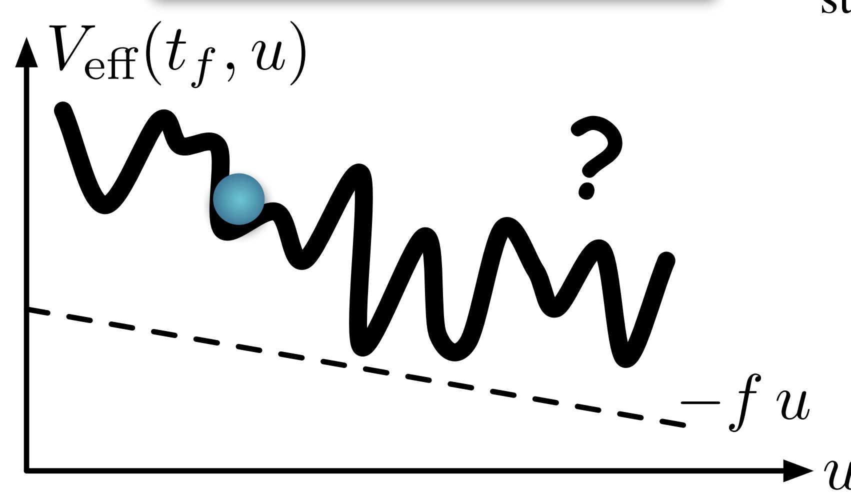
1D interface, short-range elasticity (elastic limit), random-bond disorder

$$\mu = \frac{d-2+2\zeta}{2-\zeta} \overset{(\zeta=2/3)}{=} \frac{1}{4}$$

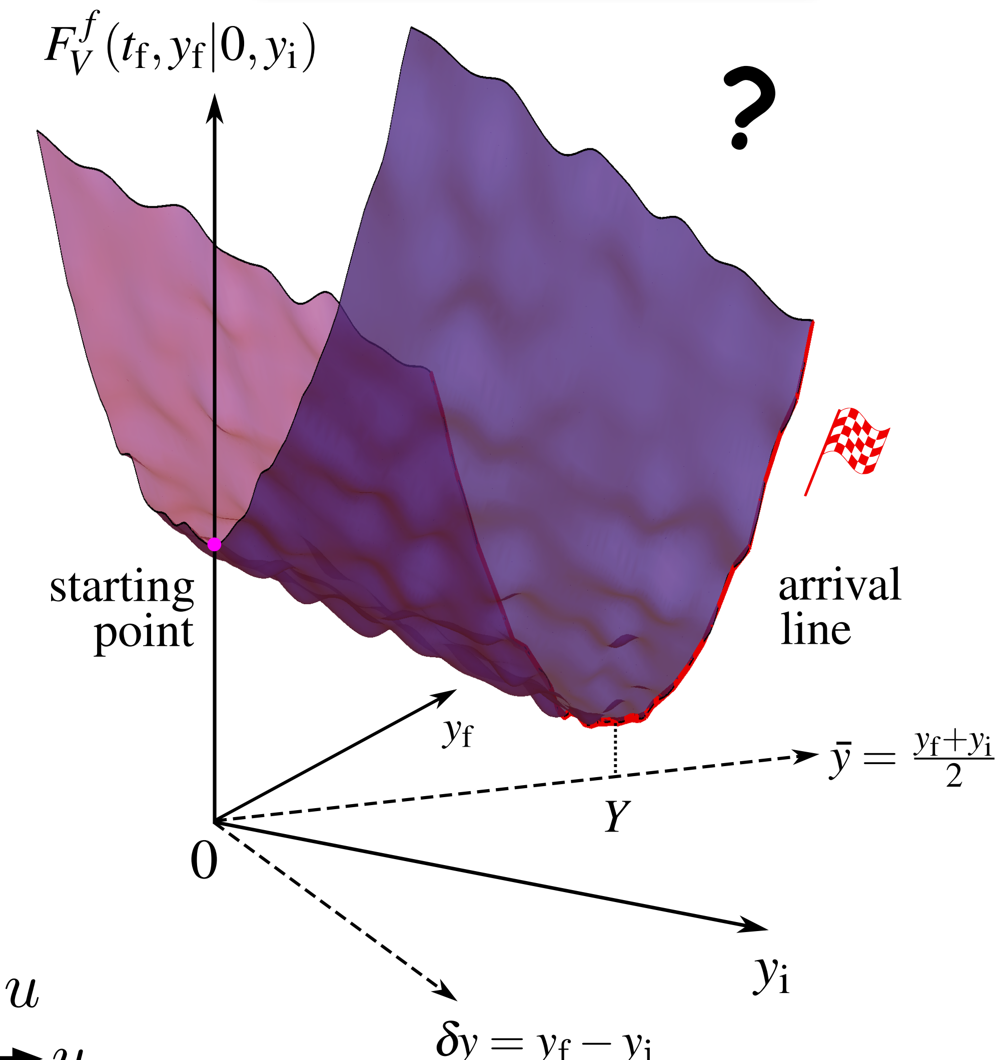
- 1D elastic interface in a 2D disordered energy landscape



1 DOF: center of mass?



2 DOFs: center of mass & elasticity?



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Quasistatic 'creep' regime - Model reduction?

- Focus on low temperature T / small force f / large system size L: creep prediction?

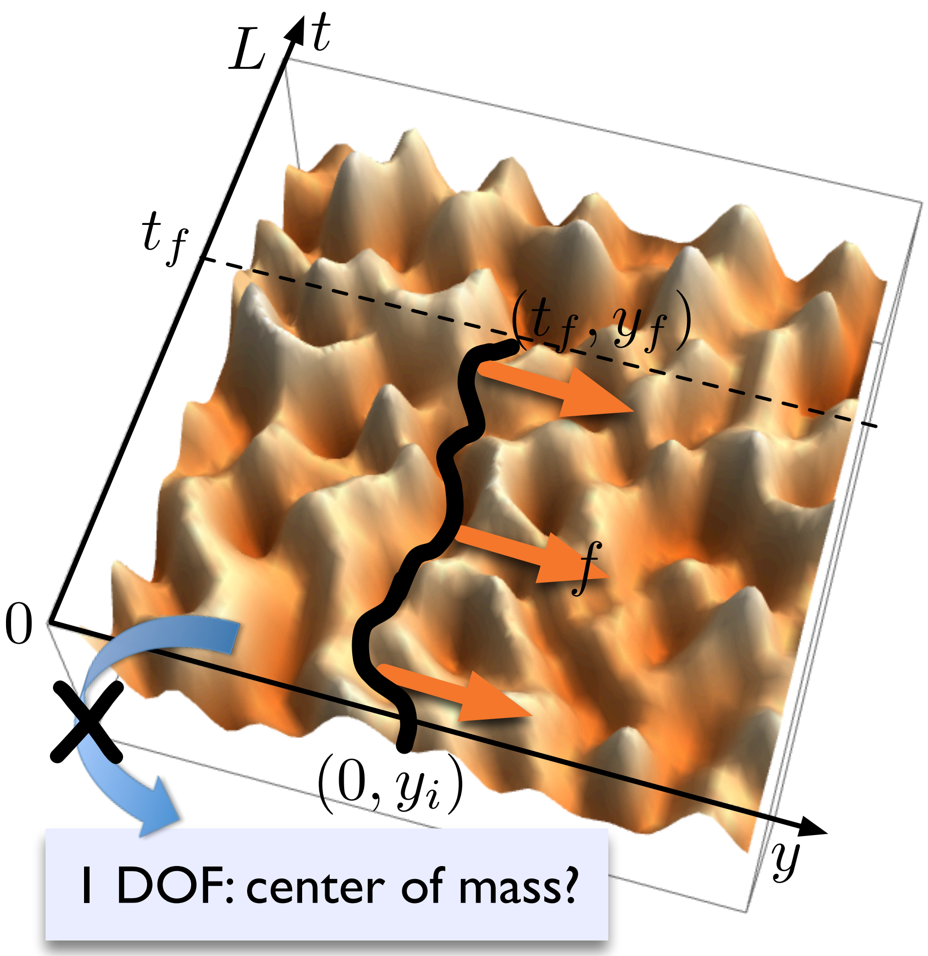
$$v_T(F) \sim e^{-\frac{U_c}{T}\left(\frac{F_c}{F}\right)^{\mu}}$$

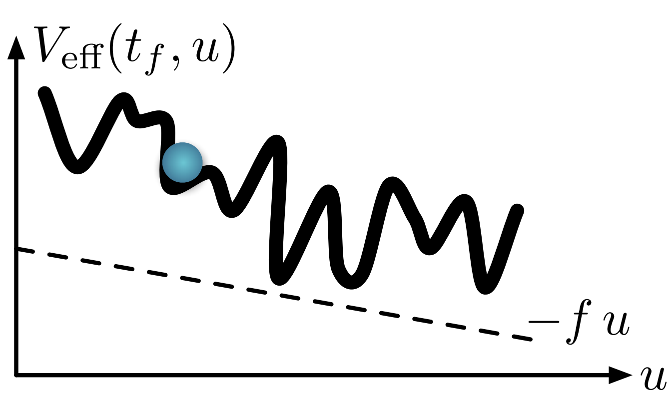
1D interface, short-range elasticity (elastic limit), random-bond disorder

$$\mu = \frac{d-2+2\zeta}{2-\zeta} \stackrel{(\zeta=2/3)}{=} \frac{1}{4}$$

- 1D elastic interface in a 2D disordered energy landscape

Why revisiting scaling arguments on effective 2 DOFs model?





- Effective 2 DOFs model allows to remove arbitrariness in scaling arguments in 1D picture
- Explicit implementation of the 'quasistatic' assumption in the language of the tilted directed polymer
- Use refined scaling properties of the effective quasistatic disorder free-energy $\bar{F}_V^f(t_f, y)$
- Saddle-point argument at vanishing force $(f \to 0)$ for the creep regime
- Validity range of assumptions / creep prediction:
  - Finite system size at vanishing force (dimensional crossover)?
  - Intermediate regime at larger forces?
  - Low-temperature dependence?

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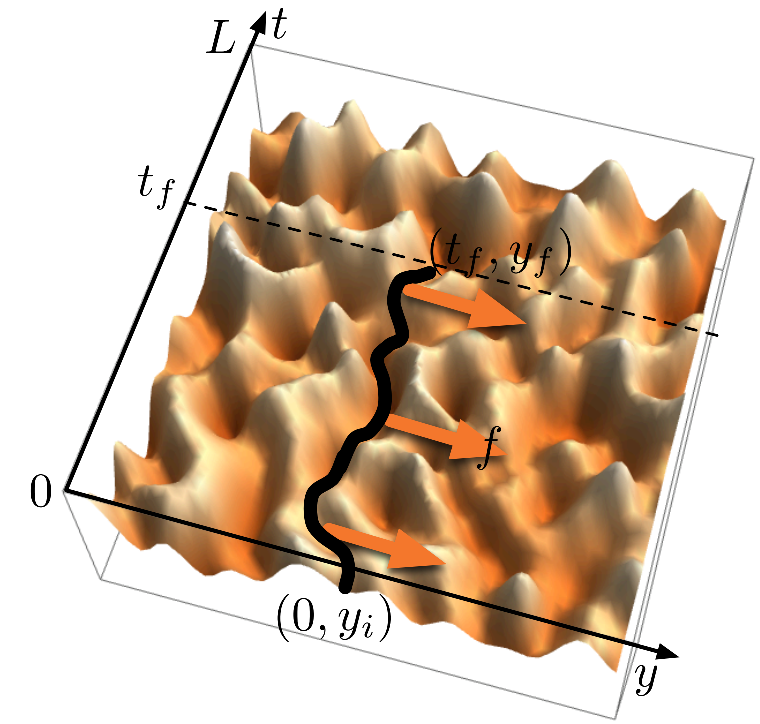
Quasistatic ‘creep’ regime — Effective model with two degrees of freedom

- Overdamped Langevin dynamics of the full interface:

$$\gamma\partial_\tau y(t,\tau) = c\partial_t^2 y - \partial_y V(t,y(t,\tau)) + f + \eta(t,\tau)$$

Thermal noise: $\langle \eta(t',\tau')\eta(t,\tau)\rangle = 2\gamma T\delta(t'-t)\delta(\tau'-\tau)$

Quenched disorder: $\overline{V(t',y')V(t,y)} = D\delta(t'-t)R_\xi(y'-y)$



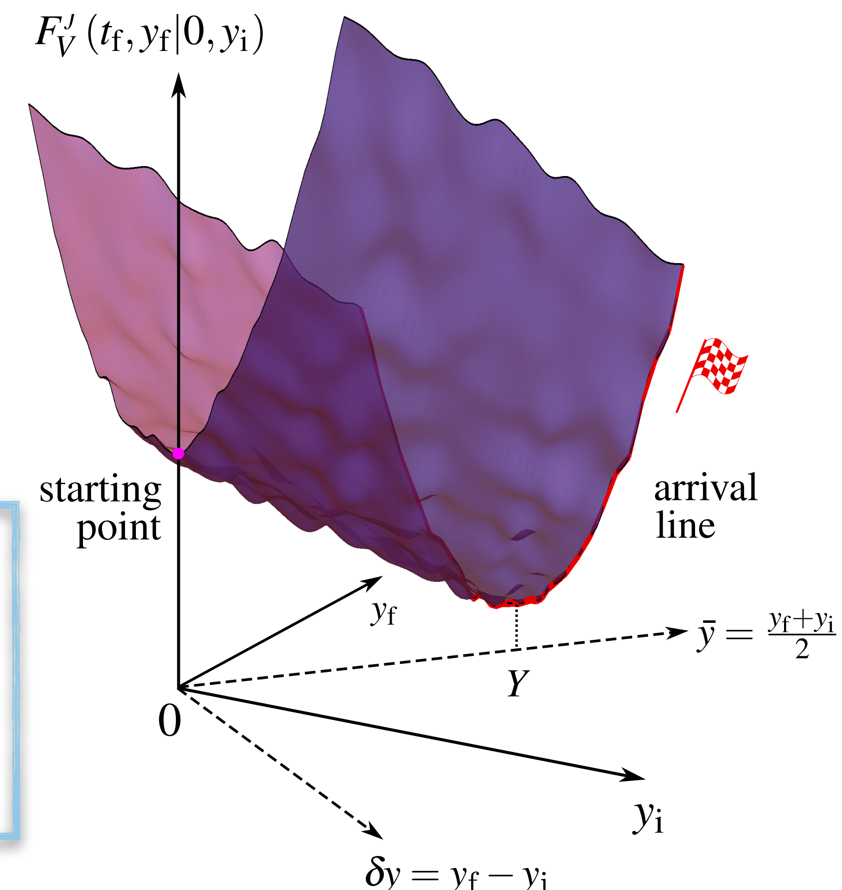
- Effective dynamics of a ‘particle’ of coordinates given by the extremities of the interface segment (y_i, y_f):

$$\tilde\gamma\partial_\tau y_{\rm i}(\tau) = -\partial_{y_{\rm i}} F_V^f(t_{\rm f},y_{\rm f}|0,y_{\rm i}) + \sqrt{2\tilde\gamma\tilde T}\,\tilde\eta_{\rm i}(\tau)$$
$$\tilde\gamma\partial_\tau y_{\rm f}(\tau) = -\partial_{y_{\rm f}} F_V^f(t_{\rm f},y_{\rm f}|0,y_{\rm i}) + \sqrt{2\tilde\gamma\tilde T}\,\tilde\eta_{\rm f}(\tau)$$

- Decomposition of the quasistatic DP free-energy:

$$F_V^f(t_{\rm f},y_{\rm f}|0,y_{\rm i}) = \frac{c}{2t_{\rm f}}(y_{\rm f}-y_{\rm i})^2 - \frac{f t_{\rm f}}{2}(y_{\rm f}+y_{\rm i}) + \bar F_V^f(t_{\rm f},y_{\rm f}|0,y_{\rm i}) + \mathrm{const}(t_{\rm f})$$

Elasticity | *Driving force* | *Effective disorder*



- Estimation of mean steady-velocity through the mean first passage time (MFPT)
- Use known scalings of the *static* disorder free-energy (from KPZ!)

E. Agoritsas, R. García-García, V. Lecomte, L. Truskinovsky, & D. Vandembroucq, *J. Stat. Phys.* 164, 1394 (2016).

Quasistatic 'creep' regime — Summary of the assumptions & their implications

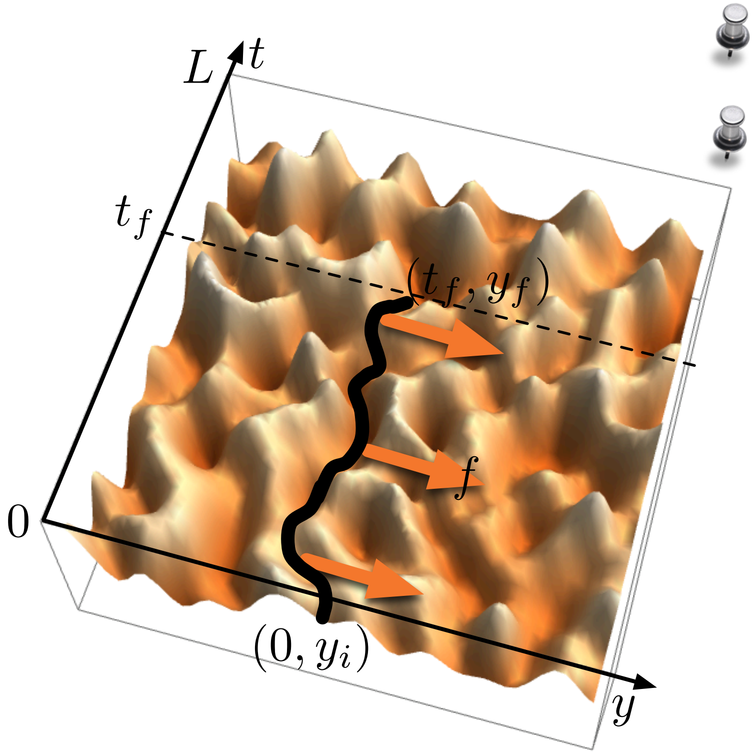
- *Low temperature* T :
  - Allows for the Arrhenius MFPT expression, based on the instanton description, as long as $T \ll U_c(f_c/f)^{1/4}$
  - Low-temperature Larkin length $\mathcal{L}_c \overset{(T \ll T_c)}{\sim} \frac{T_c^5}{cD^2} \sim \xi^{5/3}$

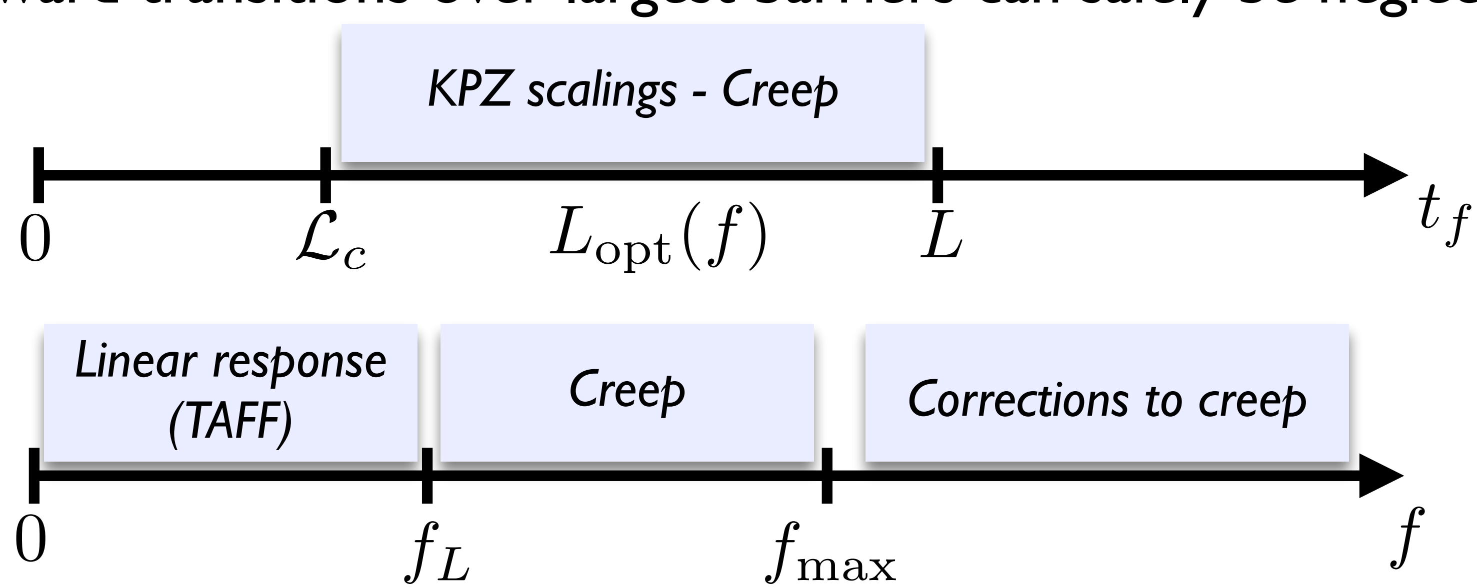
$$T_c = (\xi c D)^{1/3}$$

At higher T:
- Crossover of the Larkin length: $\mathcal{L}_c \overset{(T \gg T_c)}{\sim} \frac{T^5}{cD^2}$
- Contributions fluctuations around instanton & other transition paths

---

- *Large system size* L :
  - Possible to have $0 < \mathcal{L}_c < L_{\text{opt}}(f) < L$ so that MFPT expression dominated by Brownian scaling, allowing for rescaling free-energy $(f^{-1/4})$
- *Small force* f:
  - Small velocity ⇒ quasistatic description with static free-energy & tilt
  - Rescaling & saddle-point argument $f \to 0 \Rightarrow L_{\text{opt}}(f) \sim f^{-3/4}$
  - Backward transitions over largest barriers can safely be neglected





E. Agoritsas, R. García-García, V. Lecomte, L. Truskinovsky, & D. Vandembroucq, *J. Stat. Phys.* 164, 1394 (2016).

[TECHNICAL] *Mean first passage time (MFPT) & mean steady-state velocity*

$$y = a\hat{y} \qquad t = b\hat{t}$$

KPZ roughness scaling $\quad a = \left(\frac{\tilde{D}}{c^2}\right)^{\frac{1}{3}} b^{\frac{2}{3}}$

$$b = (c\tilde{D})^{\frac{1}{4}} f^{-\frac{3}{4}} \qquad a = \left(\frac{\tilde{D}}{cf}\right)^{\frac{1}{2}}$$

The largest barrier along the instanton starts from a minimum $(\bar{y}'', \delta y'')$ and ends in a saddle point $(y', \delta y')$ of (36) at the top of the barrier (the top of the barrier is located at local maximum along the instanton, which lies itself at a saddle of the effective potential). Generalising the MFPT expression for the particle (27) by replacing the difference of 1D potential $[V^f(y') - V^f(y'')]$ by the difference of effective potential (36) between the 2D coordinates $(\bar{y}'', \delta y'')$ and $(y', \delta y')$, the passage time writes

$$\tau_1(t_f, Y; V) \sim \frac{\gamma}{T} e^{-\frac{1}{T}\left\{\frac{c}{2t_f}\left[(\delta y'')^2 - (\delta y')^2\right] - f t_f (\bar{y}'' - \bar{y}')\right\}} \times e^{-\frac{1}{T}\left\{\bar{F}_V^f(t_f, \bar{y}'' + \delta y''/2 | 0, \bar{y}'' - \delta y''/2) - \bar{F}_V^f(t_f, \bar{y}' + \delta y'/2 | 0, \bar{y}' - \delta y'/2)\right\}}$$

Arrhenius MFPT: thermally activated

Explicitly quasistatics

One can now evaluate the disorder-averaged MFPT $\bar{\tau}_1(f)$ of the interface by integrating (39) over every possible length $t_f \in [0, L]$ of an interface segment, assuming that $L \gg \mathscr{L}_c$ with $\mathscr{L}_c$ the lengthscale above which the Brownian rescaling (42) is valid ($\mathscr{L}_c$ will be identified later on as the 'Larkin length', see Sec. 5.3). Using the rescaling (47), one has

$L \gg \mathcal{L}_c$

$$\bar{\tau}_1(f) \overset{(L > \mathscr{L}_c)}{=} \frac{1}{L}\int_0^L dt_f\, \overline{\tau_1(t_f; V)} = \frac{1}{L}\int_0^{\mathscr{L}_c} dt_f\, \overline{\tau_1(t_f; V)} + \frac{1}{L}\int_{\mathscr{L}_c}^{L} dt_f\, \overline{\tau_1(t_f; V)}$$

$$\overset{(L \gg \mathscr{L}_c)}{\sim} \frac{1}{\hat{L}}\int_{\hat{\mathscr{L}}_c}^{\hat{L}} d\hat{t}_f\, \mathbb{E}_{\hat{V}} e^{-\frac{1}{T} f^{-\frac{1}{4}}\left[\frac{\tilde{D}^3}{c}\right]^{\frac{1}{4}}\left\{\frac{(\hat{y}''_f - \hat{y}''_i)^2 - (\hat{y}'_f - \hat{y}'_i)^2}{2\hat{t}_f} - \hat{t}_f \frac{(\hat{y}''_f + \hat{y}''_i) - (\hat{y}'_f + \hat{y}'_i)}{2} + \hat{F}_{\hat{V}}^f(\hat{t}_f, \hat{y}''_f | 0, \hat{y}''_i; \xi/a) - \hat{F}_{\hat{V}}^f(\hat{t}_f, \hat{y}'_f | 0, \hat{y}'_i; \xi/a)\right\}}$$

E. Agoritsas, R. García-García, V. Lecomte, L. Truskinovsky, & D. Vandembroucq, *J. Stat. Phys.* 164, 1394 (2016).

[TECHNICAL] *Mean first passage time (MFPT) & mean steady-state velocity*

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KPZ roughness scaling $\quad a = \left(\frac{\tilde{D}}{c^2}\right)^{\frac{1}{3}} b^{\frac{2}{3}}$

$b = (c\tilde{D})^{\frac{1}{4}} f^{-\frac{3}{4}} \qquad a = \left(\frac{\tilde{D}}{cf}\right)^{\frac{1}{2}}$

Since all the dimensioned parameters have been factored out in this exponent, the saddle-point is reached at a value $\hat{t}_\mathrm{f}^\star$ of $\hat{t}_\mathrm{f}$ which is independent of the dimensioned parameters of the problem, and in particular independent of f. We emphasise that this construction requires the system to be large enough ($L \gg \mathscr{L}_\mathrm{c}$) in order that *(i)* the KPZ scaling (44) is meaningful for a large range of segment length t_f, *(ii)* we can neglect the contribution of $t_\mathrm{f} \in [0, \mathscr{L}_\mathrm{c}]$ in the MFPT, so we can self-consistently assume that the saddle point is reached at $\hat{t}_\mathrm{f}^\star \in [\hat{\mathscr{L}}_\mathrm{c}, \hat{L}]$. We refer to Sec. 5.3 for the corresponding regime of validity. We finally obtain

$$\bar{\tau}_1(f) \sim \mathbb{E}_{\hat{V}} \mathrm{e}^{-\frac{1}{T} f^{-\frac{1}{4}} \left[\frac{\tilde{D}^3}{c}\right]^{\frac{1}{4}} \left\{ \frac{(\hat{y}''_\mathrm{f} - \hat{y}''_\mathrm{i})^2 - (\hat{y}'_\mathrm{f} - \hat{y}'_\mathrm{i})^2}{2\hat{t}_\mathrm{f}^\star} - \hat{t}_\mathrm{f}^\star \frac{(\hat{y}''_\mathrm{f} + \hat{y}''_\mathrm{i}) - (\hat{y}'_\mathrm{f} + \hat{y}'_\mathrm{i})}{2} + \hat{F}_{\hat{V}}^f(\hat{t}_\mathrm{f}^\star, \hat{y}''_\mathrm{f} | 0, \hat{y}''_\mathrm{i}; \xi/a) - \hat{F}_{\hat{V}}^f(\hat{t}_\mathrm{f}^\star, \hat{y}'_\mathrm{f} | 0, \hat{y}'_\mathrm{i}; \xi/a) \right\}}$$

- Creep form of the mean steady-state velocity: $f \to 0$ saddle point with KPZ scalings

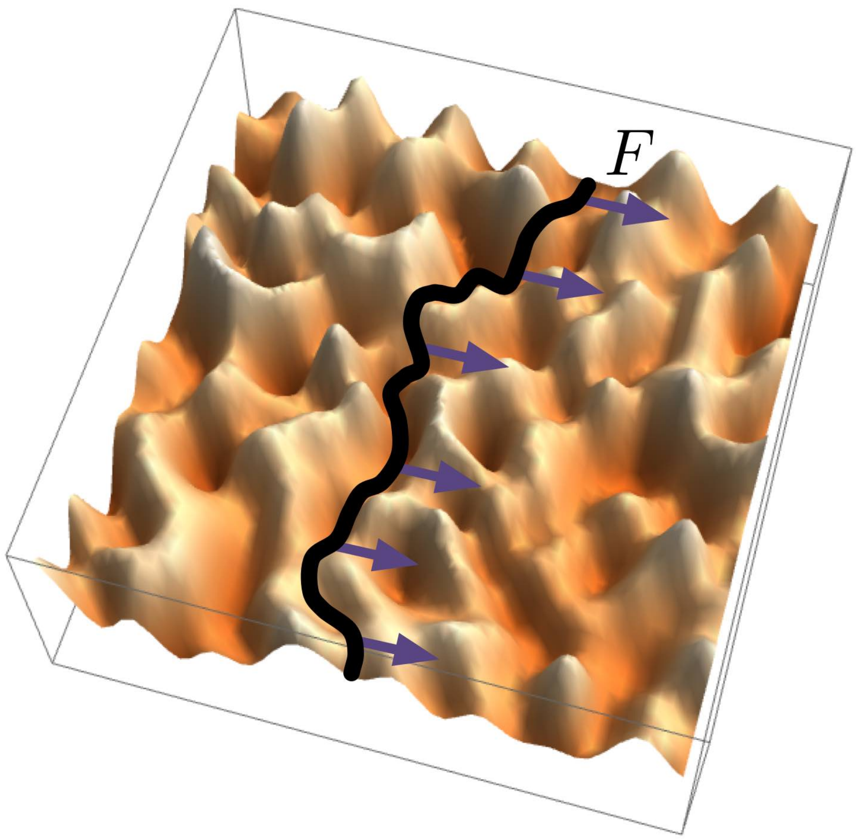
$1/\bar{\tau}_1(f)$

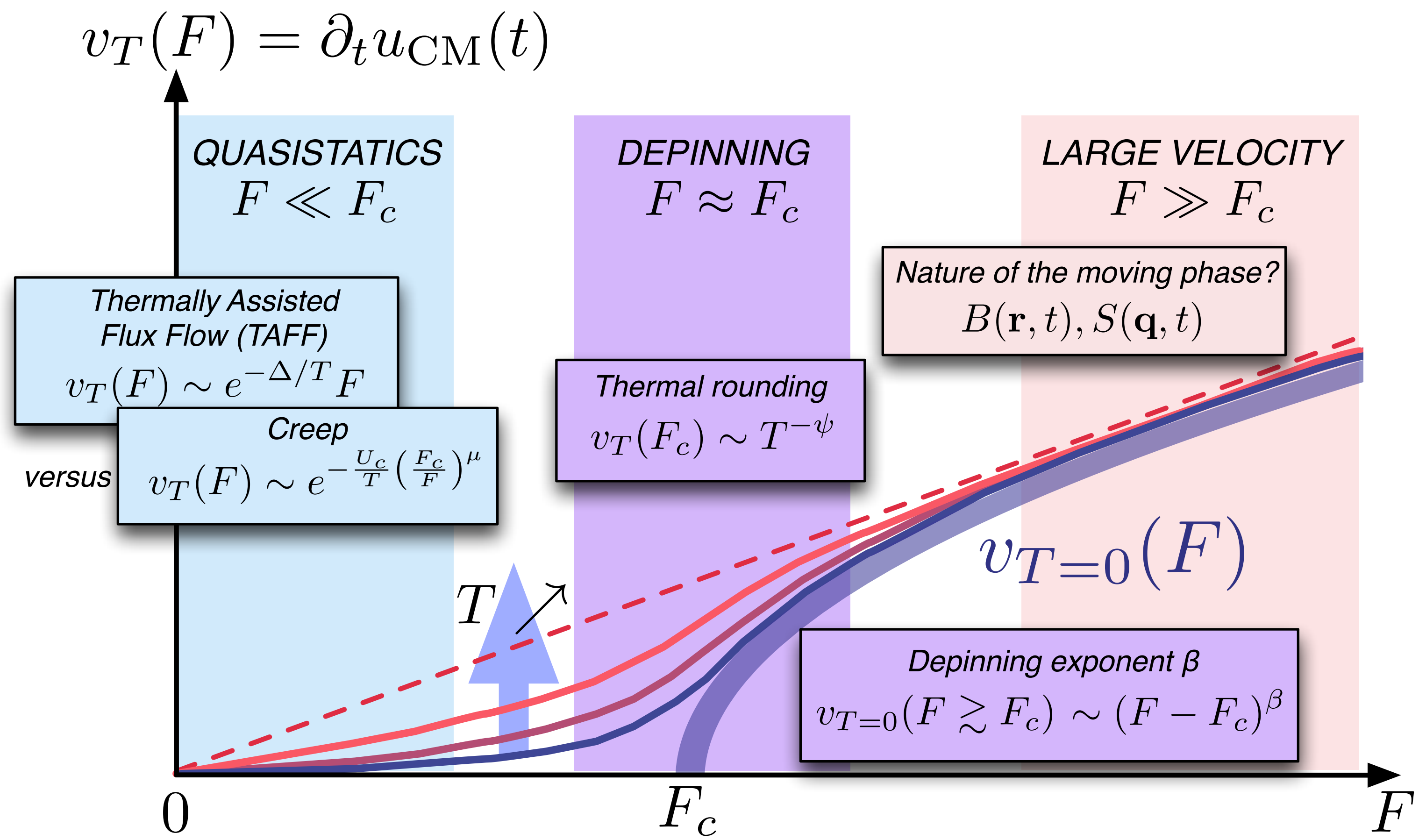
$$\bar{v}(f) \sim \mathrm{e}^{-\frac{U_\mathrm{c}}{T}\left(\frac{f_\mathrm{c}}{f}\right)^{\frac{1}{4}} \Delta\hat{F}^\star}$$

$$L_\mathrm{opt}(f) = (c\tilde{D})^{\frac{1}{4}} f^{-\frac{3}{4}}$$

E. Agoritsas, R. García-García, V. Lecomte, L. Truskinovsky, & D. Vandembroucq, *J. Stat. Phys.* 164, 1394 (2016).

@Large velocity: 'fast flow' regime





$$\gamma\, \partial_t u(z,t) = c\partial_z^2 u(z,t) + F_{\rm dis}(z,u(z)) + f_{\rm ext} + \eta_{\rm thermal}(z,t)$$

- Large force ⇒ large steady-state velocity for the center of mass

$$u(z,t) = v(t-t_0) + \delta u(z,t)$$

$$F_{\rm dis}(z,u(z,t)) = F_{\rm dis}(v(t-t_0)+\delta u(z,t)) \approx F_{\rm dis}(v(t-t_0))$$

$$\overline{F_{\rm dis}(v(t-t_0))F_{\rm dis}(v(t'-t_0))} = \Delta(v(t-t')) \begin{cases} = \dfrac{1}{v^3}\Delta_{\xi/v}(t-t') & (\text{‘RB’}) \\ = \dfrac{1}{v}\Delta_{\xi/v}(t-t') & (\text{‘RF’}) \end{cases}$$

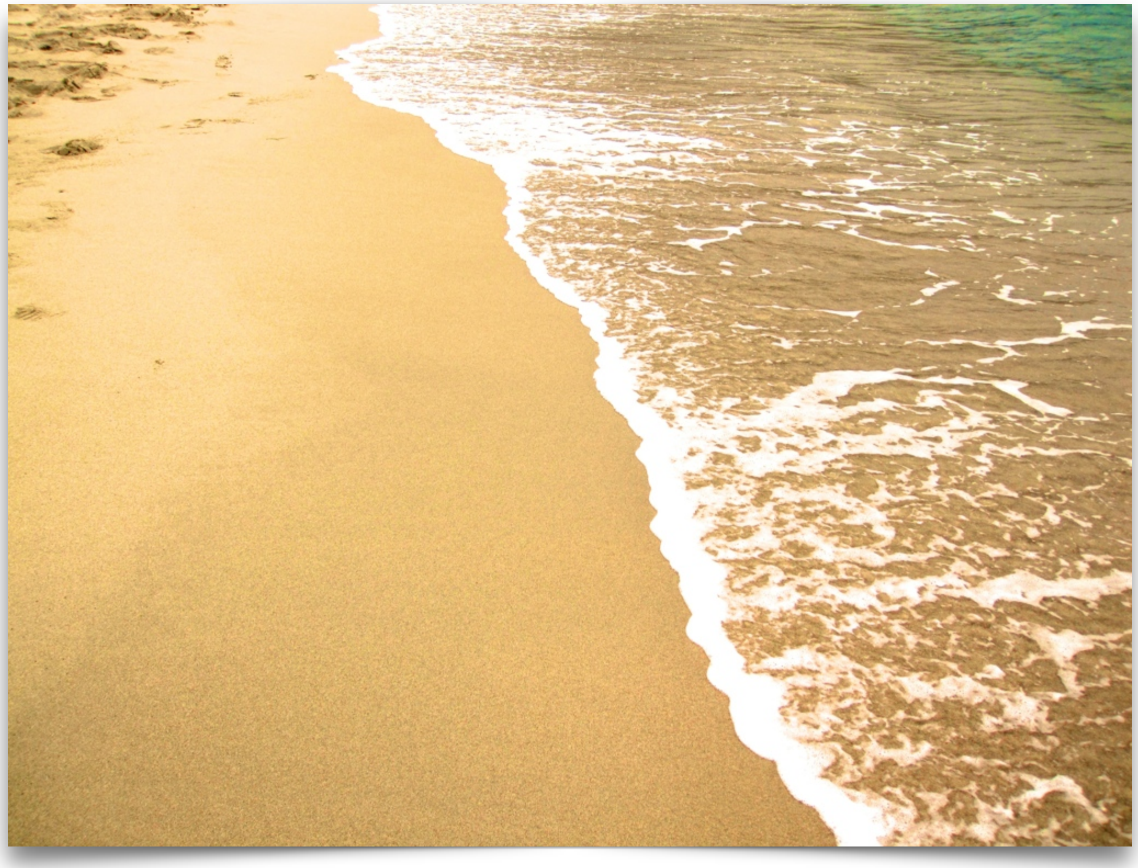
$$F_c \sim \frac{c\xi}{L_c^2}$$

P. Chauve, T. Giamarchi, P. Le Doussal, *Phys. Rev. B* 62, 6241 (2000) **[cf. Appendices B-C]**

Doctoral Training School *Kardar-Parisi-Zhang equation: new trends in theories and experiments*
April 15-26, 2024 — Ecole de Physique des Houches (France)

Interfaces in disordered systems and directed polymer

Elisabeth Agoritsas



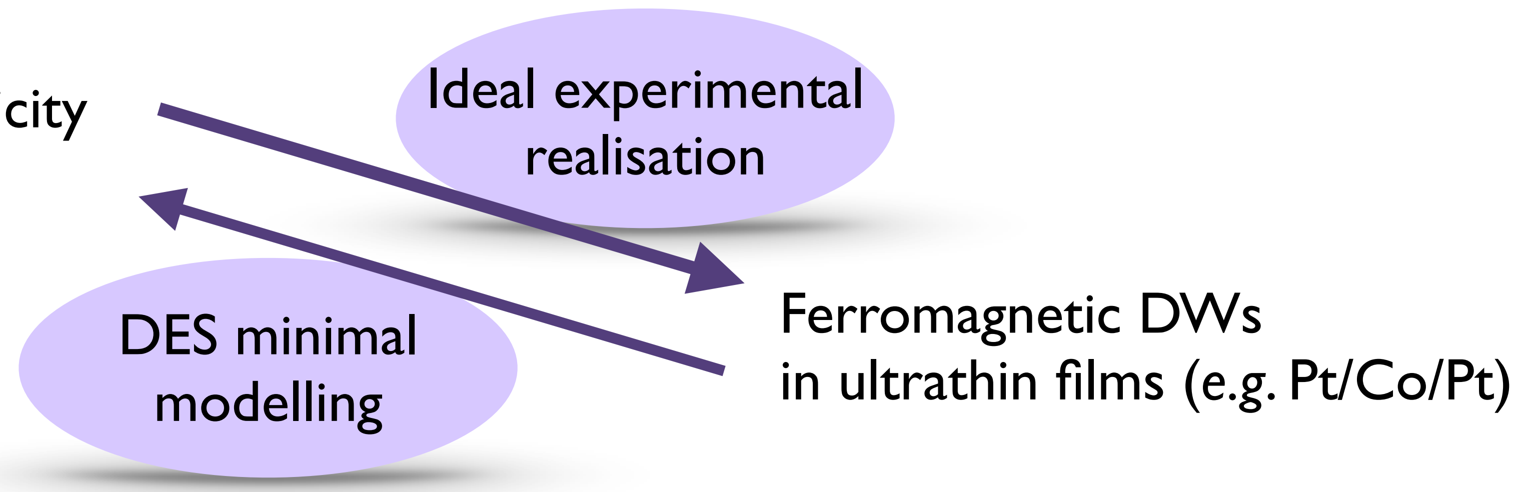
Summary (beyond the specific KPZ connections)

- Generic framework of DES systems with $\mathcal{H}_{\mathrm{DES}} = \mathcal{H}_{\mathrm{el}} + \mathcal{H}_{\mathrm{dis}}$

  ⇒ Study of disorder-conditioned features of interfaces (statics & dynamics)

- 1D interface, with short range elasticity & random-bond disorder

  (or quenched Edwards-Wilkinson dynamics)



- Focus on two features: roughness & velocity-force characteristics

  & other features accessible through DES modelling such as

  - Velocity fluctuations in the steady-state regime
  - Roughness prefactors $B(r) \sim r^{2\zeta}$, crossover lengthscales, other regimes, …
  - Whole distribution of geometrical fluctuations $\mathcal{P}(\Delta u(r))$, beyond $B(r) = \overline{\langle \Delta u(r)^2 \rangle}$
  - Avalanches statistics & spatio-temporal patterns
  - A-C dynamics

- Investigated via
  - numerics on qEW (also quenched KPZ!)
  - scaling analysis (standard Flory vs our approach)
  - Gaussian Variational Method (GVM) computation
  - special mappings when available (such as KPZ for the static fluctuations)
  - functional RG (perturbative in 4-d, non-perturbative)
  - toy models / mean-field approximations

Numerics: asymptotic 2-pt correlator of disorder free-energy

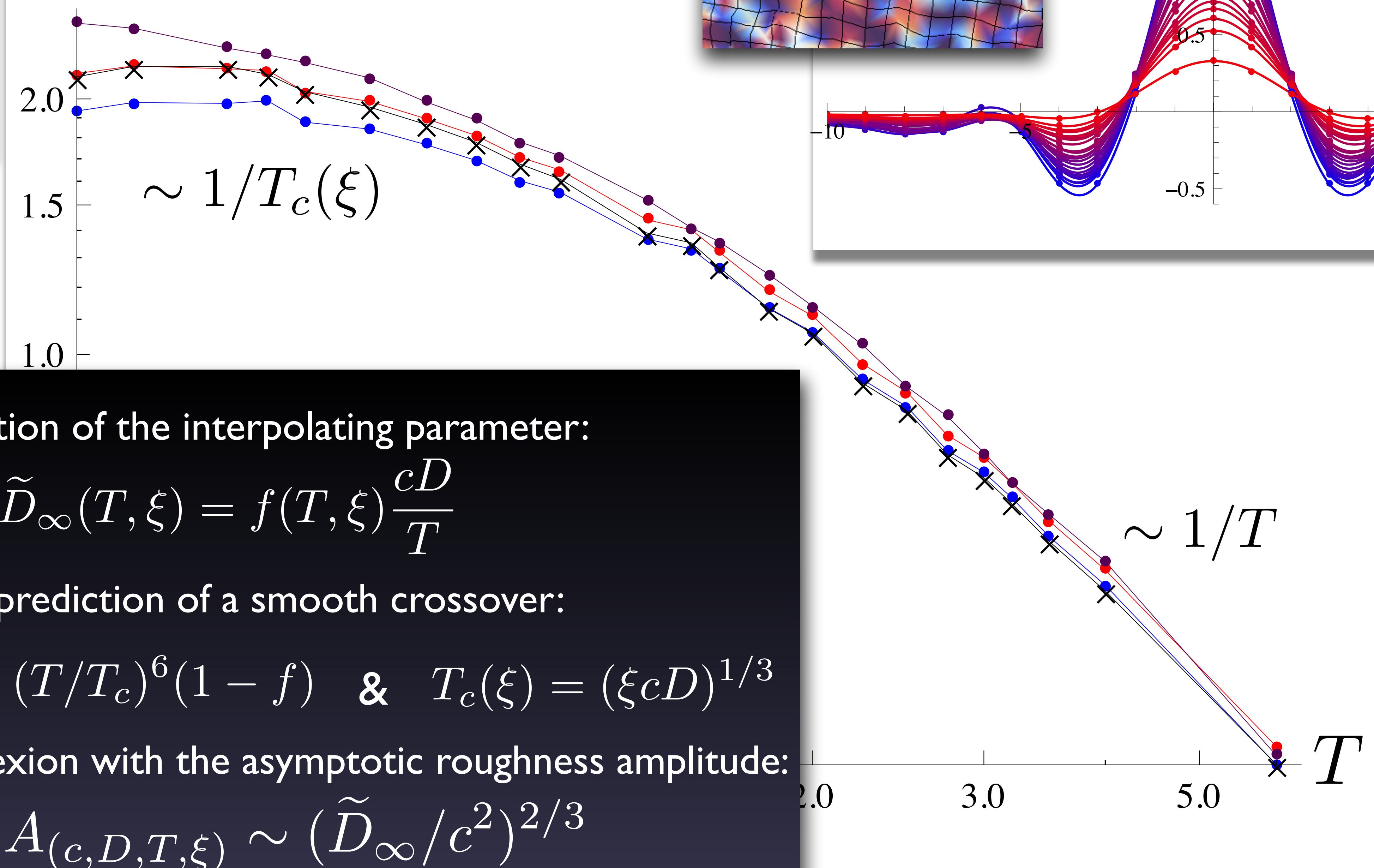
$$\bar{R}_{\text{sat}}(y) \approx \bar{R}(\infty, y) = \frac{1}{2}\partial_y^2 \bar{C}(\infty, y)$$

- Amplitude of the correlator / Maximum value:

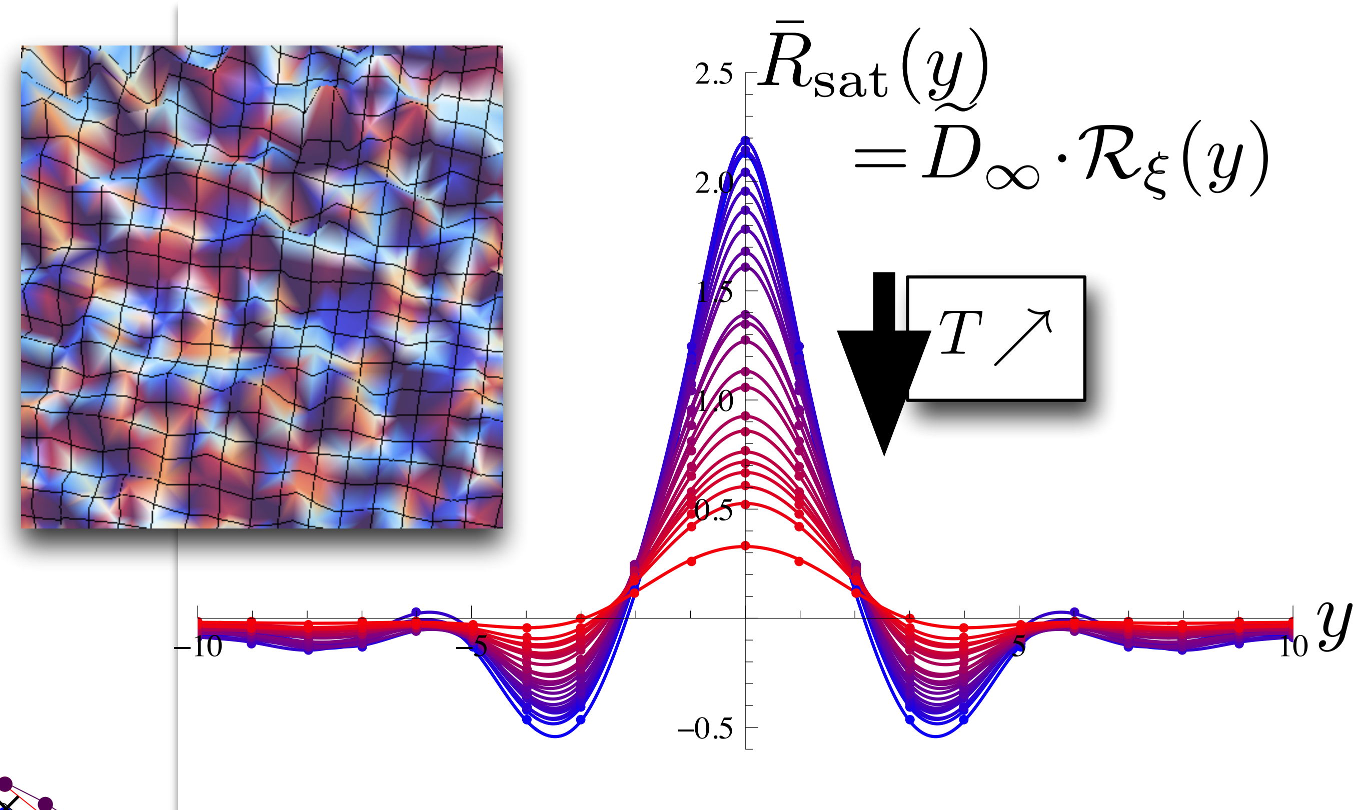
$T \approx 0$

$\widetilde{D}_\infty \sim \dfrac{cD}{T_c}$

$\mathcal{R}_{\tilde{\xi}} \approx ??$



- Shape of the asymptotic correlator:



$\xi \approx 0$

$\widetilde{D}_\infty \lesssim \dfrac{cD}{T}$

$\mathcal{R}_{\tilde{\xi}} \approx R_\xi$

- Definition of the interpolating parameter:

$$\widetilde{D}_\infty(T,\xi) = f(T,\xi)\frac{cD}{T}$$

- GVM prediction of a smooth crossover:

$$f^6 \propto (T/T_c)^6(1-f) \quad \& \quad T_c(\xi) = (\xi c D)^{1/3}$$

- Connexion with the asymptotic roughness amplitude:

$$A_{(c,D,T,\xi)} \sim (\widetilde{D}_\infty/c^2)^{2/3}$$

E. Agoritsas, V. Lecomte & T. Giamarchi, *Phys. Rev. E* 87, 042406 & 062405 (2013).

Roughness regimes & characteristic crossover scales

ROUGHNESS $B(t)$

THERMAL $B_{\rm th}(t) = \frac{Tt}{c}$

THERMAL $(T \gg T_c)$

MODIFIED LARKIN $(T \ll T_c)$

RANDOM MANIFOLD $B_{\rm RM}(t) \sim (\widetilde{D}_\infty/c^2)^{\frac{2}{3}} t^{\frac{4}{3}}$

0 — $L_c(T=0)$ — $L_c \sim \frac{T^5}{cD^2}/f^5$ — 'time' t

$T \nearrow$

0 — $t_{\rm sat}$ — $L_c(T=0)$ — 'time' t

DISORDER FREE-ENERGY

SHORT 'TIMES'

SATURATION at $|y| \lesssim \xi$

$\bar{\mathcal{P}}\left[\bar{F}_V, t\right]$

Feedback of the KPZ nonlinearity $-\frac{1}{2c}\left[\partial_y \bar{F}_V\right]^2$

Non-Gaussian features: $\partial_t \bar{R}(t,y),\ \partial_t \bar{R}_3(t,y),\ \ldots$

Non-Gaussian steady-state: $\bar{R}(\infty, y) = \widetilde{D}_\infty \cdot \mathcal{R}_\xi(y)$

Linearized evolution: Gaussian $\bar{\mathcal{P}}\left[\bar{F}_V^{\rm lin}, t\right]$

$B_{\rm dis}(t)$

Powerlaw?

~~$(\zeta_{\rm num} \approx 2 - 2.5\,?)$~~

$B_{\rm dis}(t) \sim (\widetilde{D}_\infty/c^2)^{\frac{2}{3}} t^{\frac{4}{3}}$ versus $B_{\rm th}(t) = \frac{Tt}{c}$

in the total roughness: $B(t) = B_{\rm th}(t) + B_{\rm dis}(t)$

Disordered Elastic Systems (DES) — Selective bibliography

About ~50 years of literature in theory/experiments/numerics! ⇒ See reviews with refs therein

- T. Giamarchi, Encyclopedia of Complexity and Systems Science (2009), "Disordered Elastic Media"

  Introduction to DES

- E. Agoritsas, V. Lecomte, T. Giamarchi, *Physica B* 407, 1725 (2012), "Disordered elastic systems and one-dimensional interfaces"

  Statics

- T. Giamarchi, A. B. Kolton, A. Rosso, *Lecture Notes in Physics* 688, 91 (2006) [arXiv:0503437], "Dynamics of disordered elastic systems"

  Dynamics

- K. J. Wiese, *Reports on Progress in Physics* 85, 086502 (2022), "Theory and experiments for disordered elastic manifolds, depinning, avalanches, and sandpiles"

  Very recent extended review

- D. S. Fisher, *Physics Reports* 301, 113 (1998), "Collective transport in random media: from superconductors to earthquakes"

  Lectures (1998)

- G. Blatter, M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, V. M. Vinokur, *Reviews of Modern Physics* 66, 1125 (1994), "Vortices in high-temperature superconductors"

  "Canonical review"