Random considerations on KPZ

The ”P” of KPZ

Antefacts

I was fascinated by DLA (Diffusion Limited aggregation). Apparently no upper critical dimensions.

With Zhang Yicheng we tried to study a simpler problem. Aggregation: i.e. the Eden model (*Eden model in many dimensions*). We computed an $1/d$ expansion, nothing interesting was found.

We wrote a paper *Field theories and growth models*: Eden model and DLA are related to Reggeon field but we were not able to make progress.

The Eden model has surface fluctuations. We could study them, but Zhang left in 1984 for Brookheaven.

$$\dot{\varphi}(x,t) = \nu - \Delta\varphi + \eta(x,\varphi)$$

$$\dot{\varphi} = +\Delta\varphi + (\partial_\mu \varphi)^2 + \eta(x,t)$$

1

$\phi = V + ($

$$\boxed{\mathring{\varphi}_{\partial t} = V - \Delta\varphi + \eta(t, \varphi)}$$

g_c

Kuplick $\quad \varphi \equiv Vt + \tilde{\varphi}$

$g \sim \frac{1}{V}$

$$\mathring{\tilde{\varphi}} = \cancel{V} + \Delta\varphi + \eta(x, t + g\tilde{\varphi})$$

g_c

$$+ \eta(t, t) + g\varphi\,\partial_t\eta$$

$$\boxed{\mathring{\varphi} = +\Delta\varphi + (\partial_\mu\varphi)^2 + \eta(x, t)} \quad (\omega_t L)$$

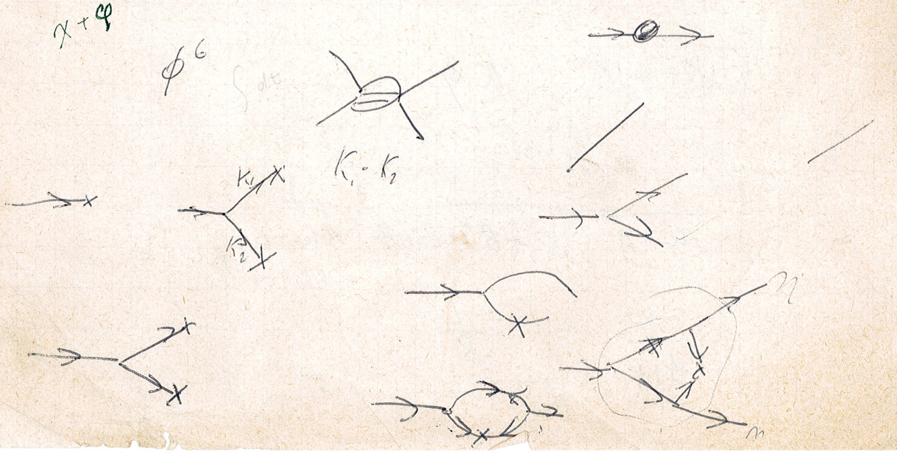
φ^2

$[\varphi] = \frac{D}{2} - 1 \qquad [\varphi] = 0$

$L(\partial_\mu\varphi) = \qquad \boxed{D = 2}$

$x + \varphi$

$\phi^6 \quad \int dt$



Interface moving through a random background

Joel Koplik and Herbert Levine

Schrödinger equation of zero interacting particles

KPZ $\to$ polymer in Random Potential $\to$ Schrödinger equation of n interacting particles in the limit $n \to 0$.

$$\boldsymbol{H} = -\frac{1}{2} \sum_{a=1,n} \Delta_a - \sum_{a=1,n,b=1,n} V(x_a - x_b),$$

$$\overline{\rho(x_1)\rho(x_2)} \equiv \int \mathrm{d}P[\rho]\rho(x_1)\rho(x_2) \propto \int \mathrm{d}x_3 \ldots \mathrm{d}x_n \, \psi(x_1, \ldots, x_n).$$

Wonderful paper of Kardar in $d = 1$: the solution with a δ potential is known for any n:

$$\Phi(x_1 \cdots x_n) = \prod_{i,k} \psi(x_i - x_k)$$

A variational approach to directed polymers (Mézard, GP)

$$\psi(x_1,\ldots,x_n) \propto \int \mathrm{d}R[\mu]\mu(x_1),\ldots,\mu(x_n).$$

$$\langle\psi|\boldsymbol{H}|\psi\rangle = \int \mathrm{d}R[\mu]\,\mathrm{d}R[\sigma]\mathfrak{H}_n[\mu,\sigma]$$

$$\mathfrak{H}_n[\mu,\sigma] = n/2 \int \mathrm{d}x \sum_{\nu=1,N} \mathrm{d}\mu/\mathrm{d}x_\nu\, \mathrm{d}\sigma/\mathrm{d}x_\nu\, I[\mu\sigma]^{n-1}$$
$$- n(n-1) \int \mathrm{d}x\,\mathrm{d}t\, \mu(x)\mu(y)\sigma(x)\sigma(y)V(x-y)I[\mu\sigma]^{n-2}$$

$$\langle\psi|\psi\rangle = \int \mathrm{d}R[\mu]\,\mathrm{d}R[\sigma]I[\mu\sigma]^n.$$

$$\int \mathrm{d}R[\sigma]\mathfrak{H}_n[\mu,\sigma] = E_n \int \mathrm{d}R[\sigma]I[\mu\sigma]^n$$

Schrödinger eq.

Half baked ideas.

The paper is full of half baked ideas.

Greens functions identities, replica symmetry breaking....

Hard problem:

$$\psi(x_1, \ldots, x_n) \propto \int \mathrm{d}R[\mu]\mu(x_1), \ldots, \mu(x_n).$$

Do we have an $R_n[\mu]$ that is independent from n. Is it unique?

RSB:

On the 1/D expansion for directed polymers with Slanina.

$$\exp\left(-\frac{1}{4}\sum_{a,b} Q^{-1}_{ab}\omega_a \cdot \omega_b\right)$$

A disperate attempt to do a too difficult computation using educated guesses.

I think that now we could to the computation without educated guesses.

Numerical Simulations: RSOS

Marinari, Pagnani, GP.

$$h(x+1) - h(x) = \{-1, 0, 1\}$$

The problem is formulated on link variables (like gauge fields). The field should be pure gauge at time 0.

We random choose the point x and we increase all the links starting from x if this is possible.

Gauge fields are represented as two bits. One reconstruct the surface when needed for measurements.

We look to the limit $t \to \infty$: the width interface w_2 scales as L^χ.

RSOS

If one looks to the h's, RSOS ha rules similar to self organized criicality (sandpiles).

Discrete space, dynamical rules. Random cellular automata with few states per site.

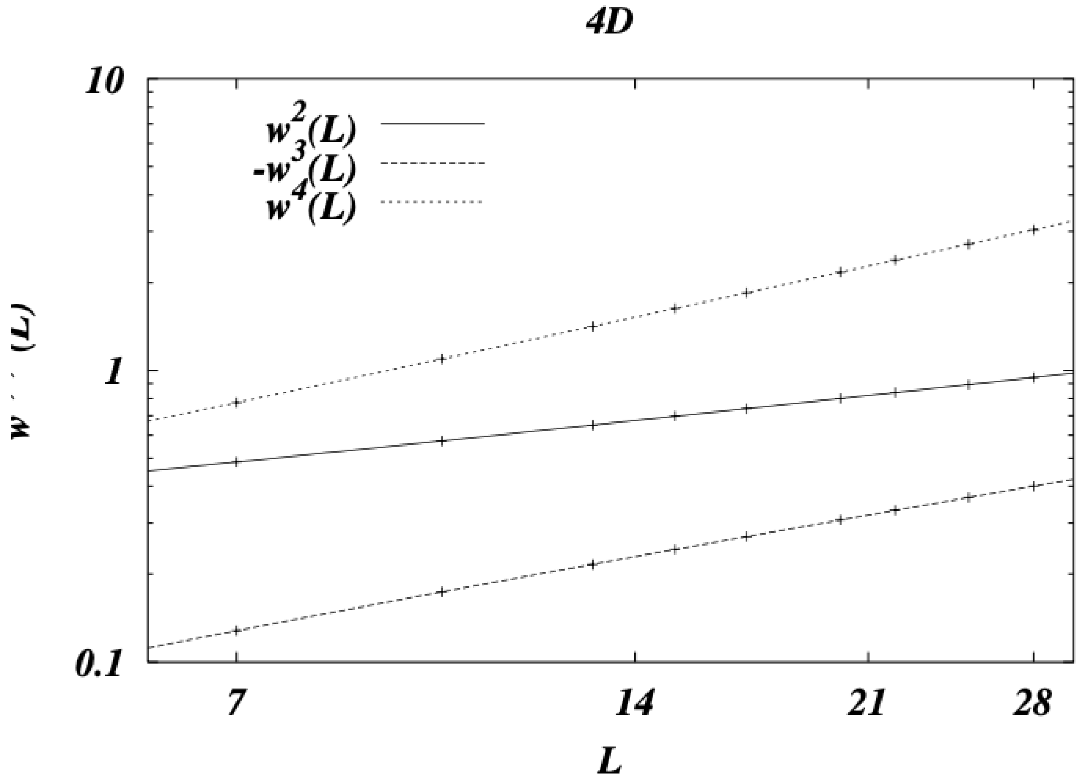
The h's have long range anticorrelations that have a power decay.

How large is the universality class?

Are there other models with this features that are not trivial in high dimensions?

$D = 4$ $L = 7 \cdots 28$: L^4 points

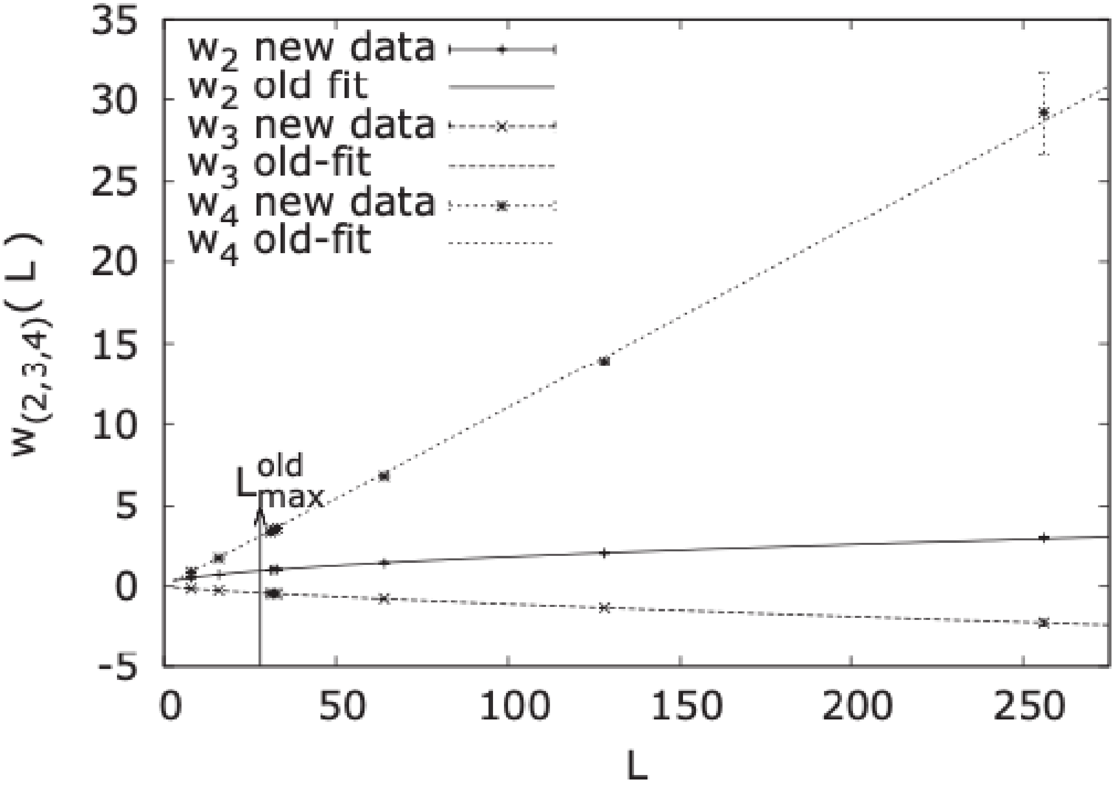
We study the cumulant of $h(x,t) - h(t)$: w_2, w_3, w_4

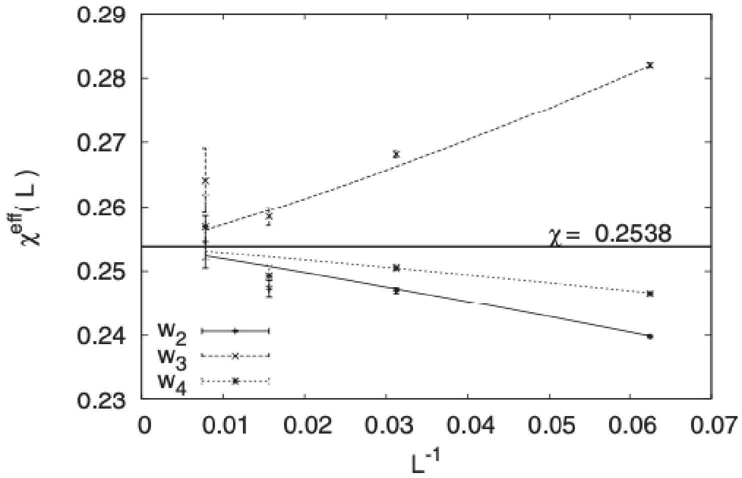


$$w_2 \simeq A_2 L^{2\chi}(1 + B_2 L^{-\omega})$$
$$w_3 \simeq -A_3 L^{3\chi}(1 + B_3 L^{-\omega})$$
$$w_4 \simeq A_4 L^{4\chi}(1 + B_4 L^{-\omega}).$$

$\chi_{D=4} = 0.255 \pm 0.003 \qquad \omega_{D=4} = 0.98 \pm 0.09.$

Vingt ans après: $D = 4$ $L = 7 \cdots 128(256)$





χ and ω

New	0.2537(8)	1.11(9)
Old	0.255(3)	0.98(9)

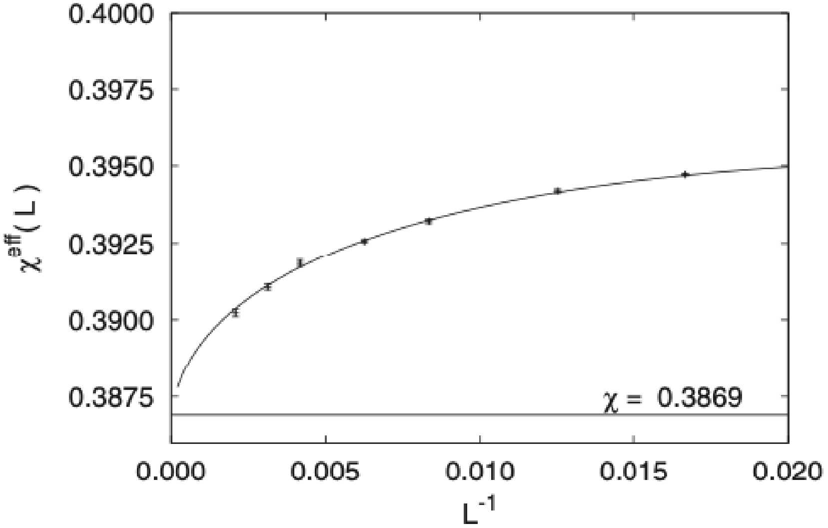
Vingt ans après: $D = 2$ same L range with much higher statistics

$$w_2 = A_2 L^{2\chi}(1 + B_2 L^{-\omega}),$$
$$w_3 = S A_2^{3/2} L^{3\chi}(1 + B_3 L^{-\omega}),$$
$$w_4 = K A_2^2 L^{4\chi}(1 + B_4 L^{-\omega}).$$

$$w_2 = A_2 L^{2\chi}(1 + B_2 L^{-\omega} + C_2 L^{-2\omega}),$$
$$w_3 = S A_2^{3/2} L^{3\chi}(1 + B_3 L^{-\omega} + C_3 L^{-2\omega}),$$
$$w_4 = K A_2^2 L^{4\chi}(1 + B_4 L^{-\omega} + C_4 L^{-2\omega}).$$

	χ	ω	A_2	B_2	C_2
FIT I	0.3893(6)	0.8(2)	0.118(1)	−0.4(2)	NA
FIT II	**0.3869(4)**	**0.57(5)**	0.1226(1)	−0.37(2)	0.6(2)

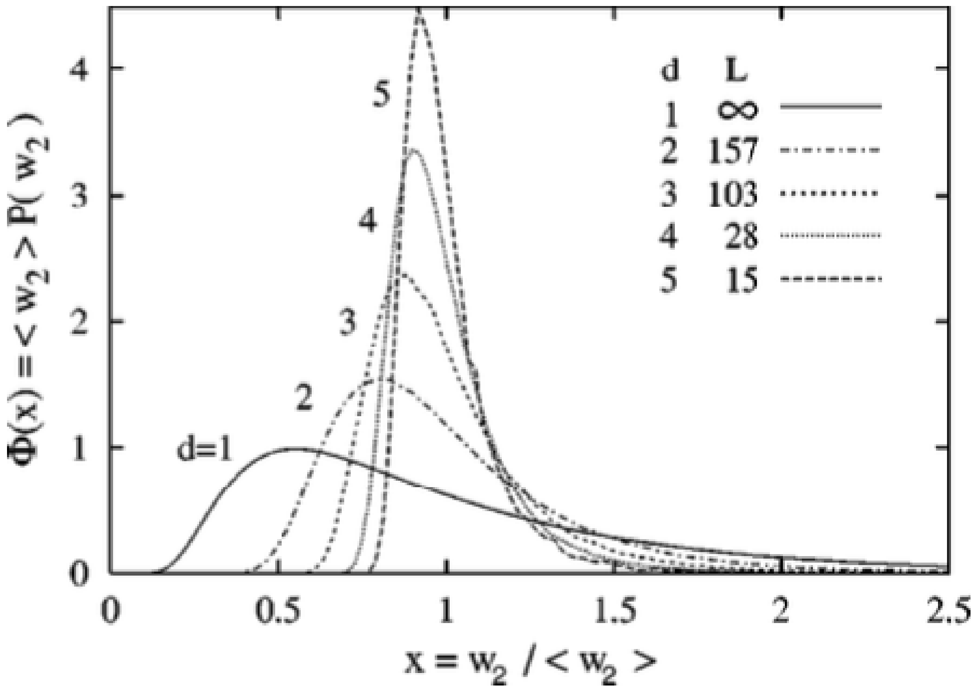
$\chi_{D=2} = 0.393 \pm 0.003 \qquad \omega_{D=2} = 1.1 \pm 0.3.$

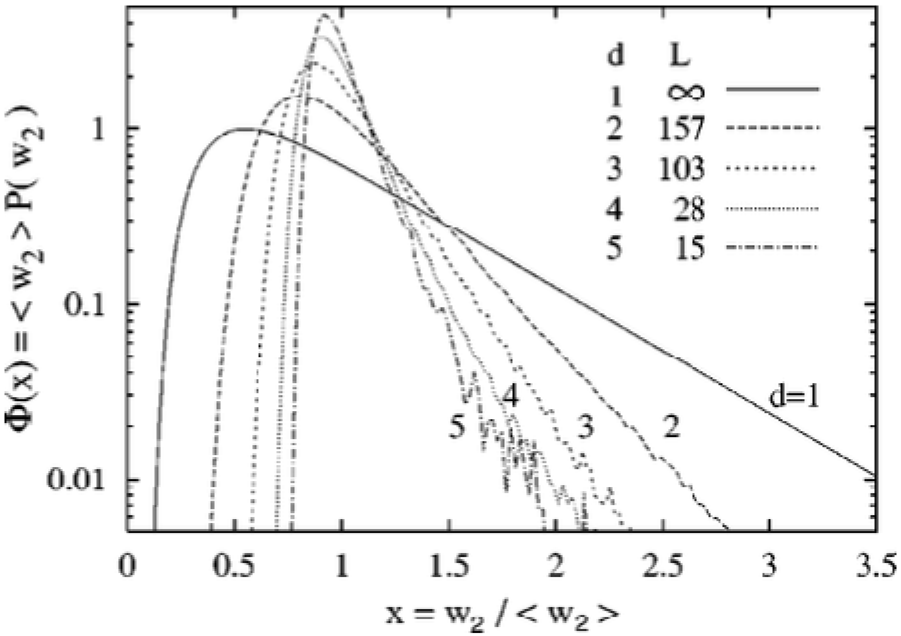


Width distributions

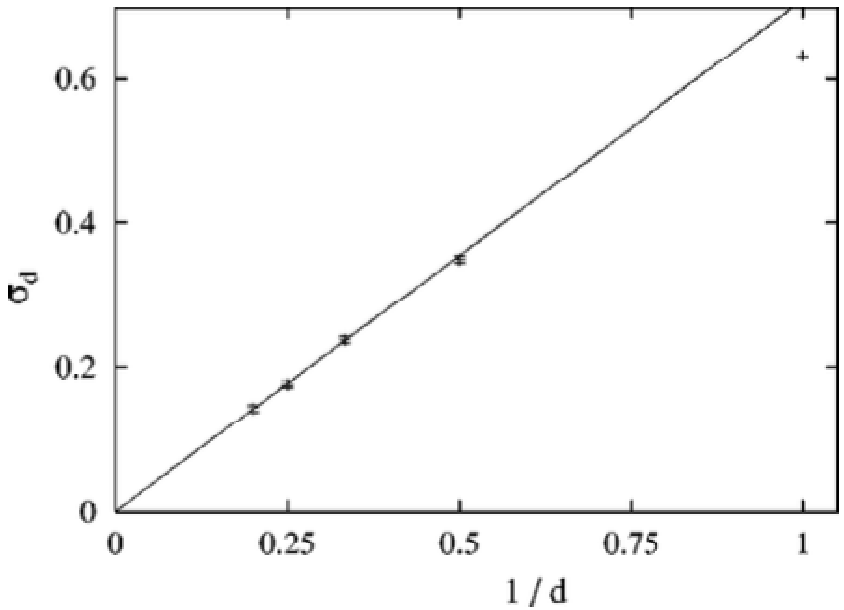
Marinari, Pagnani, GP, Racz

$$w_2 = \frac{1}{A_L} \sum_{\mathbf{r}} [h(\mathbf{r},t) - \bar{h}]^2, \qquad P_L(w_2) \approx \frac{1}{\langle w_2 \rangle_L} \Phi_d\left(\frac{w_2}{\langle w_2 \rangle_L} \right)$$





d L
1 ∞
2 157
3 103
4 28
5 15
d=1
2
3
4
5
Φ(x) = < w2 > P(w2)
x = w2 / < w2 >



σd
1 / d