





# Quantum fluids of light as a platform for non-equilibrium statistical mechanics

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### <u>Why is light different from matter ?</u>

Matter is composed by a huge number of atoms Light beam composed by a huge number of photons

- in vacuo photons travel along straight line at c
- (practically) do not interact with each other
- in standard cavity, thermalization via walls and absorption/emission

optics in vacuo typically dominated by single-particle physics

In suitable photonic structures:

- spatial confinement  $\rightarrow$  effective photon mass
- $\chi^{(3)}$  nonlinearity  $\rightarrow$  photon-photon interactions



Collective behaviour of quantum fluid of light



IC-Ciuti, Quantum Fluids of Light, RMP 85, 299 (2013)

### <u>What about mass?</u>

In vacuo: photons massless, dispersion  $\omega = c |k|$ 

In planar cavity  $\rightarrow$  confinement along *z*, free propagation along *x*, *y* 

Quantization along *z*:  $k_z^{(q)} = q \pi / L_z$ 

Massive dispersion along *x*,*y*:

$$\omega^{(q)}(\mathbf{k}_{\parallel}) = c\sqrt{[k_z^{(q)}]^2 + \mathbf{k}_{\parallel}^2} = c\sqrt{\left(\frac{q\pi}{L_z}\right)^2 + \mathbf{k}_{\parallel}^2} \simeq ck_z^{(q)} + \frac{c}{2k_z^{(q)}}\mathbf{k}_{\parallel}^2$$

Confinement gives effective photon mass  $m_{ph}c^2 = \hbar c k_z^o$ 

- Rest mass  $\rightarrow$  cut-off in the dispersion
- Inertial mass  $\rightarrow$  curvature of dispersion

IC-Ciuti, Quantum Fluids of Light, RMP 85, 299 (2013)





### <u>What about interactions?</u>

Photon-photon interactions exist in QED:

Heisenberg-Euler processes via electron-positron exchange

... but cross section ridiculously small for visible light (recent experiment in accelerator  $\rightarrow$  Nat. Phys. 2017)

How to enhance it?

Replace electron-positron pair (E ~ 1 MeV) with electron-hole pair (E ~ 1 eV)  $\rightarrow$  huge gain factor (10<sup>6</sup>)<sup>6</sup> = 10<sup>36</sup> !!

In optical language:

- $\chi^{(3)}$  nonlinearity  $\leftrightarrow$  local photon-photon interactions
- typical material  $\rightarrow$  spatially local (or quasi-local)  $\chi^{(3)}$

#### Modern exceptional media:

- Rydberg atoms
  - > Ultra-large, long-range nonlinearity in Rydberg-EIT config.
- Superconducting circuits
  - Strong coupling to macroscopic oscillation mode of superconductor device







### How to create and detect the photon gas?



Pump needed to compensate losses due to finite photon lifetime: stationary state is NOT thermodynamical equilibrium

<u>Turn a bug into a feature  $\rightarrow$  Many different pumping schemes available:</u>

- Coherent laser pump: directly injects photon BEC in cavity
- Incoherent (optical or electric) pump: BEC transition similar to laser threshold
- Exotic frequency-dependent incoherent pump  $\rightarrow$  strongly correlated states

Classical and quantum correlations of in-plane field directly transfer to emitted radiation

### <u>Mean-field theory: generalized GPE</u>

$$i\frac{d\psi}{dt} = \left|\omega_{o} - \frac{\hbar\nabla^{2}}{2m} + V_{ext} + g|\psi|^{2} + \frac{i}{2}\left|\frac{P_{0}}{1 + \alpha|\psi|^{2}} - \gamma\right| \psi + F_{ext}$$

Time-evolution of macroscopic wavefunction  $\psi$  of photon/polariton condensate

- standard terms: kinetic energy, external potential  $V_{ext}$ , interactions g, losses  $\gamma$
- under <u>coherent pump</u>: forcing term
- under <u>incoherent pump</u>: polariton-polariton scattering from thermal component give saturable amplification term as in semiclassical theory of laser

 $\rightarrow$  a sort of Complex Landau-Ginzburg equation

To go beyond mean-field theory:

• Exact diagonalization, Wigner representation, Keldysh diagrams, ...

IC-Ciuti, Quantum Fluids of Light, RMP 85, 299 (2013)

## **Part 1:**

**Non-Equilibrium condensation** 

### **<u>2006 - Non-equilibrium Bose-Einstein condensation</u>**





Momentum distribution Kasprzak et al., Nature 443, 409 (2006)



Interference Richard et al., PRL 94, 187401 (2005)



Noise (arb. u.)

#### Many features very similar to atomic BEC



The first atomic BEC M. H. Anderson et al. Science 269, 198 (1995)

0.12 mm

Interference pattern of two expanding atomic BECs M. R. Andrews, Science 275, 637 (1995)

### **<u>The devil</u>** The physicist is in the details: <u>the shape of a non-equilibrium condensate</u>



#### Experimental observations under non-resonant pump:

- condensate shape depends on pump spot size
  - wide pump spot: condensation at k=0
  - narrow pump spot: condensation on a ring of modes at finite |k|

### <u>Physical interpretation of condensation at k≠0</u>

#### **Repulsive interactions**

- outward radial acceleration
- energy conservation

 $E=k^2/2m + U_{int}(r)$ 

- $\rightarrow$  radially increasing flow velocity
- $\rightarrow$  coherent ballistic flow



M. Wouters, IC, and C. Ciuti, PRB 77, 115340 (2008)

#### Narrow spot:

- ballistic free flight outside pump spot  $U_{int}(r)=0$
- emission mostly on free particle dispersion

#### Later expts confirm mechanism –

(J. Bloch's group @ LPN in 1D)

### T-reversal breaking:

- allowed by non-equilibrium
- allows for non-zero current
- also visible as  $n(k) \neq n(-k)$



### **Low-d effects: quasi-BEC → Jacqueline's talk**

(bs)

#### Quasi-condensation features:

- Hohenberg-Mermin-Wagner theorem:
  - at equilibrium, no BEC in d<3
- Non-equilibrium condensation or lasing:
  - no BEC in 1D: exponential decay of g<sup>(1)</sup>(x) (Graham-Haken, 1970; Wouters-IC, PRB 2006)
  - debate in 2D: KPZ nonlin. destroy BKT phase? (Dagvadorj et al., PRX 2015; Altman et al., PRX 2015; Zamora et al., PRX 2017)

#### Experiment @ C2N Palaiseau:

- 1D lattice with array of semiconductor micropillars
- measure space-time coherence function  $g^{(1)}(\Delta x, \Delta t)$
- Coherence only in limited space/time, then smoothly decays → quasi-BEC effect

Fontaine, Squizzato, Baboux, Amelio, Lemaître, Morassi, Sagnes, Le Gratiet, Harouri, Wouters, IC, Amo, Richard, Minguzzi, Canet, Ravets, Bloch, *Observation of KPZ universal scaling in a 1D polariton condensate*, Nature 2022



### **<u>KPZ universality features</u>**



→ Quasi-BEC dynamics involves non-equilibrium effects & interactions between Bogoliubov excitations

Fontaine et al., Observation of KPZ universal scaling in a 1D polariton condensate, Nature 2022

## **<u>Part 1bis:</u>** <u>**Condensation + Topology = ?**</u>

### **2017 - Topological lasing** *a.k.a. non-equilibrium BEC in a topological edge state*

<u>In other words</u>: what happens if gain added to topological photonics model ?





St. Jean, et al., Nat. Phot. '17 <u>System:</u> 1D SSH array of micropillar cavities for exciton-polaritons under incoherent pump

Bahari et al., Science 2017 <u>System:</u> 2D photonic crystal slab, amplification by QWs, magnetic field to break T







Bandres et al., Science 2018 <u>System:</u> array of Si-based ring resonators with optically pumped III-V amplifier layer. Tai-Ji shape to break inversion symmetry

Early theoretical work by Conti & Pilozzi, Solnyshkov, Nalitov & Malpuech... Other expts: Khajavikhan's group, PRL 2018; Klembt et al., Nature 2018; Dikopoltsev et al. Science 2021,...

What new physics in these systems? What technological applications?

### **Topological lasing in 2D models: why interesting?**



#### Topologically trivial system:

- pumping many cavities gives complicate many-mode emission because of unavoidable fabrication disorder
- hard to preserve coherence and fully exploit gain
- serious technological problem for high-power semiconductor laser applications



#### Topological system:

- Non-trivial band topology

   → chiral edge states, unidirectionally
   propagating around (finite) system
- 2D Topolaser operation into edge mode when edge only is pumped (WEG)
- Chiral motion  $\rightarrow$  phase locks many sites

First proposal of topological photonics: Haldane & Raghu, 2006. First expt: Wang et al., Nature 2009 (Soljacic's group @ MIT)

Figures from M. Secli's Msc thesis @ UniTN, 2017 and Secli *et al.*, Phys. Rev. Research 2019 See also: Harari et al., Science 2018.

### **Coherence of topolaser emission (I)**

#### Important fundamental & applied questions:

- What are ultimate limitations of coherence?
- How robust is coherence to disorder?
- What advantage over standard lasers?

#### Laser operation in spatially extended system:

- Linearized theory not enough, crucial role of nonlinearities
- Kardar-Parisi-Zhang model of non-equilibrium stat mech (Canet/Minguzzi, Altman/Diehl, Gladilin/Wouters)
- spatio-temporal scaling properties of phase-coherence

### Topological laser:

- One-dimensional edge state gives effective 1D dynamics
- KPZ spatio-temporal scaling of g<sup>(1)</sup>(x,t)
- Most promising for experiments  $\rightarrow$  periodic boundary conditions





Closely related to polariton quasi-BECs @ C2N Q. Fontaine et al., Nature 2022

I. Amelio and IC, PRX 10, 041060 (2020)

### **Coherence of topolaser emission (II)**



In the presence of static disorder:

- Non-Topological: weak disorder suppresses temporal coherence (mode fragmentation, multimode emission, localization, etc.)
- Topological: robust spatio-temporal coherence, chiral propagation travels through/around defects without backscattering.



#### Technologically important in (semiconductor) laser technology:

Allows to phase lock many individual lasers  $\rightarrow$  strong intensity and high coherence <u>Next steps:</u> extend theory to Class-B lasers. Control instabilities and maintain single-mode emission <u>Fundamental question</u>  $\rightarrow$  effect of convective/absolute instability on coherence properties

I. Amelio and IC, Theory of the coherence of topological lasers, PRX 10, 041060 (2020)

### **Beyond Schawlow-Townes linewidth**

#### Basic laser theory:

- single mode approximation  $\rightarrow$  phase diffusion
- diffusion rate  $\sim 1/N_{phot}$ 
  - $\rightarrow$  Schawlow-Townes linewidth (with Henry factor)

#### Spatially extended, yet small device:

- many modes available for fluctuation dynamics
- linear Bogoliubov regime

   → fluctuation modes decoupled from each other
- only global phase matters  $\rightarrow$  ST linewidth  $\sim N_{phot}$

#### Spatially large device:

- Fluctuations grow large in IR sector (smaller restoring force)
- Fluctuation modes coupled by KPZ-nonlinear terms

$$\partial_t \phi = \frac{1}{2m} \left[ -\frac{\Gamma^{-1}}{2m} \partial_x^4 \phi + \alpha \partial_x^2 \phi - (\partial_x \phi)^2 \right] + \sqrt{\frac{D(1+\alpha^2)}{\dot{n}_0 \dots}} \xi_1$$

• Bogoliubov modes renormalized  $\rightarrow$  different scaling of linewidth  $\sim (N_{phot})^{1/2}$ 

I. Amelio, A. Chiocchetta, IC, Kardar-Parisi-Zhang universality in the linewidth of non-equilibrium 1D Quasi-condensates, PRE (2023)



## **Part 2:**

### **Superfluidity & collective excitation modes**

### **2008 - Superfluid light (under coherent pump)**



#### Figure from LKB-P6 group:

A.Amo, J. Lefrère, S.Pigeon, C.Adrados, C.Ciuti, IC, R. Houdré, E.Giacobino, A.Bramati, *Observation of Superfluidity of Polaritons in Semiconductor Microcavities*, Nature Phys. **5**, 805 (2009)

Theory: IC and C. Ciuti, PRL 93, 166401 (2004).

### **<u>Very non-trivial collective excitation modes - theory</u>**

#### Under a coherent pump:



Depending on detuning / interaction energy, different regimes: gapless, gapped, dynamically unstable... Physically: in contrast to equilibrium, GPE oscillation frequency fixed by pump, not by interaction energy

$$i\frac{d\psi}{dt} = \left[\omega_o - \frac{\hbar\nabla^2}{2m} + V_{ext} + g|\psi|^2\right]\psi + F_{ext}e^{-i\omega_p t}$$

IC and C. Ciuti, PRL 93, 166401 (2004); RMP 85, 299 (2013)

### **Experiment (a) LKB under coherent pump**



Collective excitation modes  $\rightarrow$  peak in (k, $\omega$ ) transmittivity of weak probe

Claude, Jacquet... IC ... Glorieux, High-Resolution Coherent Probe Spectroscopy of a Polariton Quantum Fluid, PRL **129**, 103601 (2022)

## Part 2bis:

**Goldstone mode of a condensate** 

### Sound in photon BECs: non-equilibrium effects

Polariton BEC regime under incoherent pump (i.e. polariton lasing)

Linearize GPE around steady state

- Reservoir R mode at  $-i \gamma_R$
- → Condensate density and phase modes at:

$$\omega_{\pm}(k) = -rac{i\Gamma}{2} \pm \sqrt{[\omega_{Bog}(k)]^2 - rac{\Gamma^2}{4}}$$

with:

$$\omega_{Bog}(k) = \sqrt{rac{\hbar k^2}{2m_{LP}} \left(rac{\hbar k^2}{2m_{LP}} + 2\,\mu
ight)},$$



• Goldstone mode (+) associated to U(1) symmetry breaking is diffusive

M. Wouters and IC, *Excitations in a non-equilibrium polariton BEC*, Phys. Rev. Lett. 99, 140402 (2007)
 Similar results in: M. H. Szymanska, J. Keeling, P. B. Littlewood, PRL 96, 230602 (2006)
 Experiment @ LKB: Claude, Jacquet... arXiv:2310.11903 (Bramati, Glorieux, Giacobino's group)



### <u>Experimental observation @ LKB (I)</u>



- Condensate phase locked with phase fixing beam
- Condensate @ finite  $v \rightarrow$  easier spectroscopy

Claude et al., Observation of the diffusive Nambu-Goldstone mode of a non-equilibrium phase transition, arXiv:2310.11903

### **Experimental observation @ LKB (II)**



Claude et al., Observation of the diffusive Nambu-Goldstone mode of a non-equilibrium phase transition, arXiv:2310.11903

### <u>Experimental observation @ LKB (III)</u>



Wouters, IC, PRA 2006

With phase fixing beam:

- U(1) symmetry explicitly broken
- Gapped dispersion
- Goldstone mode disappears, replaced by broad mode



Claude et al., Observation of the diffusive Nambu-Goldstone mode of a non-equilibrium phase transition, arXiv:2310.11903

### **Consequences on superfluidity**



Long-range coherence  $\rightarrow$  metastability of supercurrents (mode stability of ring lasers)

#### Interaction with defect: naïf Landau argument

- Landau critical velocity  $v_{L} = \min_{k} [\omega(k) / k] = 0$  at non-equilibrium BEC
- Any moving defect expected to emit phonons

M. Wouters, IC, Superfluidity and Critical Velocities in Nonequilibrium BECs, PRL 105, 020602 (2010).

### **But nature is always richer than expected...**

Steady-state  $\rightarrow$  well defined  $\omega$ Defect  $\rightarrow$  k not a good quantum number

(Complex) k vs.(real) w dispersion

#### Low v :

- emitted  $k_{\parallel}$  purely imaginary
- no real propagating phonons
- perturbation localized around defect

#### <u>Critical velocity $v_c < c$ :</u>

- corresponds to bifurcation point
- decreases with  $\Gamma / \mu$

#### High v:

- emitted propagating phonons:
  - $\rightarrow$  Cerenkov cone
  - $\rightarrow$  parabolic precursors
- spatial damping of Cerenkov cone





M. Wouters, IC, Superfluidity and Critical Velocities in Nonequilibrium BECs, PRL 105, 020602 (2010).

## Part 4: <u>The future:</u> <u>High-dimensional</u>

<u>and/or</u> strongly interacting fluids of light

### How to create synthetic dimensions for photons?



Different modes of ring resonators  $\rightarrow$  synthetic dimension *w* Tunneling along synthetic *w*:

- strong beam modulates resonator  $\varepsilon_{ij}$  at  $\omega_{FSR}$  via optical  $\chi^{(3)}$
- neighboring modes get linearly coupled
- phase of modulation  $\rightarrow$  hopping phase along synthetic *w*

Extends Fan's idea of synthetic gauge field via time-dependent modulation (Nat. Phys. 2008)

Theory: T. Ozawa, N. Goldman, O. Zilberberg, H. M. Price, and IC, Synthetic Dimensions in Photonic Lattices: From Optical Isolation to 4D Quantum Hall Physics, PRA (2016) Many experimental realizations: Fan, St.-Jean, Cardano/Marrucci, ...

### 1+1 array: chiral edge states & optical isolation

- 1 (physical) + 1 (synthetic) dimensions: Hofstadter model
  - Bulk topological invariant  $\rightarrow$  Chern number
    - > measured via Integer Quantum Hall effect
  - Chiral states on edges:
    - > Physical edges along x
    - > Synthetic edges via design of  $\varepsilon(\omega)$ 
      - (e.g. inserting absorbing impurities in chosen sites)  $\rightarrow$  topologically protected optical isolator



Absorbing row of sites





### 3+1 array: 4D Quantum Hall physics

У

4D magneto-electric response Nonlinear integer QH effect

> Lateral shift of photon intensity distribution in response to external synth-E and synth-B:

- > only present with both E & B
  - > proportional to 2<sup>nd</sup> Chern



T. Ozawa, N. Goldman, O. Zilberberg, H. M. Price, and IC, *Synthetic Dimensions in Photonic Lattices: From Optical Isolation to 4D Quantum Hall Physics*, PRA 93, 043827 (2016) See also recent charge pumping experiments with atoms ( Lohse et al. Nature '18) and light (Zilberberg et al. Nature '18)

#### Natural question in this school:

- add gain to the model
- probe KPZ & roughening transition in high-d
- understand it in terms of mode-locked laser operation

### **Strong photon interactions: photon blockade**

Driven-dissipative Bose-Hubbard model:

$$H_0 = \sum_i \hbar \omega_\circ \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \hbar rac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) + \sum_i F_i(t) \hat{b}_i^\dagger + h.c.$$

- Array of single-mode cavities at  $\omega_0$ , tunnel coupling J, losses  $\gamma$
- Polariton interactions: on-site interaction U due to optical nonlinearity
- If U >> γ & J, coherent pump resonant with 0→1, but not with 1→2.
   Photon blockade → Effectively impenetrable photons
   Opposite regime than non-interacting photons of Maxwell's eqs.

Single-cavity blockade observed in many platforms since the 2000s,



IC-Ciuti, Quantum Fluids of Light, RMP 85, 299 (2013)



Fluid of spin excitations in lattice of Rydberg atoms. (Broways, Lukin,...)



### <u>Many-cavity system</u>

#### Scale up to large systems:

- Fabricate array of cavities with strong interactions
- Find a way to stabilize desired many-body state

#### Non-Markovian pump:

- Inverted emitters  $\rightarrow$  Lorentzian emission line around  $\omega_{at}$
- Photon injection only active if many-body transition is near resonance, otherwise losses dominate
- For P >> γ photons injected until band is full (MI) or many-body gap develops (FQH)
- Many-body gap blocks excitation to higher states and larger N

 $\rightarrow$  <u>desired correlated state gets stabilized !</u>





<u>General idea:</u> Kapit, Hafezi, Simon, PRX 2014 Lebreuilly et al. CRAS (2016)

Umucalilar-IC, PRA 2017 Lebreuilly, Biella et al., PRA 2017

### **Mott insulator of light**

- Most naive non-Markovian master equation: frequency-dependent emission → rescaled jump operators
- driven-dissipative steady state stabilizes strongly correlated many-body states e.g. Mott-insulator, FQH...
- resembles low-T equilibrium
- (in principle) no restriction to small N<sub>ph</sub> only requirement → many-body energy gap



$$\bar{\mathcal{L}}_{em}(\rho_{ph}) = \frac{\Gamma_{em}}{2} \sum_{i=1}^{k} \left[ 2\bar{a}_{i}^{\dagger}\rho_{ph}\bar{a}_{i} - \bar{a}_{i}\bar{a}_{i}^{\dagger}\rho_{ph} - \rho_{ph}\bar{a}_{i}\bar{a}_{i}^{\dagger} \right]$$
$$\left\langle f' \left| \bar{a}_{i}^{\dagger} \right| f \right\rangle = \frac{\Gamma_{pump}/2}{\sqrt{(\omega_{at} - \omega_{f',f})^{2} + (\Gamma_{pump}/2)^{2}}} \left\langle f' \right| a_{i}^{\dagger} \left| f \right\rangle$$



Lebreuilly, Biella et al., 1704.01106 & 1704.08978 Related work in Kapit, Hafezi, Simon, PRX 2014

First experimental observation of a Mott insulator state of impenetrable photons

### <u>Conclusions</u>

Quantum fluids of light  $\rightarrow$  ideal platform for non-equilibrium statistical mechanics

#### Non-equilibrium condensation (aka lasing):

- Peculiar shape of condensate under localized pump
- KPZ features in coherence function of emission
- Topological lasing  $\rightarrow$  spatially homogeneous, periodic boundary conditions

#### Superfluid light:

- Superfluid probed in frictionless flow past an obstacle
- Wide range of collective & hydrodynamic excitation modes
- Diffusive Goldstone mode of non-equilibrium condensate, gapped when symmetry explicitly broken

#### The future:

- Strongly interacting quantum fluids of light  $\rightarrow$  quantum non-equilibrium statistical mechanics
- High-dimensional systems beyond  $d=2 \rightarrow$  promising platform for statistical models in high-d?

### If you wish to know more...



REVIEWS OF MODERN PHYSICS, VOLUME 85, JANUARY-MARCH 2013

#### **Quantum fluids of light**

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I. Carusotto, C. Ciuti, Rev. Mod. Phys. 85, 299 (2013)



PhD positions soon available

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REVIEWS OF MODERN PHYSICS, VOLUME 91,

#### **Topological photonics**

Ozawa, Price, Amo, Goldman, Hafezi, Lu, Rechtsman, Schuster, Simon, Zilberberg, <u>IC</u>, RMP **91**, 015006 (2019)

Non-equilibrium Bose–Einstein condensation in photonic systems

Jacqueline Bloch<sup>©</sup><sup>1</sup><sup>∞</sup>, Iacopo Carusotto<sup>©</sup><sup>2∞</sup> and Michiel Wouters<sup>3∞</sup> Review article on Nat. Rev. Phys. (2022)

We acknowledge generous financial support from:

nature

physics



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Horizon 2020 European Union funding for Research & Innovation



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Funded by the European Union NextGenerationEU

#### FOCUS | REVIEW ARTICLE

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### Photonic materials in circuit quantum electrodynamics

lacopo Carusotto<sup>1</sup>, Andrew A. Houck  $^{\odot 2}$ , Alicia J. Kollár $^{3,4}$ , Pedram Roushan $^5$ , David I. Schuster $^{6,7}$  and Jonathan Simon  $^{\odot 4,7}$ 

Review article on Nature Physics (2020)

### **Coherence of topolaser emission (II)**



#### Coherence of laser emission:

- Physical system necessarily finite
   → crossover from stretched-exp to exp decay
   at long times for given size N<sub>x</sub>
- crossover from Schawlow-Townes  $\tau_c$  to KPZ at long times for increasing size  $N_x$
- imposes fundamental limitation to  $\tau_{_{c}}$
- physics similar to non-topological 1D chains, but...



I. Amelio and IC, *Theory of the coherence of topological lasers*, PRX 10, 041060 (2020)

### **Mott-superfluid phase transition: theory**



Exact description of non-Markovianity of emitter

- $\rightarrow$  explicit inclusion of two-level emitters:
- Markovian incoherent pump  $\Gamma_{\rm p}$
- Coupling to cavity mode  $\Omega_{R} \rightarrow$  emission irreversible via  $\Gamma$ 's
- Frequency-dependent emission of linewidth  $\Gamma_{\rm P}$ Biella, Lebreully et al., 1704.08978

#### Superfluid-insulator non-equilibrium phase transition







#### Interesting behaviour of collective excitation modes across transition:

- Linearized Gutzwiller approach; visible in transmission/reflection/FWM
- Gap closes in Mott insulating phase approaching critical point
- Diffusive Goldstone mode in superfluid
- Similar physics as in polariton BECs (Wouters, Szymanska/Keeling, Diehl, expt: Bramati)

F. Caleffi, M. Capone, IC, Collective excitations of a strongly-correlated non-equilibrium photon fluid across the Mott/superfluid phase transition, arXiv:2211.07246; Fabio Caleffi, PhD thesis @ SISSA (2022)