# Kardar Parisi Zhang universal scaling in the coherence of polariton condensates

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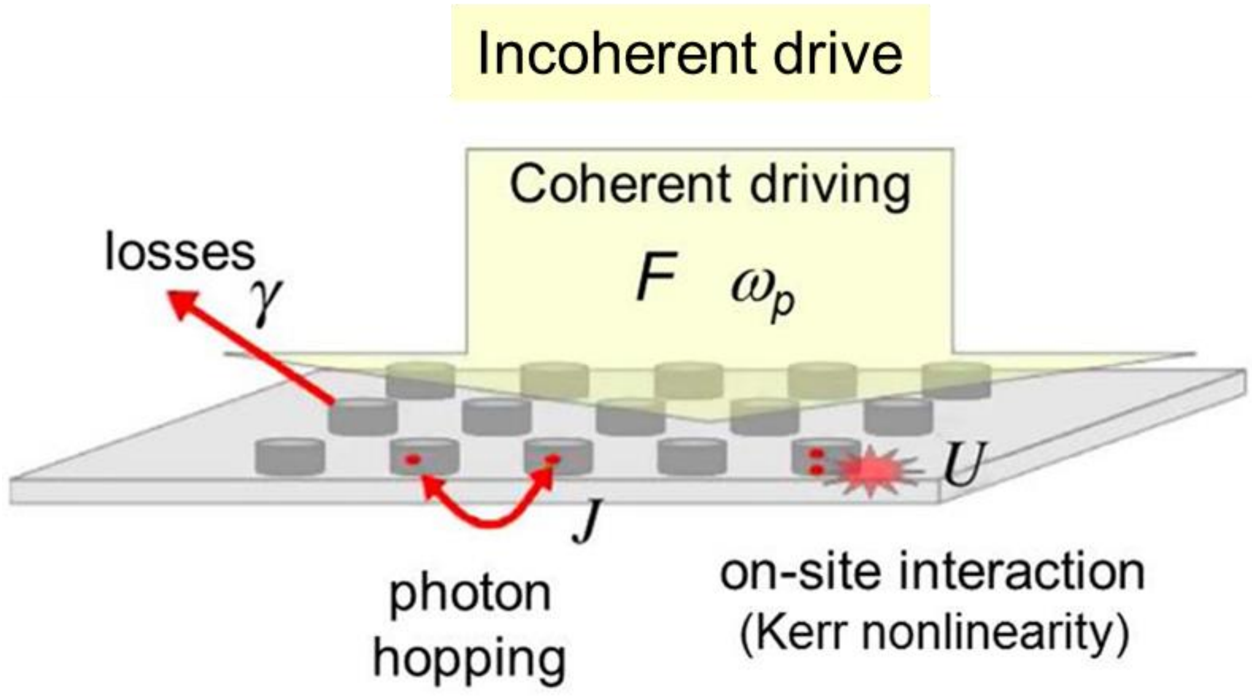
M. Wouters

# Driven dissipative non-linear lattices

## Driven dissipative Bose Hubbard

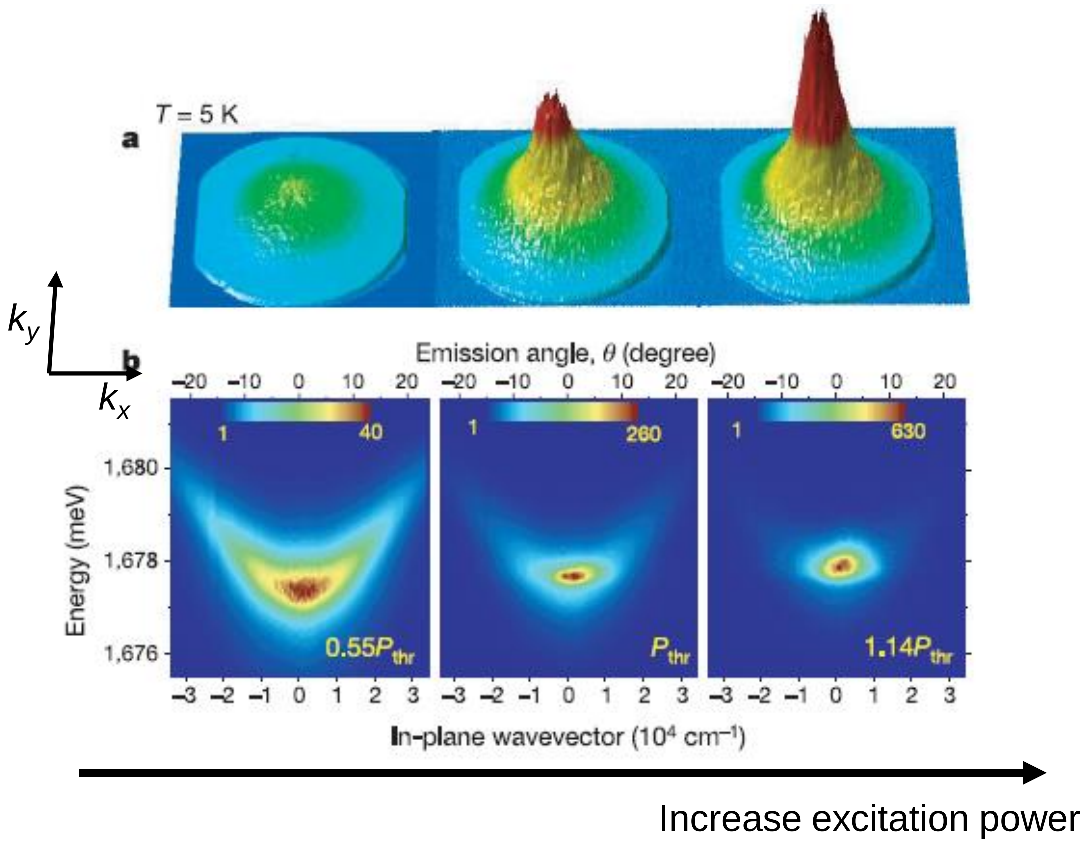



- Mean field
- Noise induced by drive and dissipation
- Quantum correlations

C. Ciuti & I. Carusotto, Rev. Mod. Phys. **85**, 299 (2013)

# Bose–Einstein condensation of exciton polaritons

J. Kasprzak[1], M. Richard[2], S. Kundermann[2], A. Baas[2], P. Jeambrun[2], J. M. J. Keeling[3], F. M. Marchetti[4], M. H. Szymańska[5], R. André[1], J. L. Staehli[2], V. Savona[2], P. B. Littlewood[4], B. Deveaud[2] & Le Si Dang[1]




Benoid Deveaud

Le Si Dang

Kasprzak *et al.* Nature, **443**, 409 (2006)

See also H. Deng et al. Science (2002), R. Balili et al., Science (2007)

# Polariton superfluidity: resonant drive

Iacopo  Carusotto

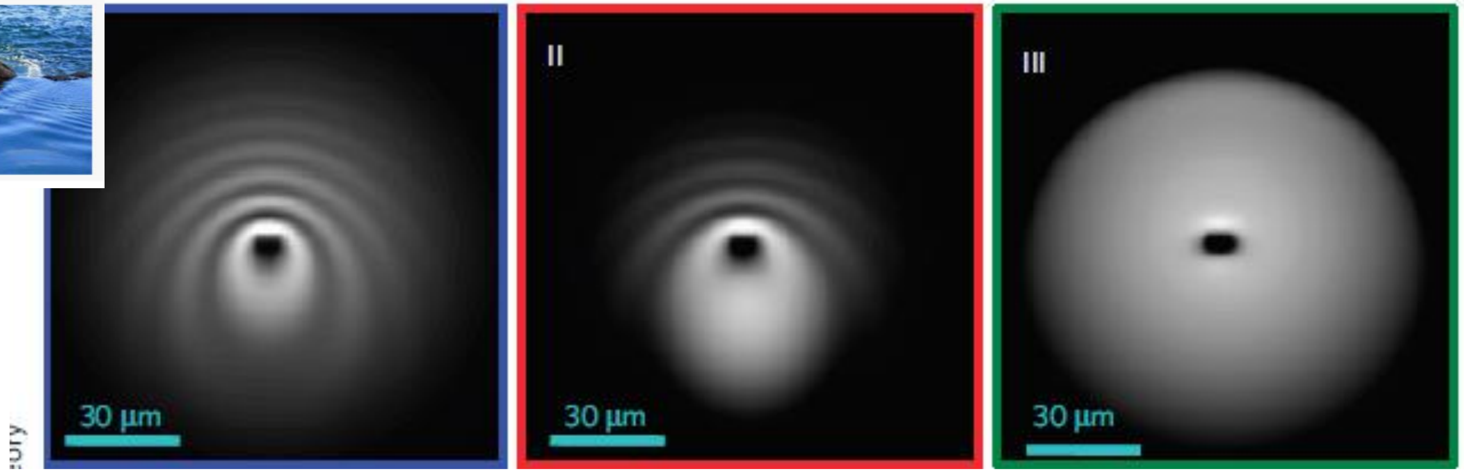
Cristiano Ciuti

Alberto Amo

Alberto Bramati

Elisabeth Giacobino

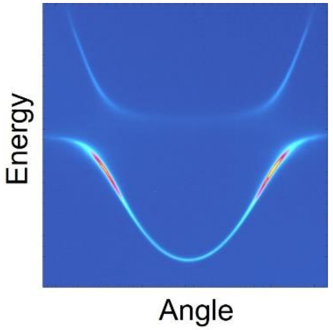
$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(x) + U|\psi|^2 - i\frac{\gamma}{2}\right]\psi + iF(x)e^{-i(\omega t - k_p x)}$$

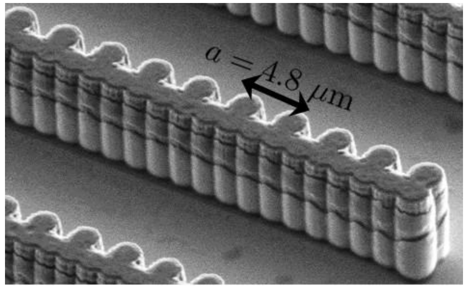



C. Ciuti and I. Carusotto PRL 242, 2224 (2005)
A. Amo et al. Nature Physics **5**, 805 (2009)
C. Ciuti & I. Carusotto, Rev. Mod. Phys. **85**, 299 (2013)

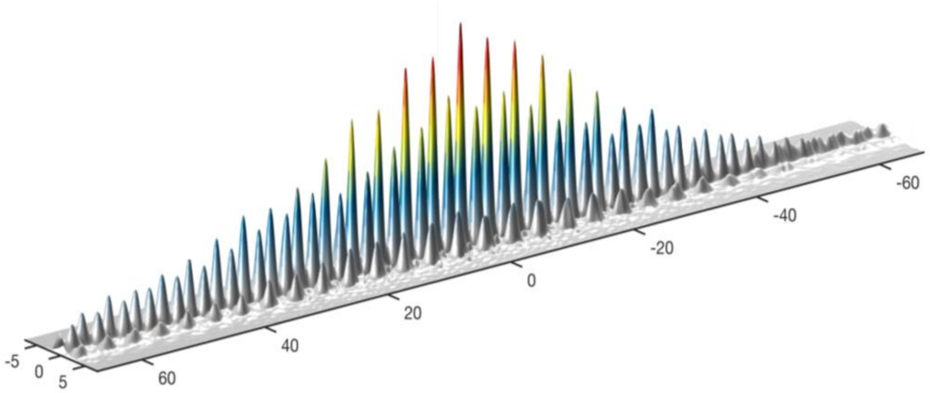
# Outline of the talk

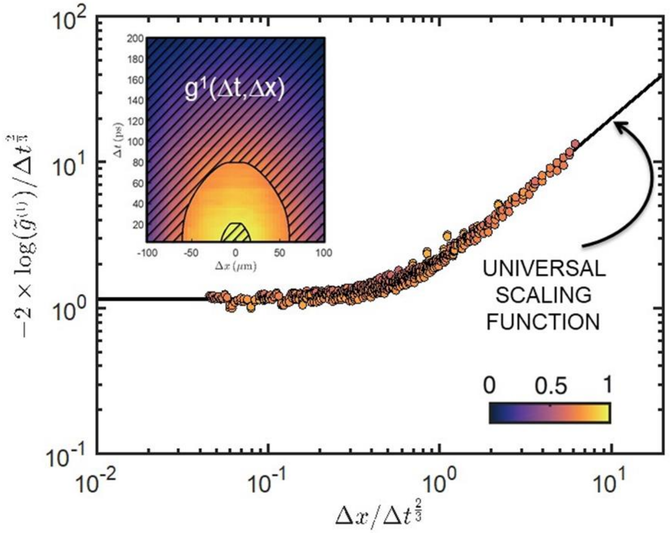
- Introduction : synthetic polariton matter



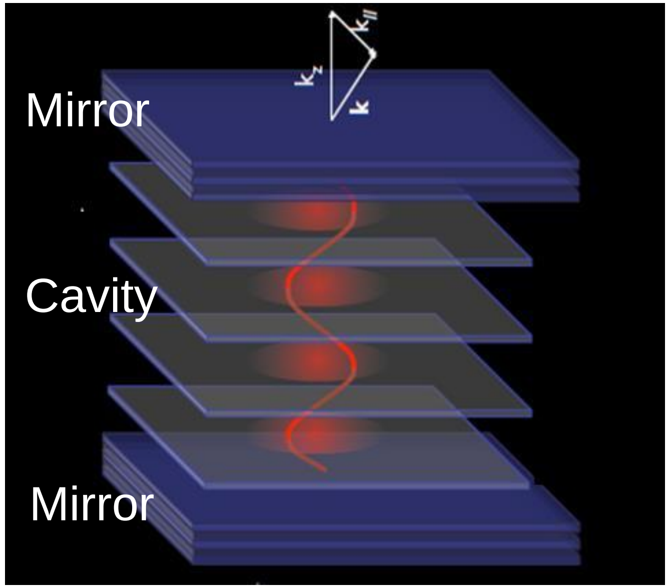




- Polariton condensates belong to the Kardar Parisi Zhang universality class

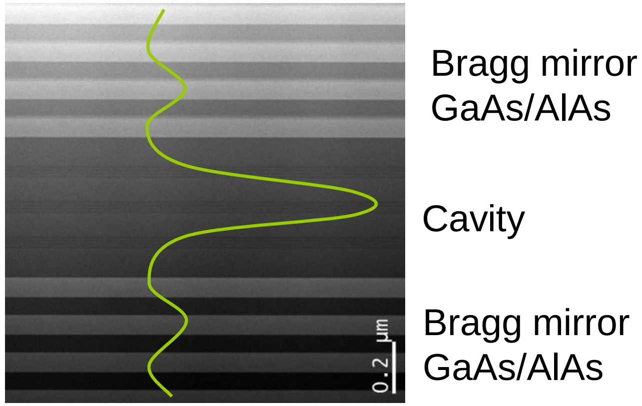


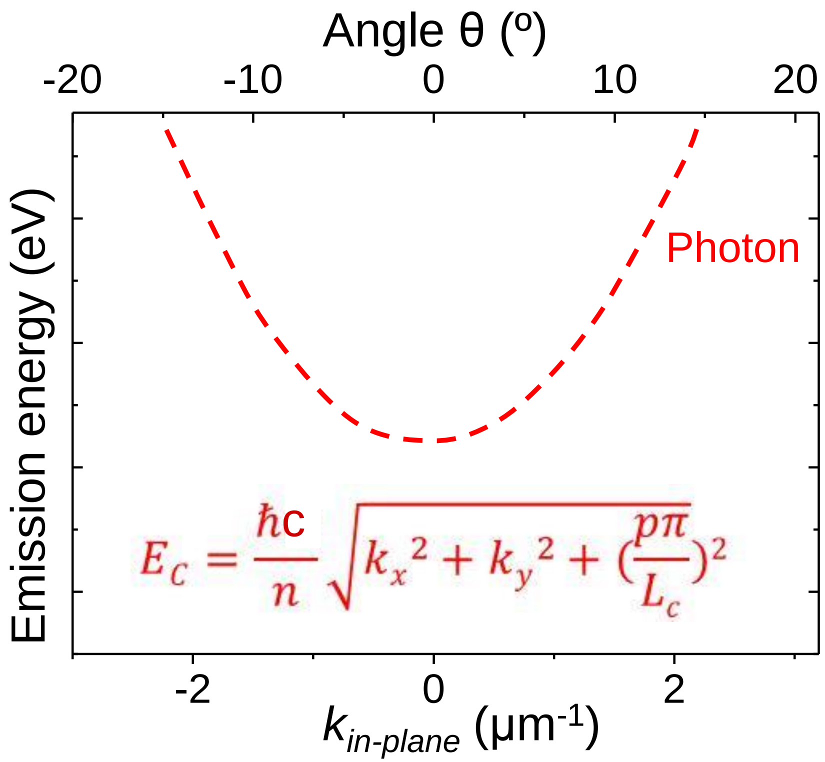
# Microcavity polaritons




**Optical cavity**



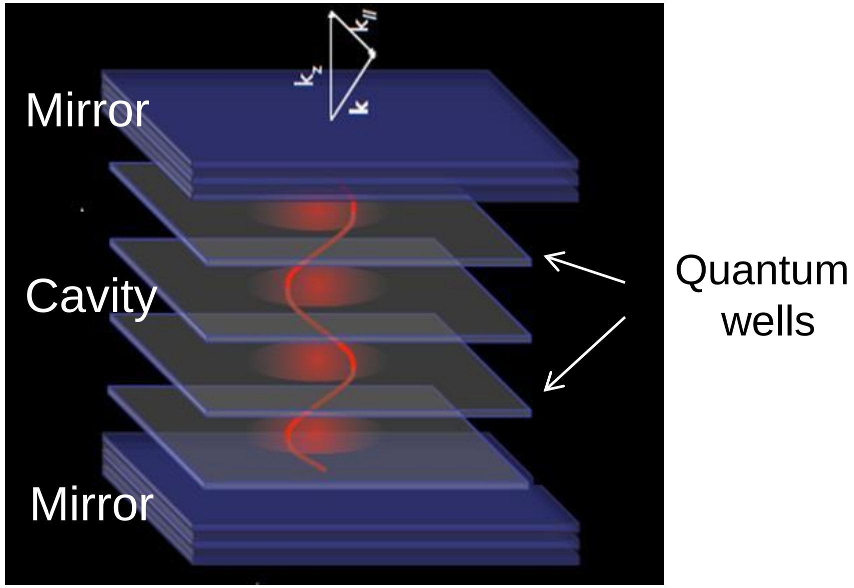

TEM, G. Patriarche, LPN

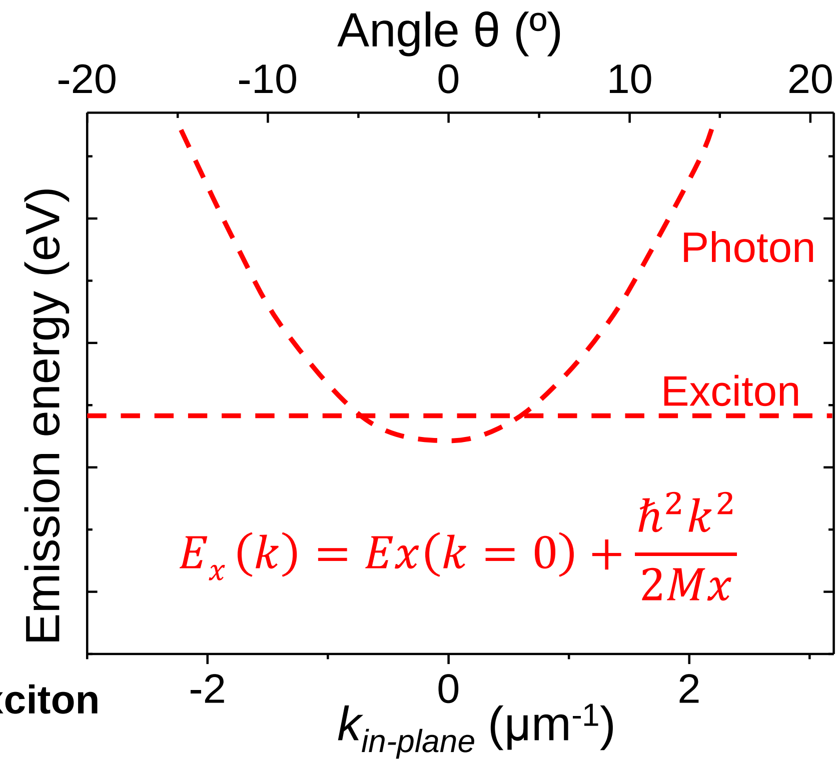



$$E_C(k) = Ec(k=0) + \frac{\hbar^2 k^2}{2Mp_{hot}}$$

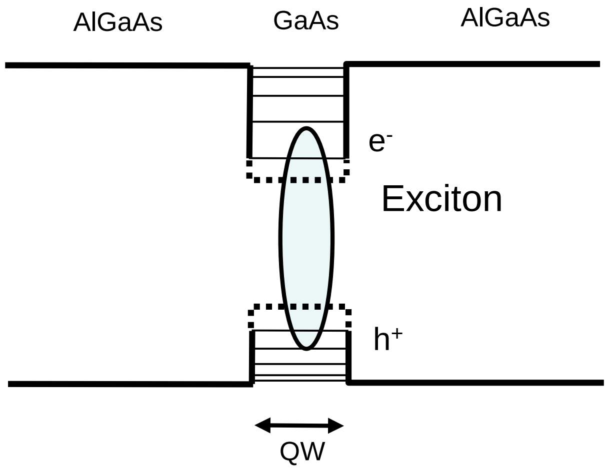
with $M_{phot} = \frac{p^2\pi^2\hbar^2}{L_c{}^2 n^2}$

# Microcavity polaritons

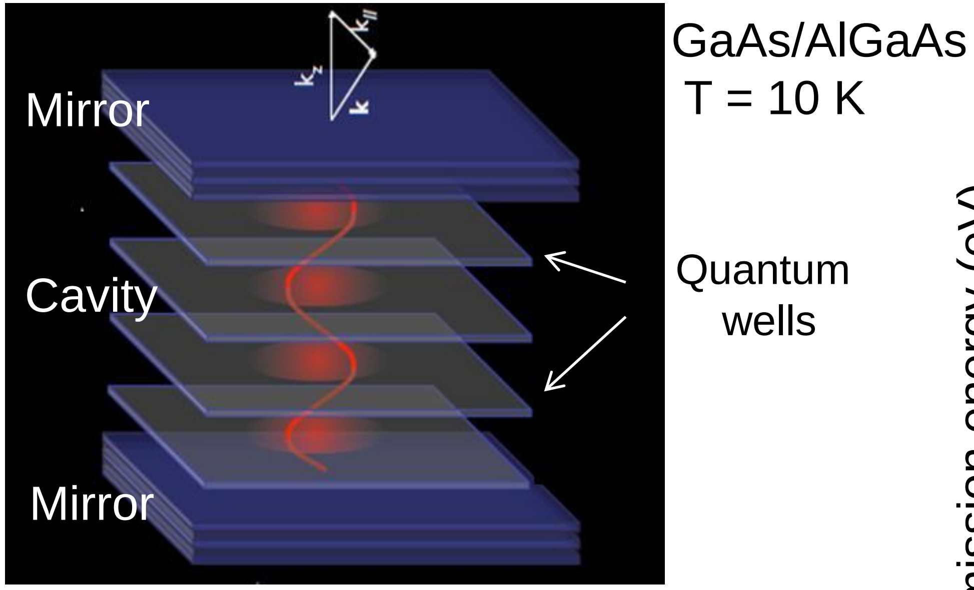


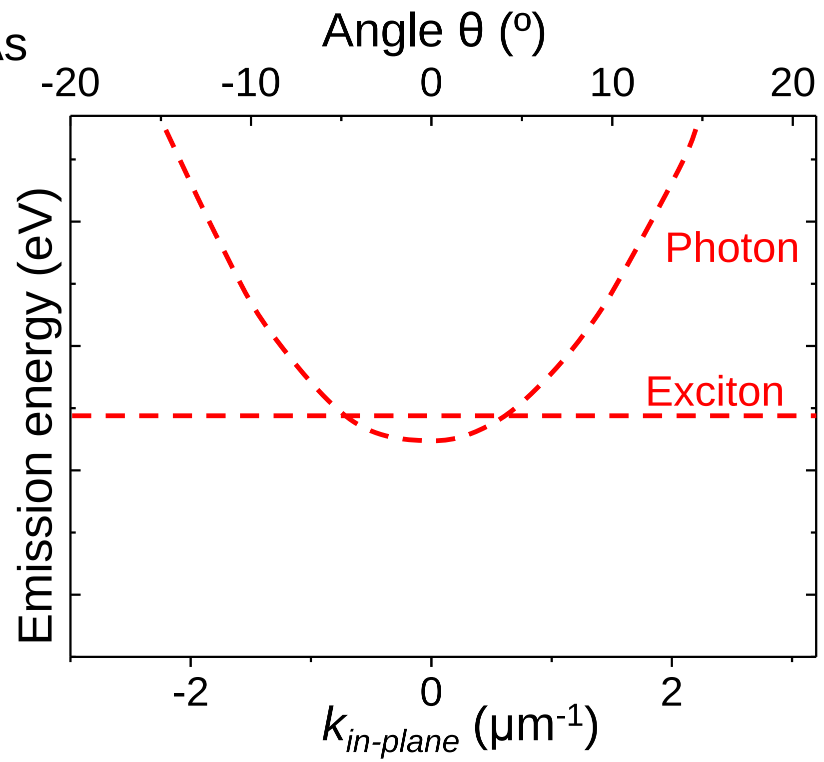




**Quantum well exciton**



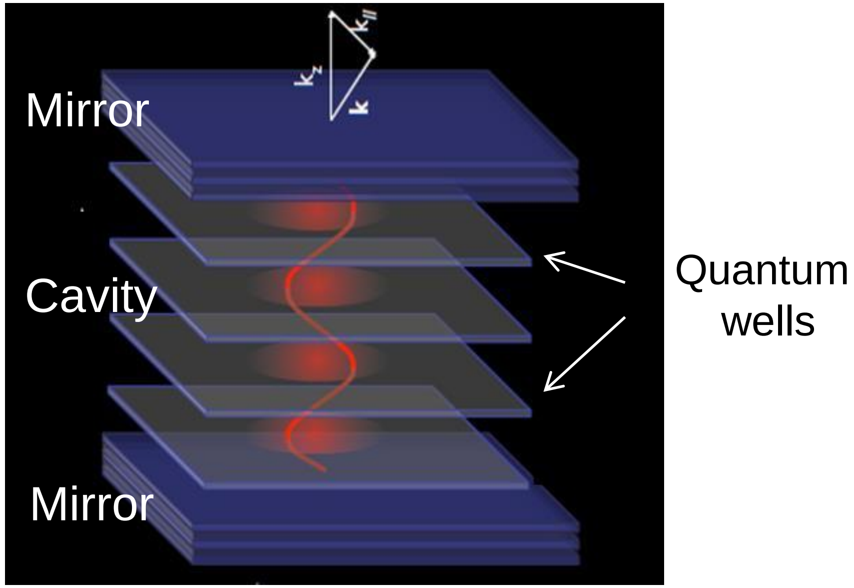
# Microcavity polaritons

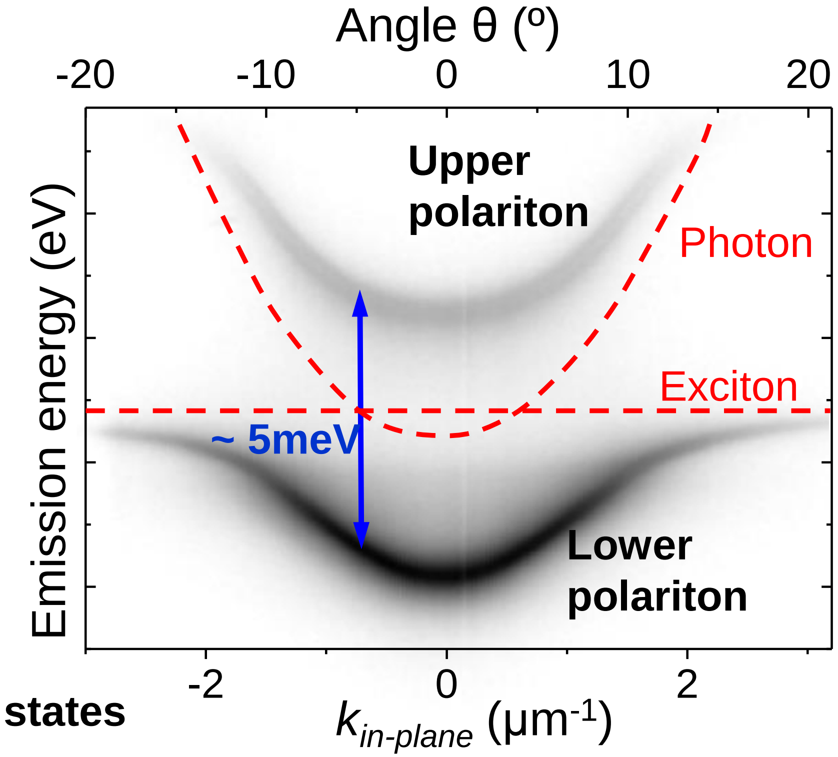


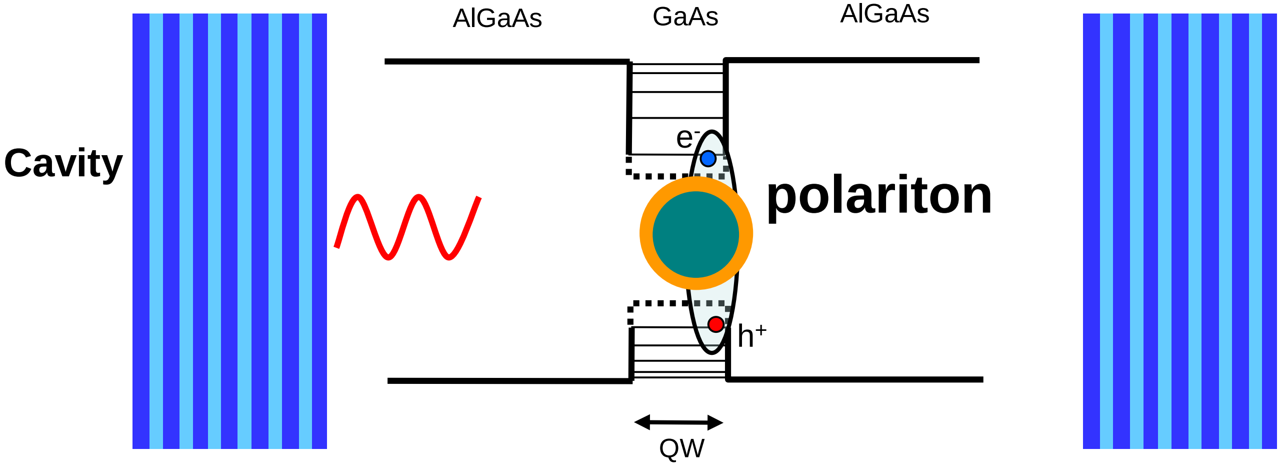

$$\hat{H} = \hat{H}_C + \hat{H}_X + \sum_{\mathbf{k}} \frac{\hbar\Omega_R}{2}\left(\hat{a}^\dagger_{\mathbf{k}}\hat{b}_{\mathbf{k}} + \hat{b}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{k}}\right)$$

# Microcavity polaritons



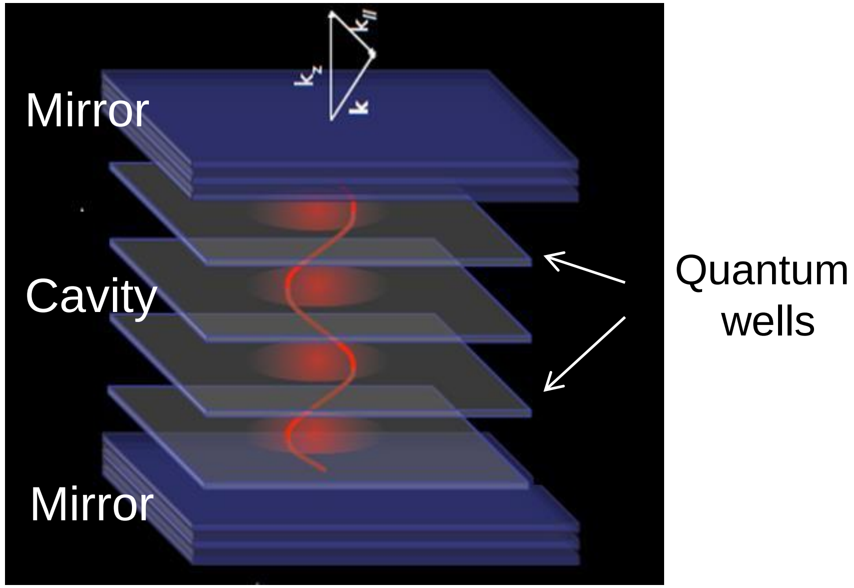

**Microcavity polaritons : mixed exciton-photon states**

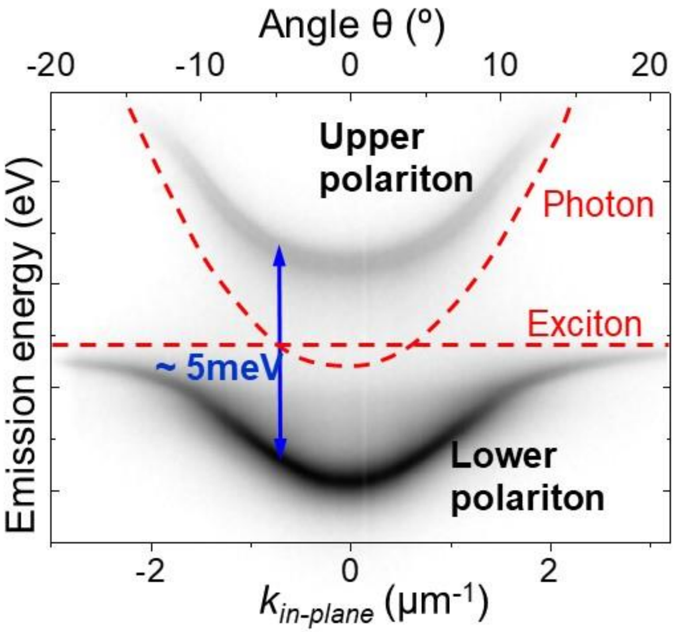



Courtesy D.Sanvitto

Claude Weisbuch
PRL **69**, 3314 (1992)

# Microcavity polaritons




**Microcavity polaritons : mixed exciton-photon states**

**Properties** $$|1\mathrm{LP}_{\mathbf{k}}, 0_{\mathbf{k}}\rangle = (\cos\theta_{\mathbf{k}}\hat{a}^{\dagger}_{\mathbf{k}} + \sin\theta_{\mathbf{k}}\hat{b}^{\dagger}_{\mathrm{k}})\,|0_{\mathbf{k}}, 0_{\mathbf{k}}\rangle$$

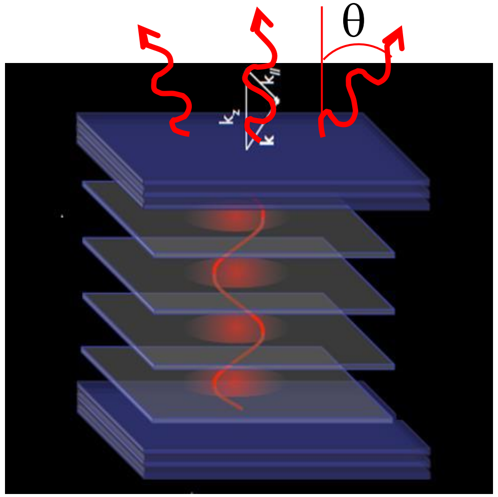
Photons Excitons

- Photonic component ➡ Confinement in microstructures; Dissipation
- Excitonic component ➡
  - Interactions - $\chi^{(3)}$ (dominated by exchange)
  - Gain (lasing)
  - Sensitivity to magnetic field

Claude Weisbuch PRL **69**, 3314 (1992)

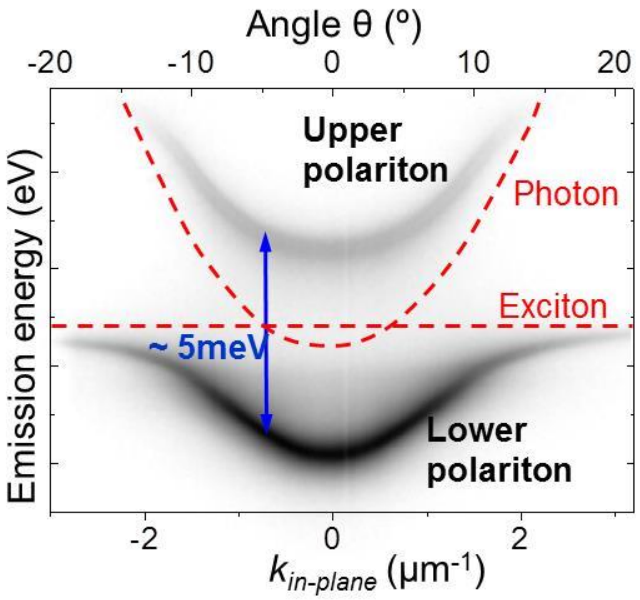
C. Ciuti & I. Carusotto, Rev. Mod. Phys. **85**, 299 (2013)

# Probing polariton states

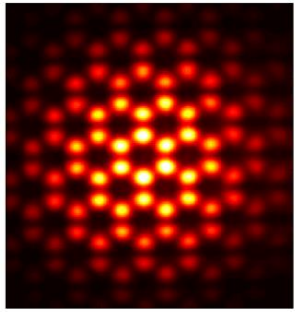



$k_{//} = \omega/c \sin(\theta)$

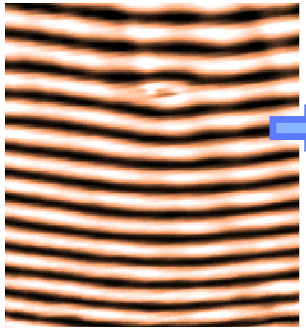
## Imaging of k-space




## Imaging of real space

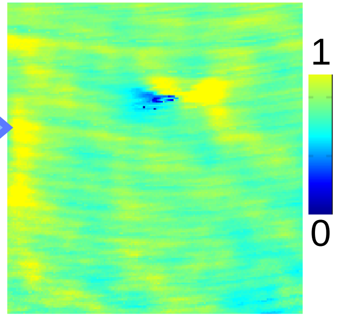


Density



Interferometry

- vortices
- solitons




Coherence

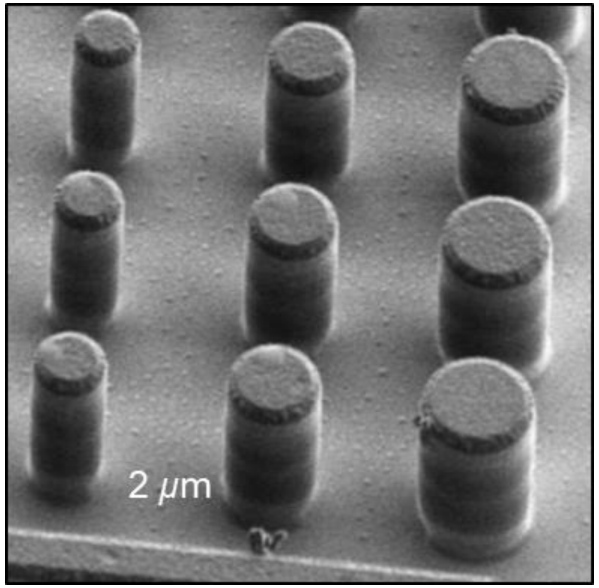
$g^{(1)}$

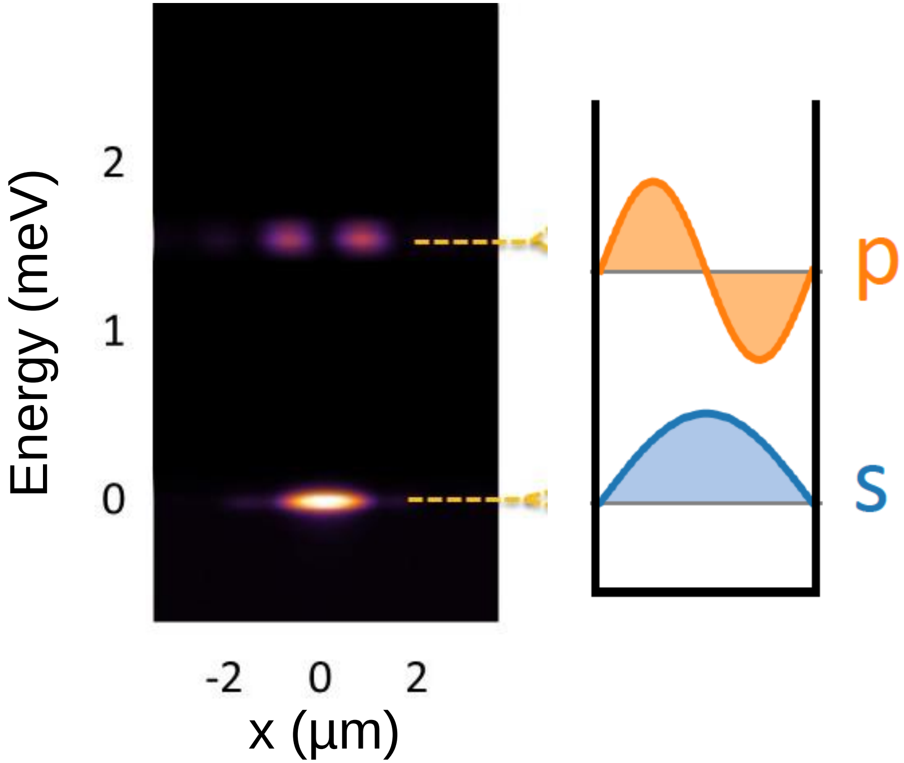
$g^{(2)}$

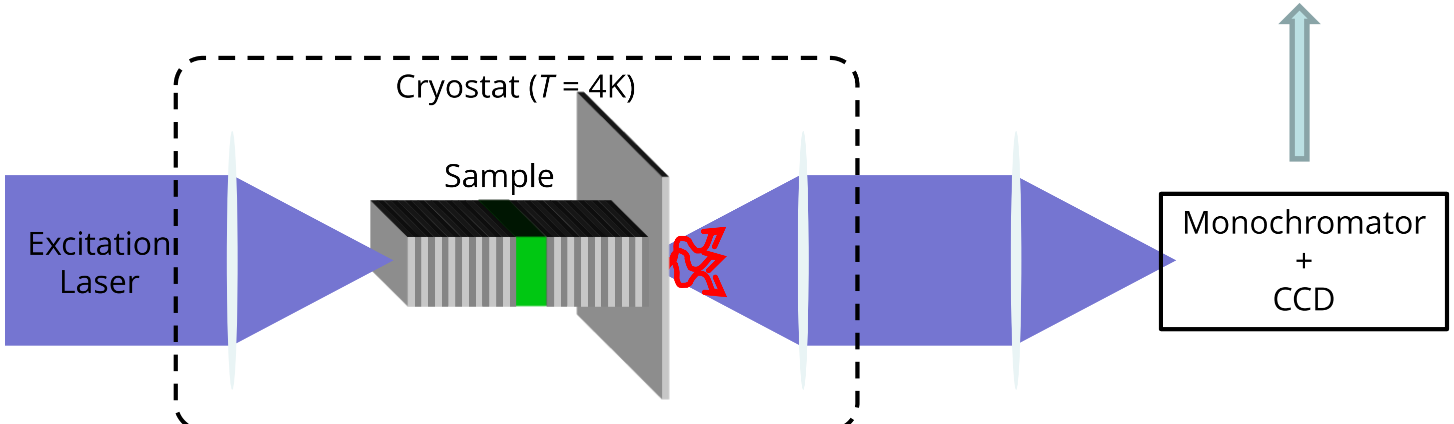
C. Ciuti & I. Carusotto, Rev. Mod. Phys. **85**, 299 (2013)

# Lattices of coupled micropillars

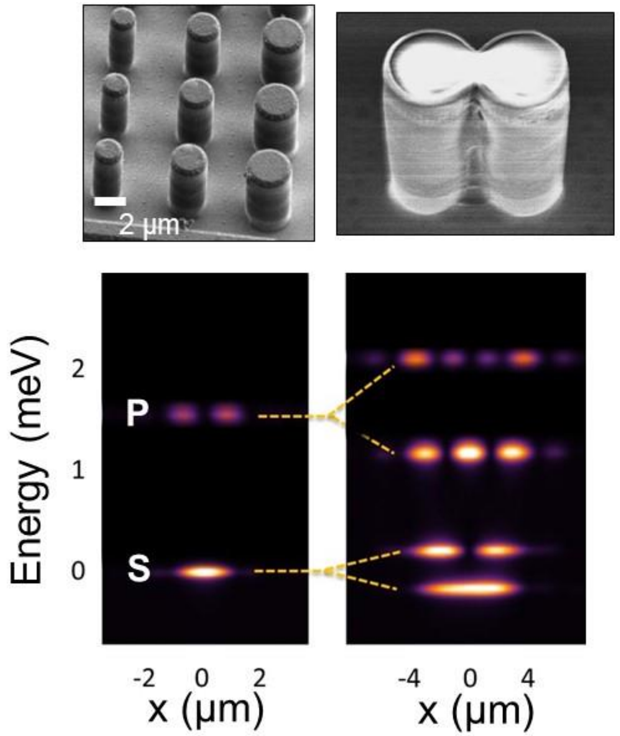
Building block

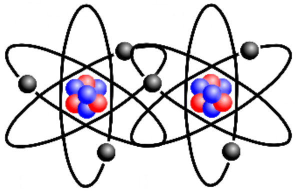


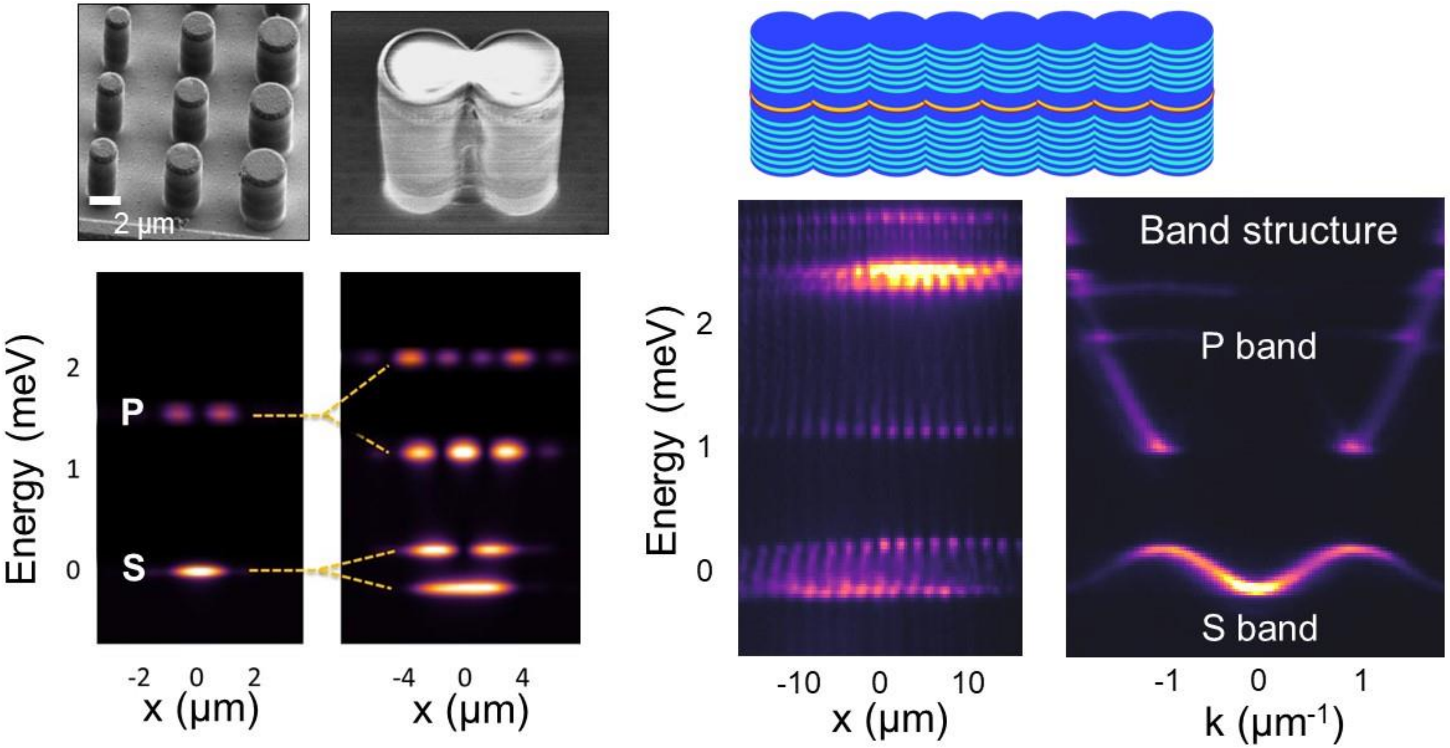
# Lattices of coupled micropillars



C. Ciuti & I. Carusotto, Rev. Mod. Phys. **85**, 299 (2013)
Compte Rendus Physique Vol. 17, Issue 8, Pages 805-956 (2016) Physique des polaritons:  Edité par A. Amo, J. Bloch and I. Carusotto

# Lattices of coupled micropillars

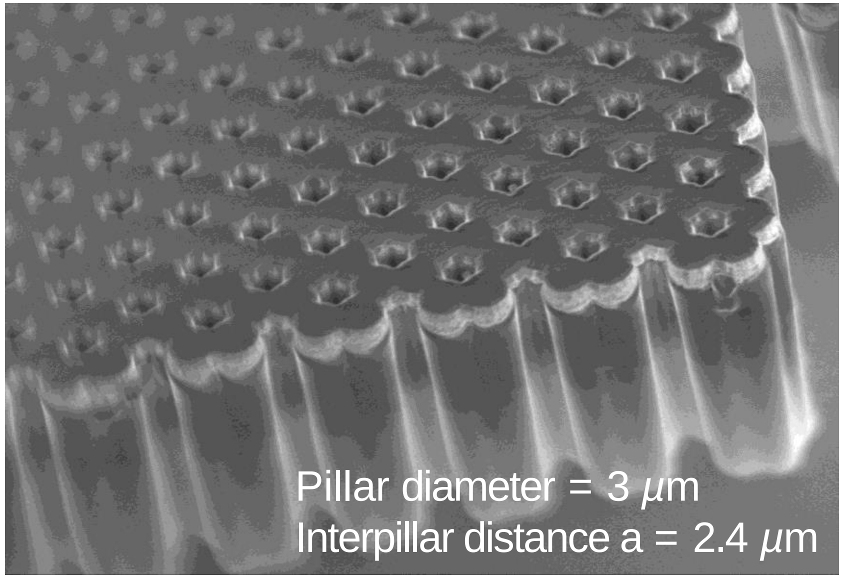


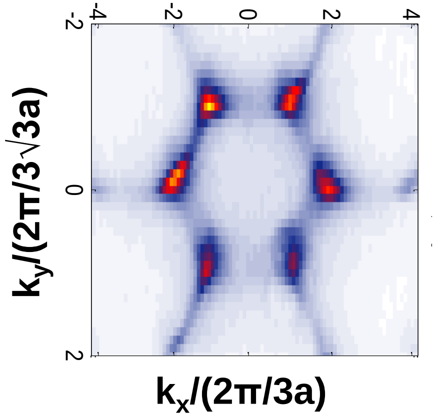

**Correspondance** : Wavefunction = electric field
Spin = Polarisation

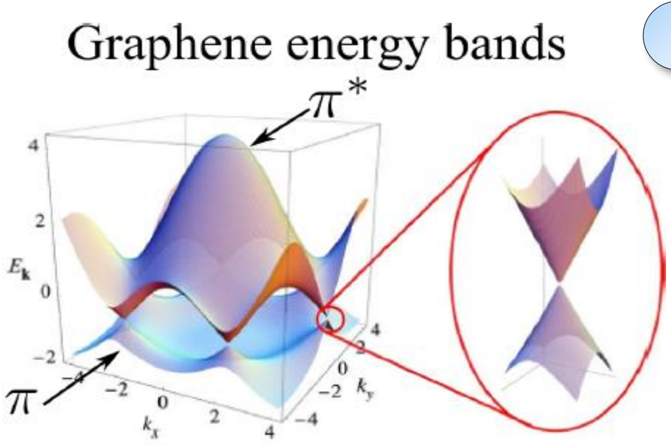
$$i\hbar\frac{\partial\psi_n}{\partial t} = \left(\hbar\omega_n - i\frac{\gamma_n}{2}\right)\psi_n + g\,|\psi_n|^2\psi_n - \sum_m J_{n,m}\psi_m + F_n e^{-i\omega t+\varphi_n}$$

C. Ciuti & I. Carusotto, Rev. Mod. Phys. **85**, 299 (2013)
Compte Rendus Physique Vol. 17, Issue 8, Pages 805-956 (2016) Physique des polaritons: Edité par A. Amo, J. Bloch and I. Carusotto

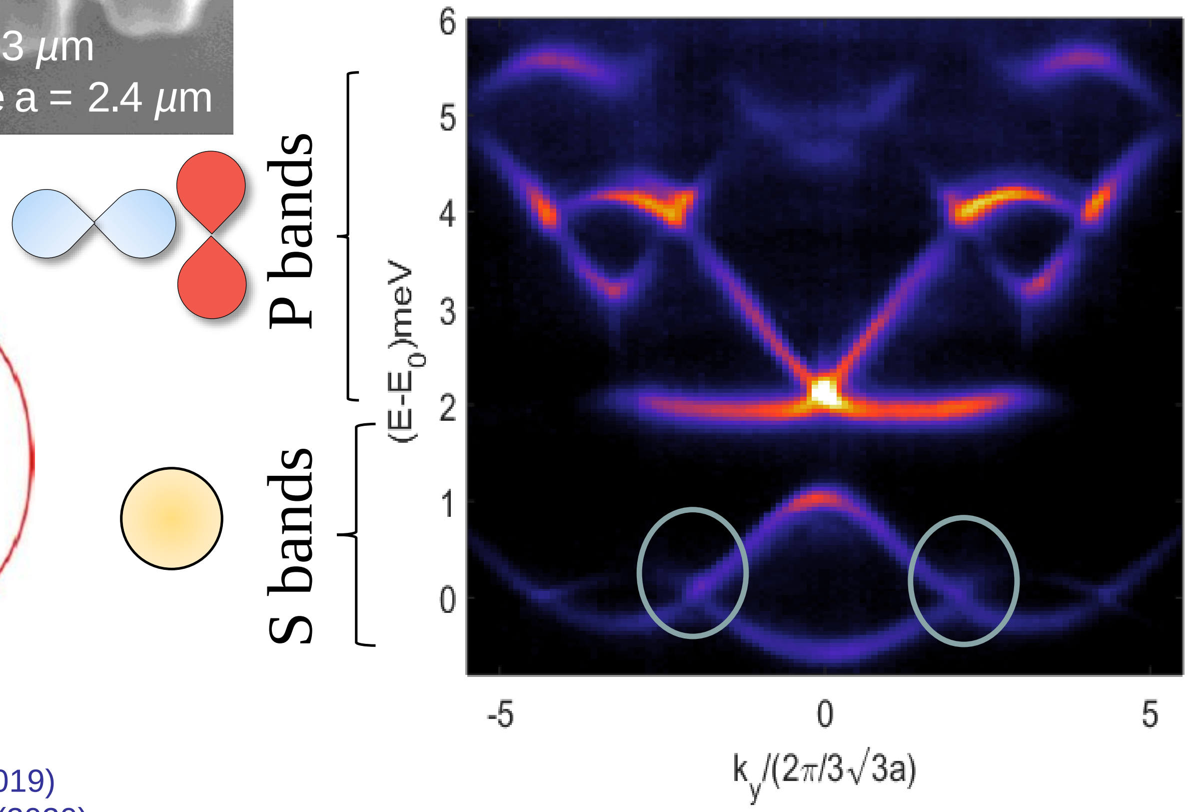
# Polariton honeycomb lattice



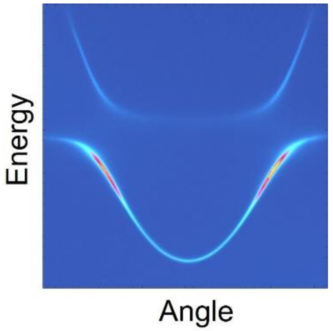

Castro Neto et al., Rev. Mod. Phys. 81 (2009)

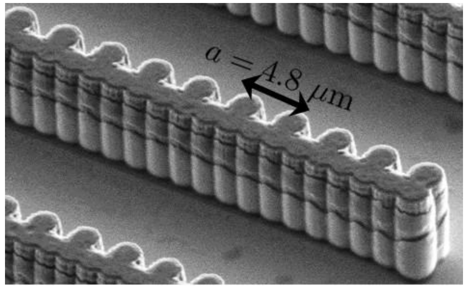



Jacqmin *et al.*, PRL **112,** 116402 (2014)
M. Milićević et al., Phys. Rev. X 9, 31010 (2019)
B. Real et al., Phys. Rev. Lett. 125, 186601 (2020)

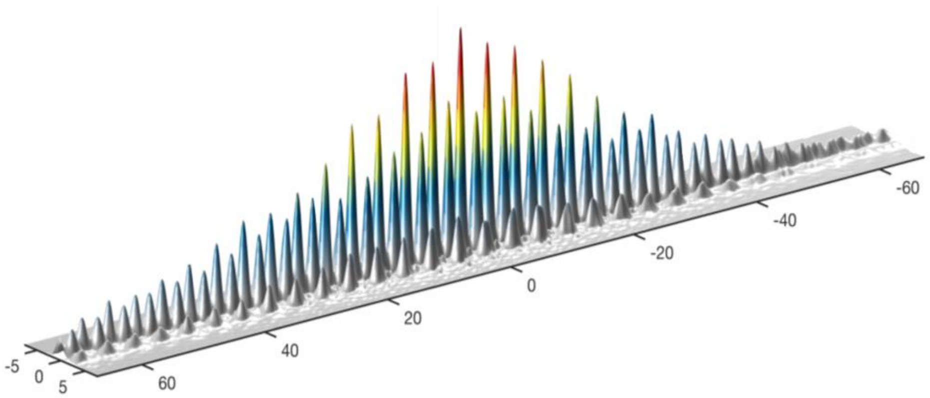
# Outline of the talk

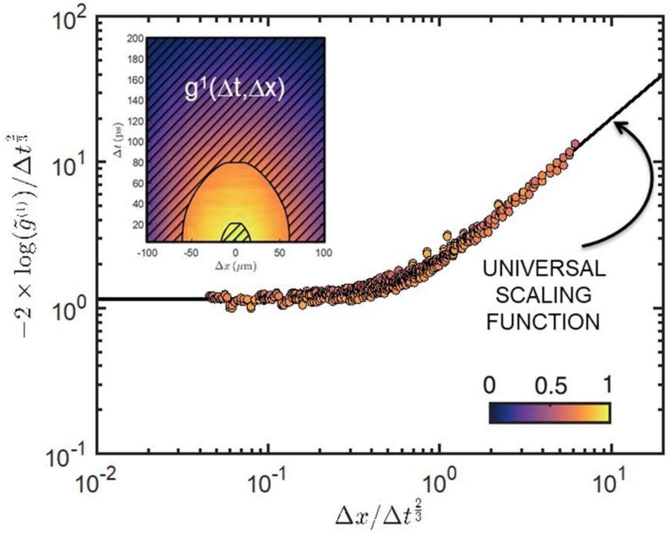
- Introduction : synthetic polariton matter



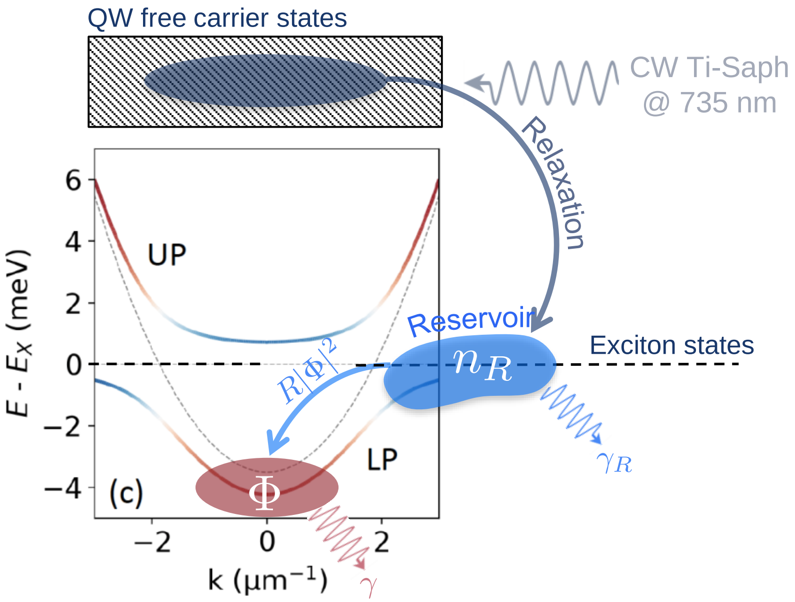



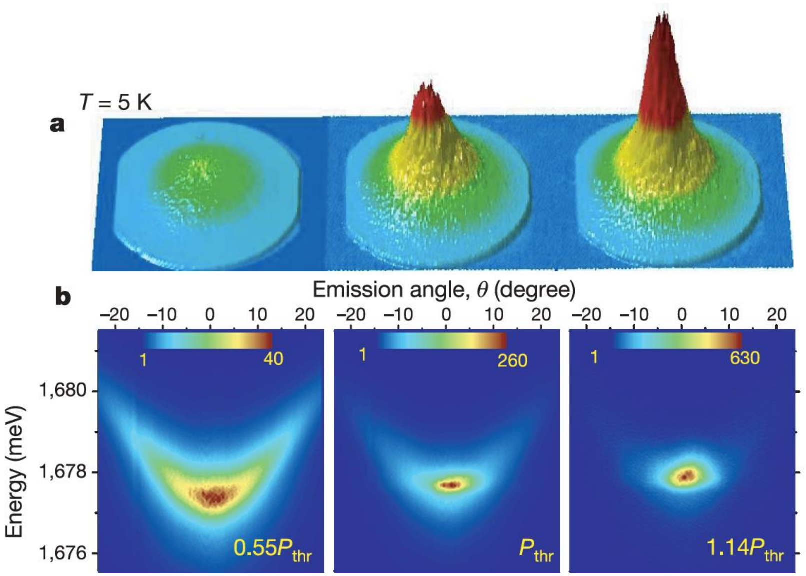

- Polariton condensate belong to the Kardar Parisi Zhang universality class



# Polariton Bose Einstein condensation




Kasprzak *et al.* Nature, **443**, 409 (2006)

See also H. Deng et al. Science (2002), R. Balili et al., Science (2007)

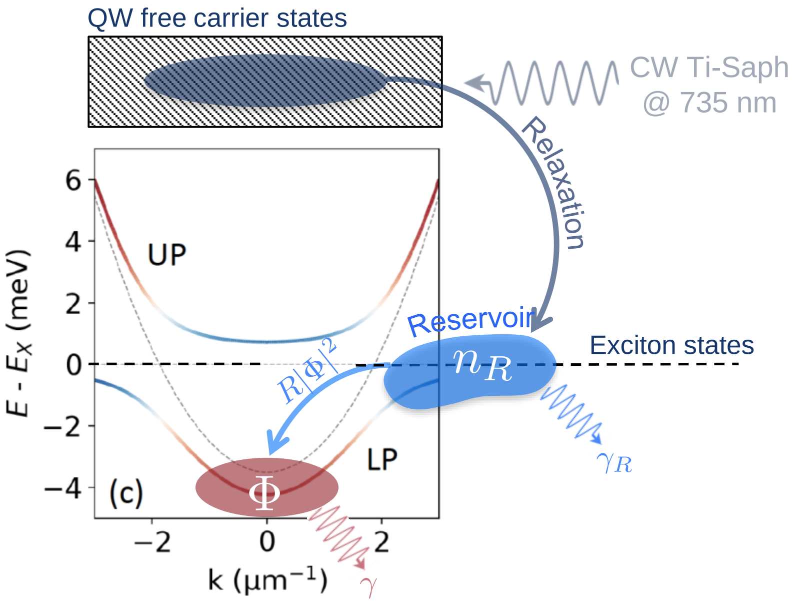
Similarities with atomic BEC BUT

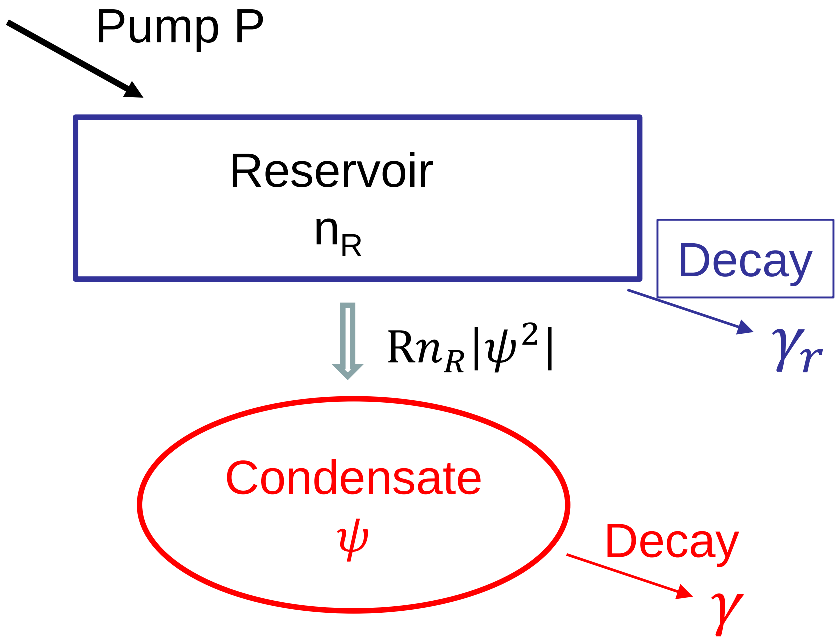
Driven dissipative system
Out of equilibrium

Does it make a difference?

J. Bloch, I. Carusotto and M. Wouters, *Spontaneous coherence in spatially extended photonic systems: Non-Equilibrium Bose-Einstein condensation, Nature Review Physics (2022) (https://doi.org/10.1038/s42254-022-00464-0)*

# Polariton Bose Einstein condensation




MEAN FIELD DESCRIPTION OF THE POLARITON FLUID

(Incoherent Pumping = GPE + Reservoir)

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m^*}\nabla^2 + g\,|\psi|^2 + 2g_R n_R + i\frac{\hbar}{2}(Rn_R - \gamma(\mathbf{k}))\right]\psi + \xi$$

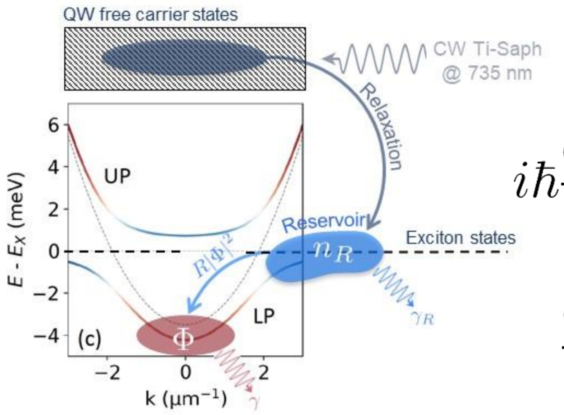
*noise*

$$\frac{\partial n_R}{\partial t} = P - (\gamma_R + R\,|\psi|^2)n_R$$

J. Bloch, I. Carusotto and M. Wouters, *Spontaneous coherence in spatially extended photonic systems: Non-Equilibrium Bose-Einstein condensation, Nature Review Physics (2022) (https://doi.org/10.1038/s42254-022-00464-0)*

# Phase coherence in a polariton condensate

E. Altman, S. Diehl, M. Wouters 2015




$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m^*}\nabla^2 + g\left|\psi\right|^2 + 2g_R n_R + i\frac{\hbar}{2}(Rn_R - \gamma(\mathbf{k}))\right]\psi + \xi$$

$$\frac{\partial n_R}{\partial t} = P - (\gamma_R + R\left|\psi\right|^2)n_R$$

- Density-phase representation : $\psi(\mathbf{x},t) = \sqrt{\rho(\mathbf{x},t)}e^{-i\omega_0 t + i\theta(\mathbf{x},t)}$
- Assume different time scales for density and phase fluctuations :

$$\partial_t\theta = \nu\nabla^2\theta + \frac{\lambda}{2}(\nabla\theta)^2 + \eta$$

**This is the celebrated Kardar Parisi Zhang equation!!**

E. Altman, *et al.*, PRX **5**, 011017 (2015)
K. Ji, *et al.*, PRB **91**, 045301 (2015)
L. He, et al., PRB **92**, 155307 (2015)

# Phase coherence in a polariton condensate

Microscopic model

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left[E(\hat{k}) - \frac{i\hbar}{2}\gamma(\hat{k}) + g|\psi(x,t)|^2 + 2g_R n_R(x,t) + \frac{i\hbar}{2}R n_R(x,t)\right]\psi(x,t) + \xi(x,t)$$

$$\frac{\partial}{\partial t}n_R(x,t) = P(x) - \left(\gamma_R + R|\psi(x,t)|^2\right) n_R(x,t)$$

$$\gamma(k \simeq 0) \simeq \gamma_0 + \gamma_2 k^2$$

KPZ equation

$$\partial_t\theta = \left[\frac{\gamma_2}{2} - u\frac{g_R}{\hbar R}\frac{\hbar}{m}\right]\nabla^2\theta - \left[\frac{\hbar}{2m} + u\frac{g_R}{\hbar R}\gamma_2\right](\nabla\theta)^2 + \eta$$

$$\equiv \nu\nabla^2\theta + \frac{\lambda}{2}(\nabla\theta)^2 + \eta$$

$$u = 2 - p\frac{g}{g_R}\frac{\gamma_R}{\gamma_0}$$

$$\langle\eta(x,t)\eta(x',t')\rangle = \frac{\xi_0}{\rho_0}\left[1 + 4\left(u\frac{g_R}{\hbar R}\right)^2\right]\delta(x-x')\delta(t-t')$$

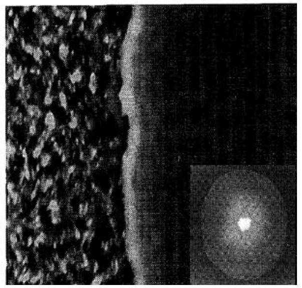
$$\equiv 2D\delta(x-x')\delta(t-t'),$$

Detailed derivation of the maping in : Q. Fontaine, et al, Nature 608, 687 (2022) sup mat

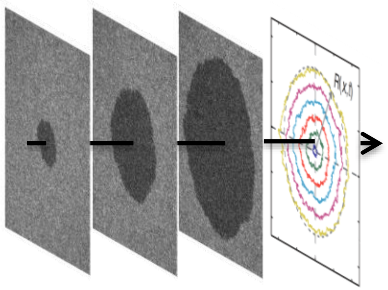
# Kardar-Parisi-Zhang theory of interface stochastic growth


Kardar, Parisi and Zhang, *PRL* (1986)

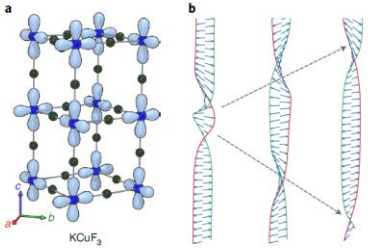
Frost on a window



Bacteria



Liquid crystals

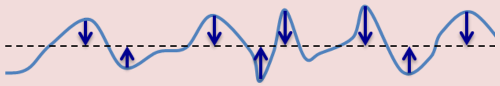



1D antiferromagnet (Nature Phys. 2021)
See also D. Wei, Science 376 716 (2022)

$$\frac{\partial h}{\partial t} = \nu \boldsymbol{\nabla}^2 h + \frac{\lambda}{2}\left(\boldsymbol{\nabla} h\right)^2 + \eta(\boldsymbol{x}, t)$$

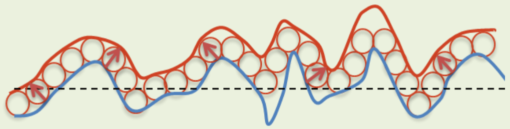
## Diffusion term

- *smoothens the surface*



## Growth term (nonlinear)

- *Orthogonal to the surface*
- *Non-equilibrium*



## Stochastic term (fluctuations)

- *White Gaussian noise*

$$\langle \eta(\boldsymbol{x}, t) \rangle = 0$$

$$\langle \eta(\mathbf{x})\eta(\mathbf{x}') \rangle = \delta(\mathbf{x} - \mathbf{x}')$$

# Landmark signatures of KPZ physics


Kardar, Parisi and Zhang, *PRL* (1986)

- Self-organized **scale invariance**

$\Longrightarrow$ Critical exponents (universal)

$$C(\mathbf{x},t) = \langle h(\mathbf{x},t)h(0,0)\rangle - \langle h(\mathbf{x},t)\rangle \langle h(0,0)\rangle \propto \begin{cases} t^{2\beta} & (\mathbf{x}=0) \\ x^{2\chi} & (t=0) \end{cases}$$

$$C(\mathbf{x},t) \propto t^{2\beta}\mathcal{F}_{\mathrm{KPZ}}\left(\kappa\frac{|\mathbf{x}|}{t^{1/\mathcal{Z}}}\right)$$

→ KPZ **universal scaling function** (tabulated)

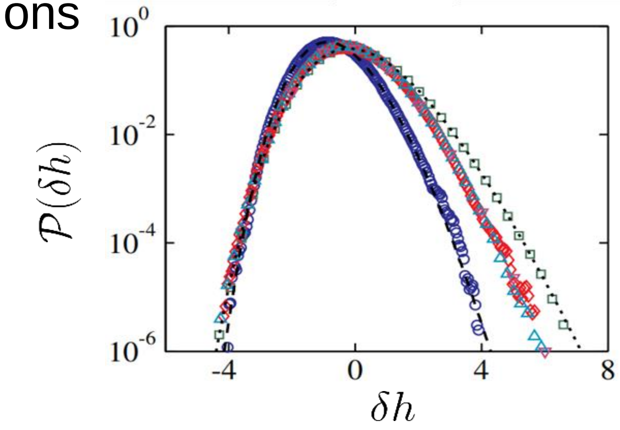
1D KPZ universality class

$\beta = 1/3$ $\qquad \chi = 1/2$

$\mathcal{Z} = \chi/\beta = 3/2$

- Non-Gaussian probability distribution of height fluctuations

$$\delta h(t) = \frac{h(\mathbf{x_0},t) - v_\infty t}{(\Gamma t)^{1/3}}$$

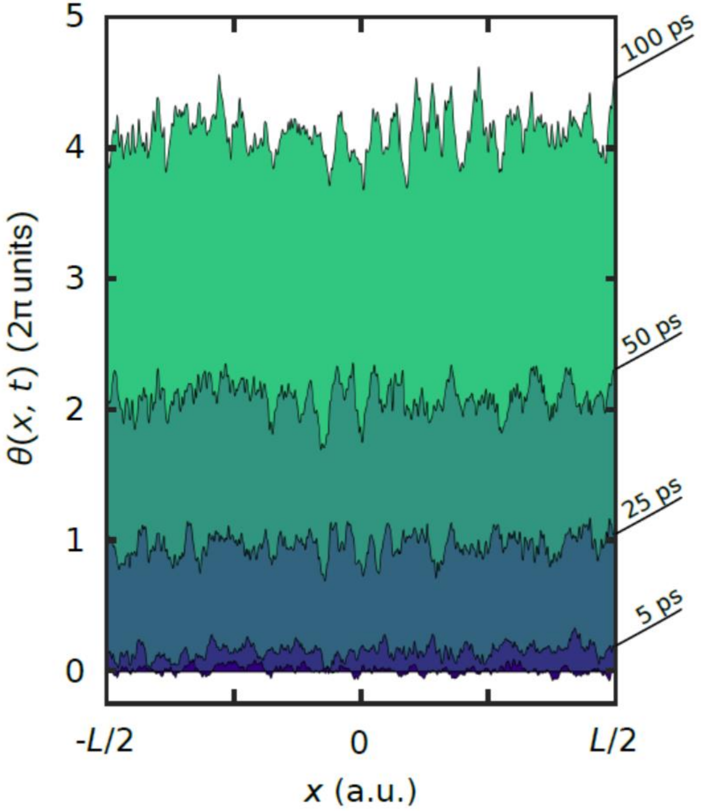



K. A. Takeuchi, PRL **110**, 210604 (2013)

T. Halpin-Healy, & Y.-C. Zhang, Phys. Rep. **254**, 215 (1995)
J. Krug, Adv. Phys. **46**, 139 (1997)
K. A. Takeuchi, Physica A **504**, 77 (2018)

# Polariton condensates

$$\partial_t \theta = \nu \nabla^2 \theta + \frac{\lambda}{2}(\nabla\theta)^2 + \sqrt{D}\eta$$




- The phase front behaves as a growing interface

KPZ scaling expected in the spatio-temporal correlations of the phase

$$\mathrm{Var}\left[\Delta\theta(\Delta x, \Delta t)\right]$$

Instantaneous phase : difficult to access in the experiment …

How to probe KPZ scaling ???

E. Altman, *et al.*, PRX **5**, 011017 (2015)
K. Ji, *et al.*, PRB **91**, 045301 (2015)

L. He, et al., PRB **92**, 155307 (2015)

# Polariton condensates

We can measure amplitude amplitude correlations of the field (first order coherence :

$$g^{(1)}(\Delta x, \Delta t) = \frac{\langle \psi^*(x,t_0)\psi(-x,t_0+\Delta t)\rangle}{\sqrt{\langle \rho(x,t_0)\rangle}\sqrt{\langle \rho(-x,t_0+\Delta t)\rangle}}$$

- If phase fluctuations are independent of density fluctuations and for small density fluctuations : $g^{(1)}(\Delta x, \Delta t) = \Big\langle \exp\left[i\Delta\theta(\Delta x, \Delta t)\right] \Big\rangle$

- For small phase fluctuations :

$$|g^{(1)}(\Delta x, \Delta t)|^2 \simeq \exp(-\langle \Delta\theta(\Delta x, \Delta t)^2\rangle + \langle \Delta\theta(\Delta x, \Delta t)\rangle^2) \equiv \exp(-\mathrm{Var}\left[\Delta\theta(\Delta x, \Delta t)\right])$$

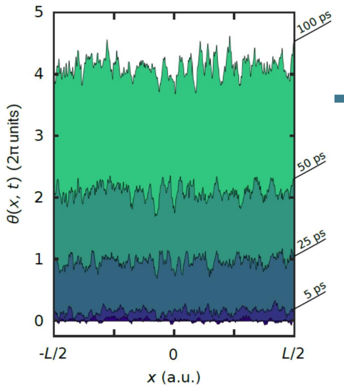
$$\boxed{\mathrm{Var}\left[\Delta\theta\right] \simeq -2\log\left(\left|g^{(1)}\right|\right)}$$

➢ In 1D: $-2\log\left(\left|g^{(1)}(\Delta x, \Delta t)\right|\right) \sim \begin{cases} \Delta t^{2\beta} & (\Delta \mathrm{x} = 0) \\ \Delta x^{2\chi} & (\Delta \mathrm{t} = 0) \end{cases}$

$\beta = 1/3 \qquad \chi = 1/2$

See derivation in Q. Fontaine, *et al*, Nature 608, 687 (2022)

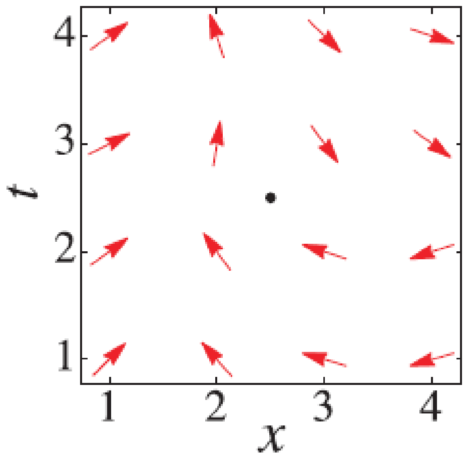
# KPZ physics in polariton condensates

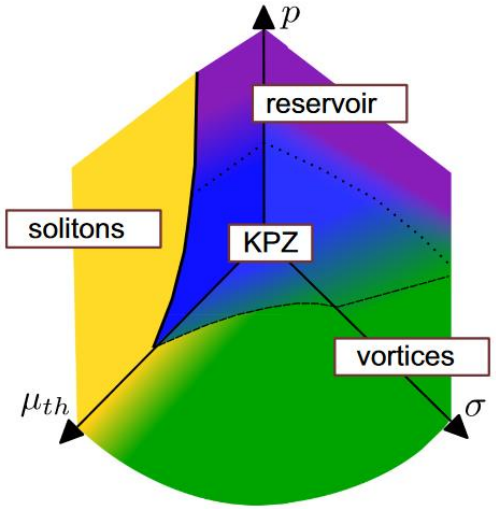



The phase is a compact variable : $\theta\epsilon[0, 2\pi]$

- Even **in 1D** : Space time vortex

**Effective 2D system:**




L. He, L.M. Sieberer and S. Diehl, Phys. Rev. Lett. 118, 085301 (2017)

F. Vercesi et al. Phys. Rev. Research 5, 043062 (2023)

- **In 2D**: Space time AND spatial vortices

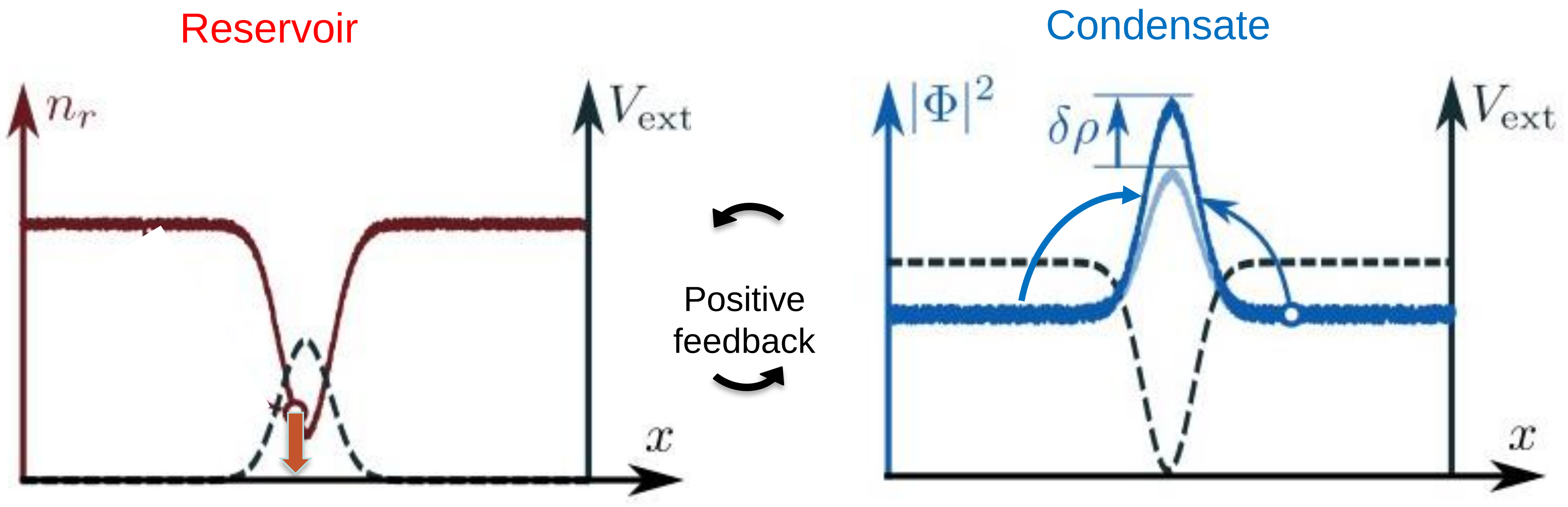
KPZ scaling in 2D open condensates? $\Longrightarrow$ Still actively debated

E. Altman, et al., PRX 5, 011017 (2015)
A. Zamora, et al., PRX 7, 041006 (2017)

Q. Mei, et al., PRB 103, 045302 (2021)
A. Ferrier, et al., PRB 105, 205301 (2022)

# KPZ physics in polariton condensates

Why no explored experimentally so far ?

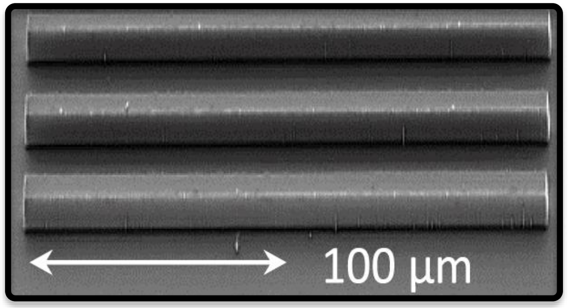


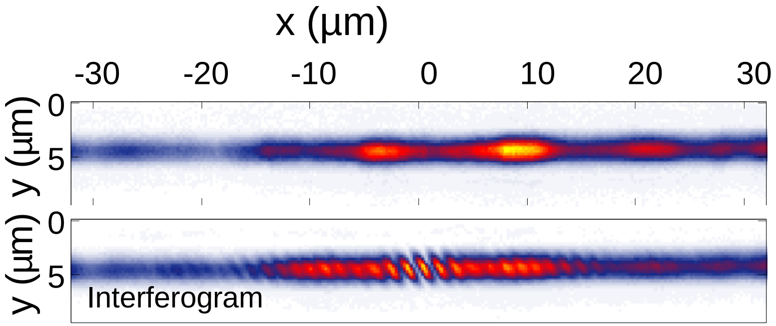

Repulsive interactions between polaritons and reservoir excitons ⇒ Amplication of fluctuations

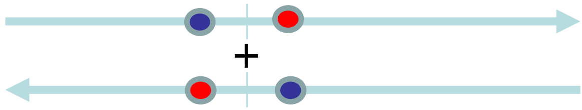
Effective attractive interactions within the condensate mediated by the reservoir

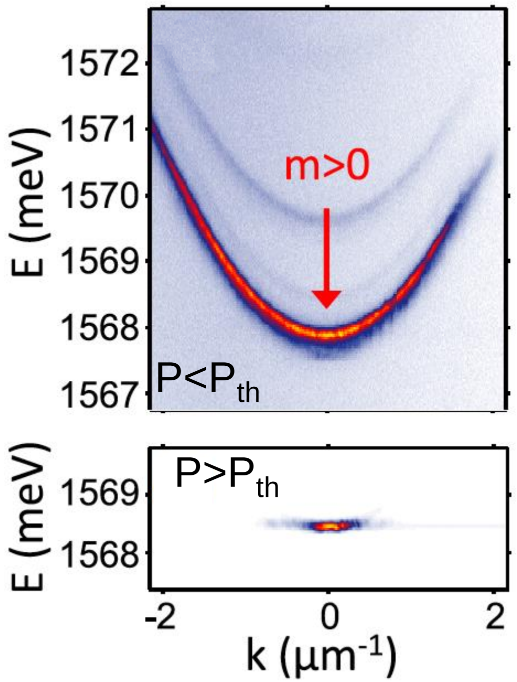
# Modulation unstability of polariton condensates

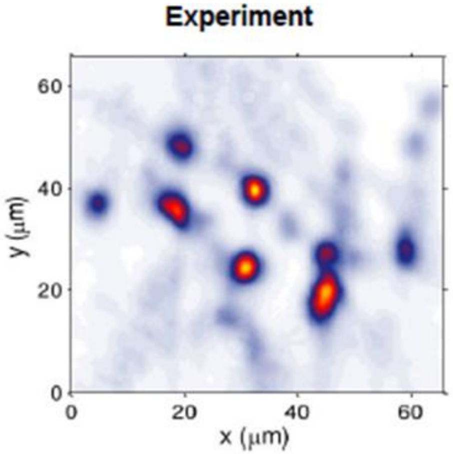
1D



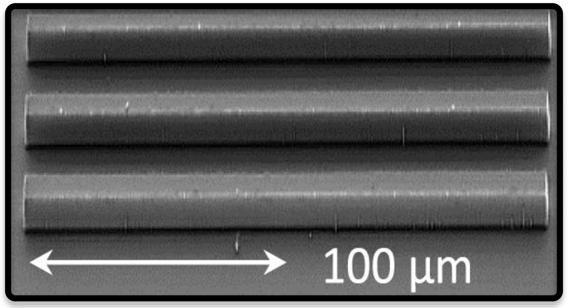



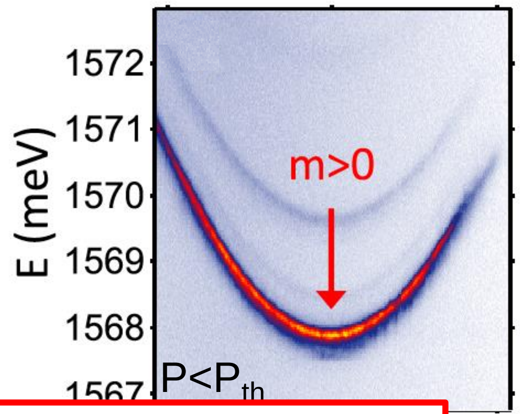

2D

F. Baboux, *et al.*, Optica **5**, 1163 (2018)

# Modulation unstability of polariton condensates

1D

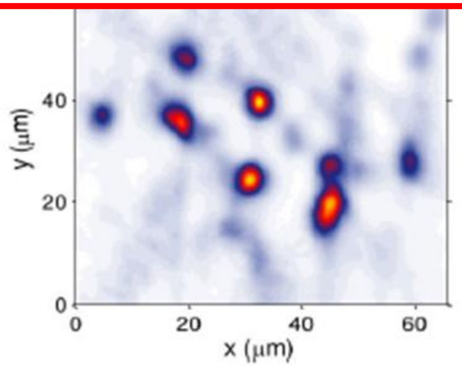



How to tame this unstability ?

By changing the sign of the effective interactions with the condensate :

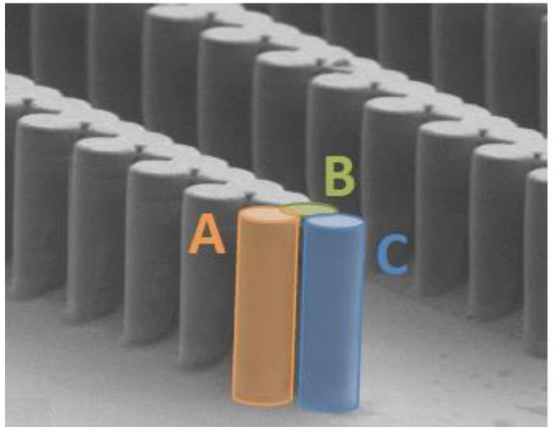
Condensation on negative mass bands !!!

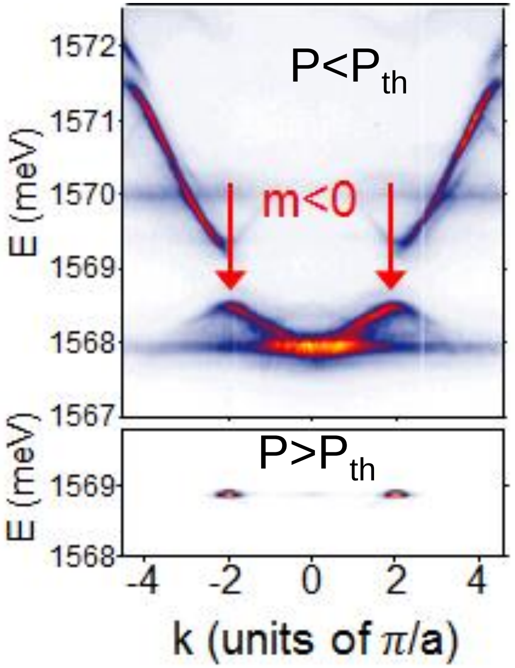


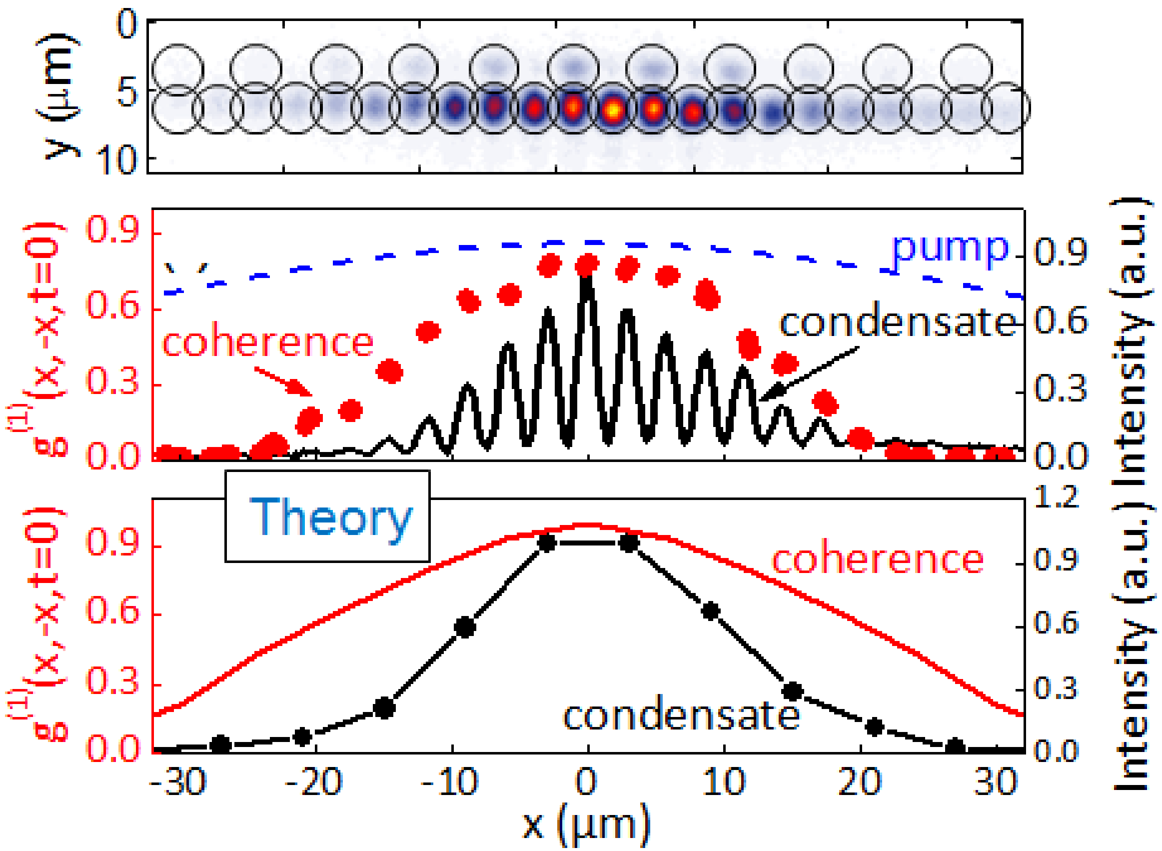

F. Baboux, *et al.*, Optica **5**, 1163 (2018)

# Extended stable polariton condensates

*1D polariton lattice*

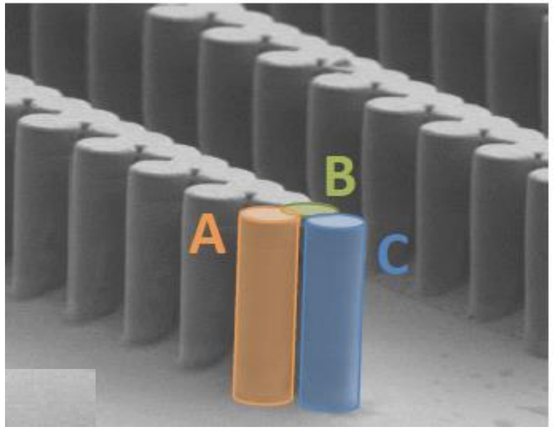


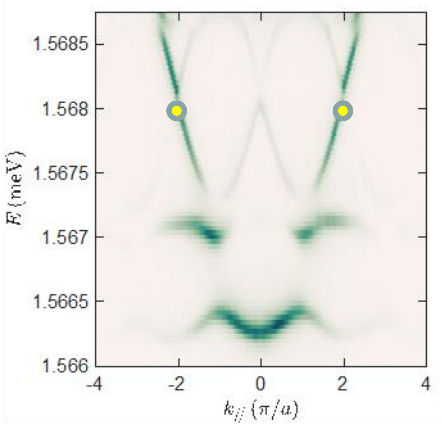



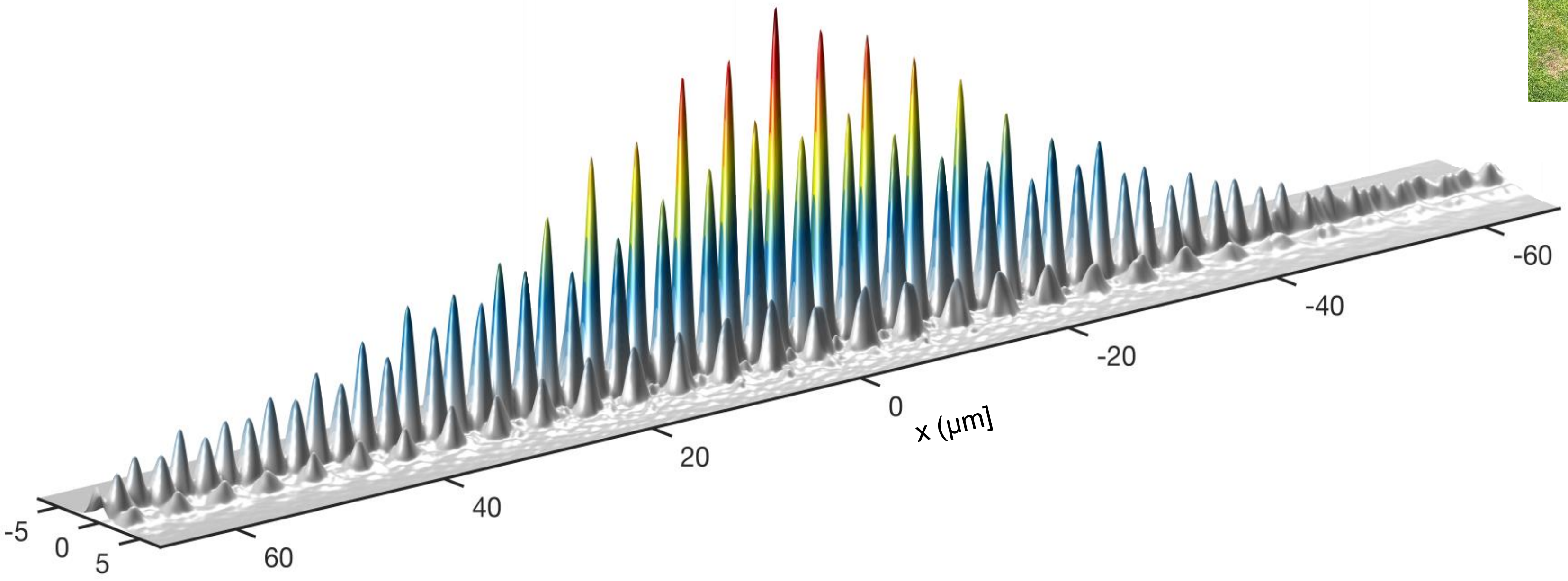




Stable extended polariton condensates : F. Baboux et al., Optica 5, 1163 (2018)

# KPZ physics in 1D polariton condensates



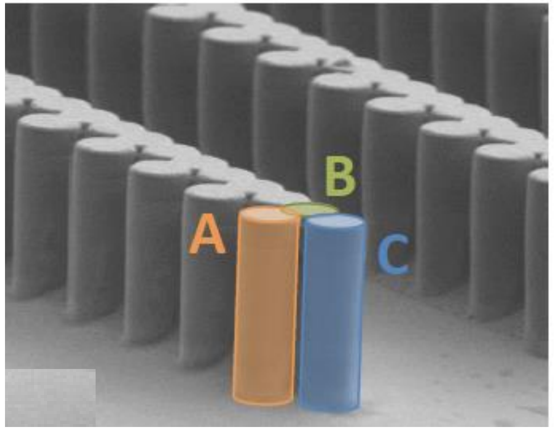



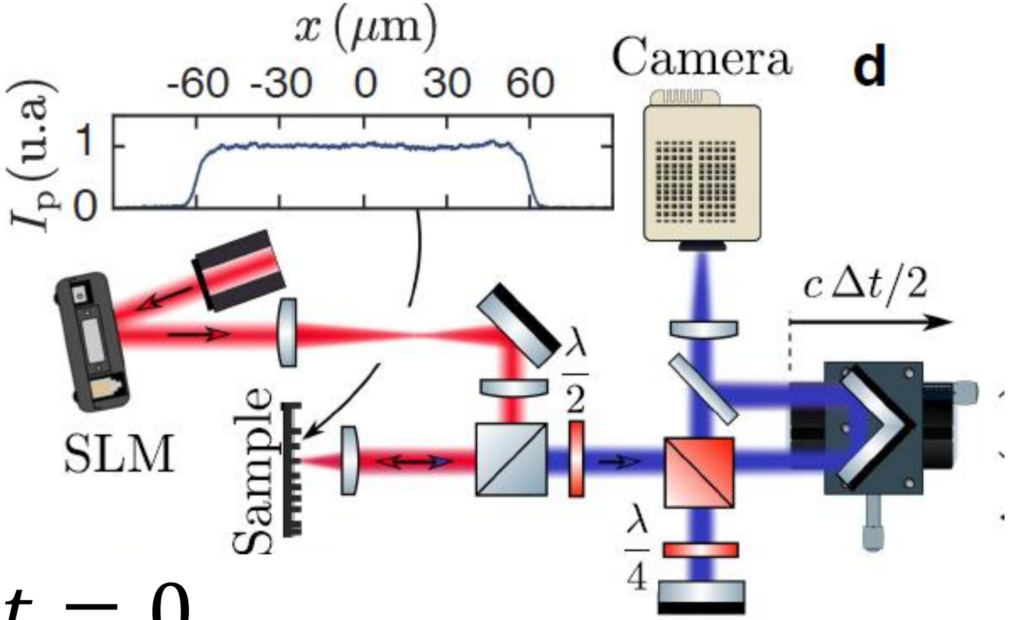



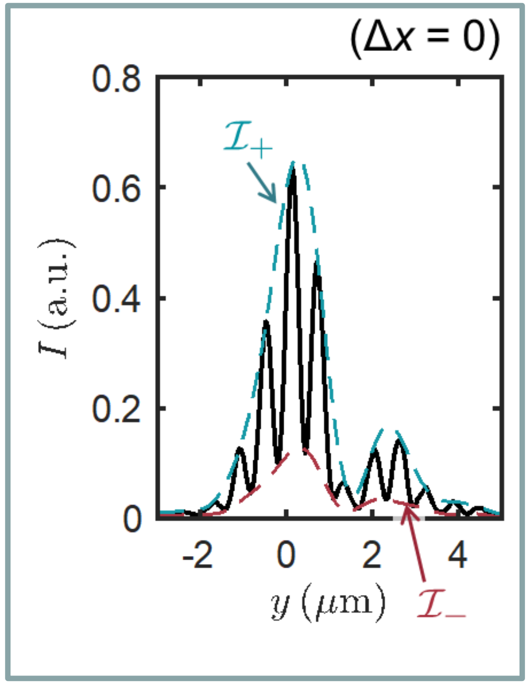

Quentin Fontaine

Q. Fontaine, *et al*, Nature 608, 687 (2022)

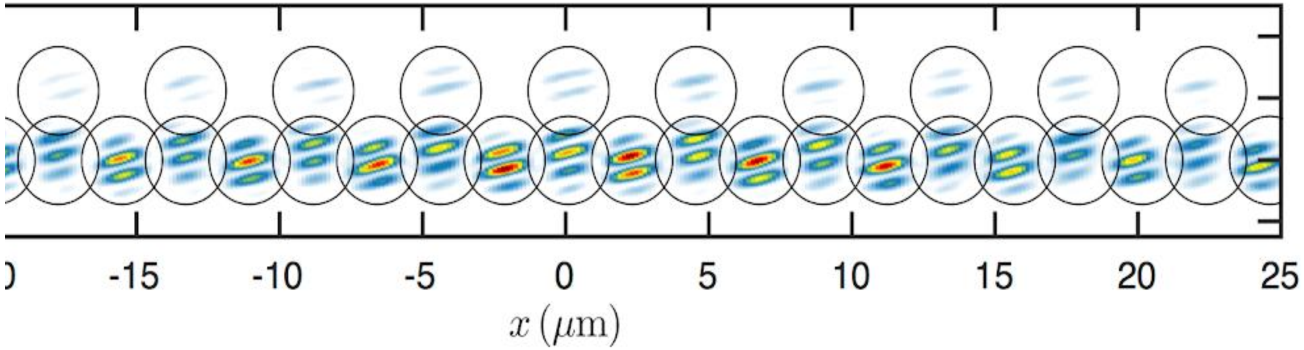
# KPZ physics in 1D polariton condensates



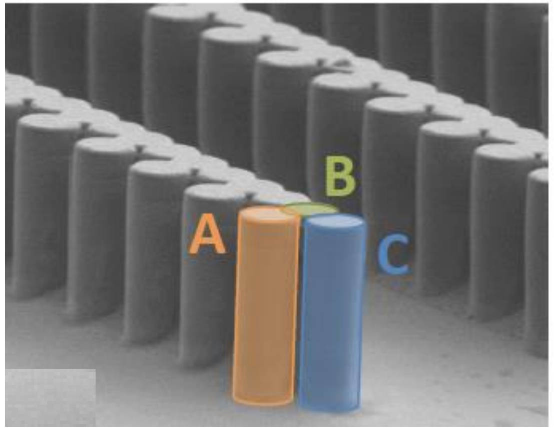



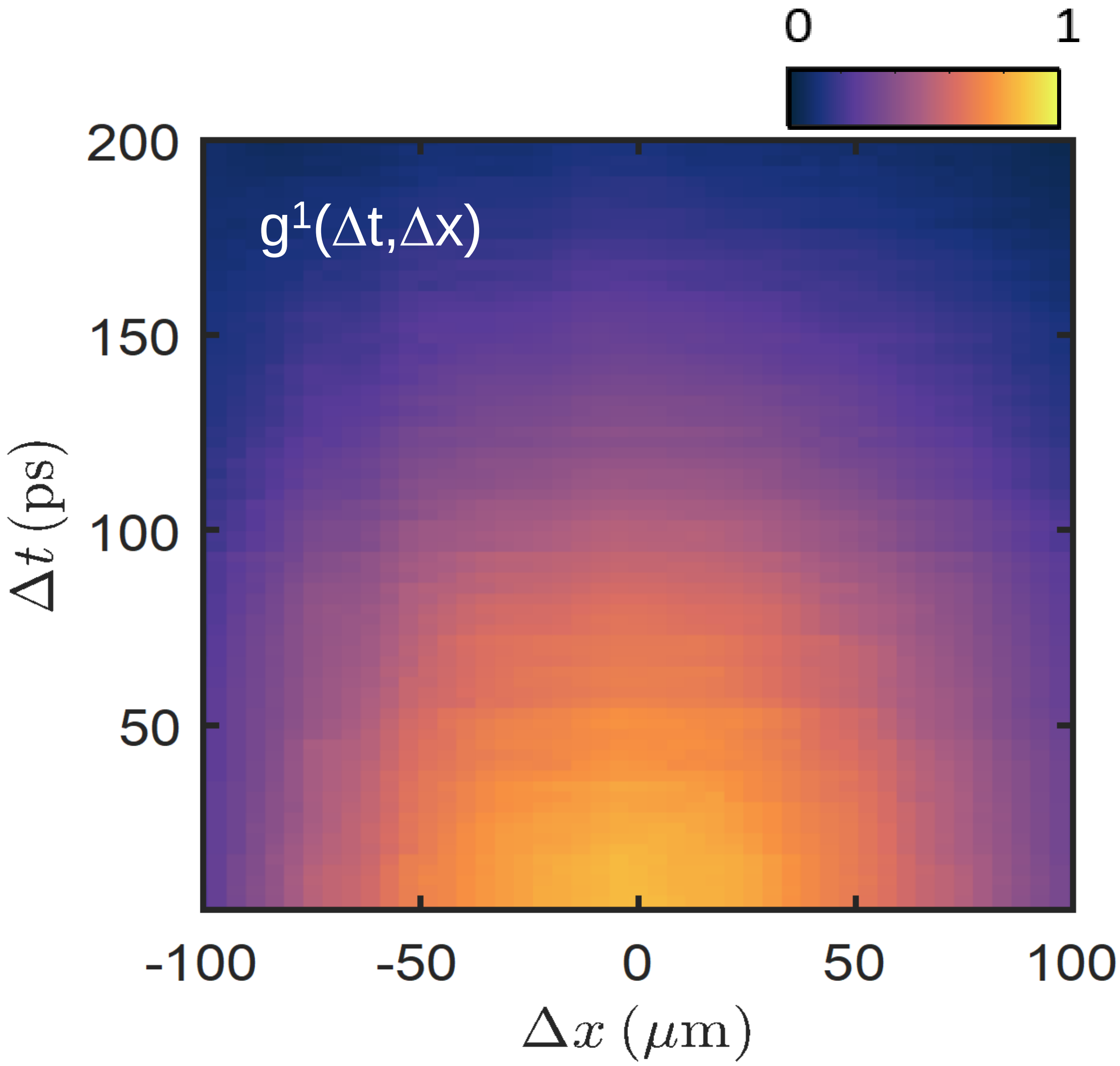

$\Delta t = 0$




$$\frac{\mathcal{I}_+ - \mathcal{I}_-}{\mathcal{I}_+ + \mathcal{I}_-} = \frac{2\sqrt{\mathcal{I}(\boldsymbol{r})\,\mathcal{I}(-\boldsymbol{r})}}{\mathcal{I}(\boldsymbol{r}) + \mathcal{I}(-\boldsymbol{r})}\,|g^{(1)}(\boldsymbol{\Delta r}, \Delta t)|$$

# KPZ  physics in 1D polariton condensates

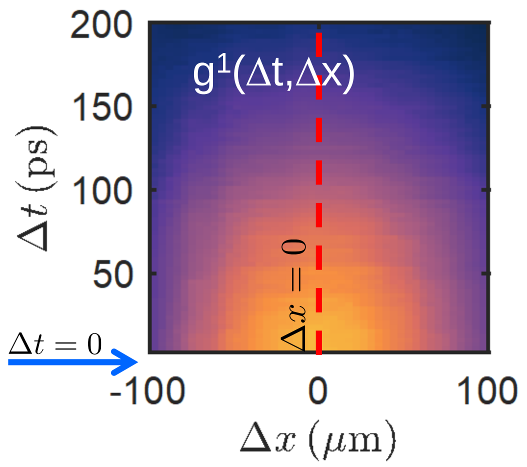


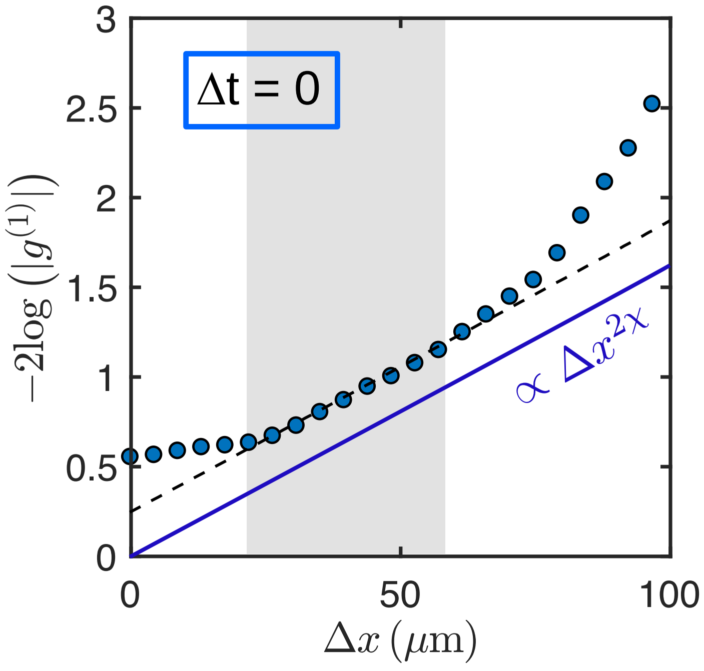



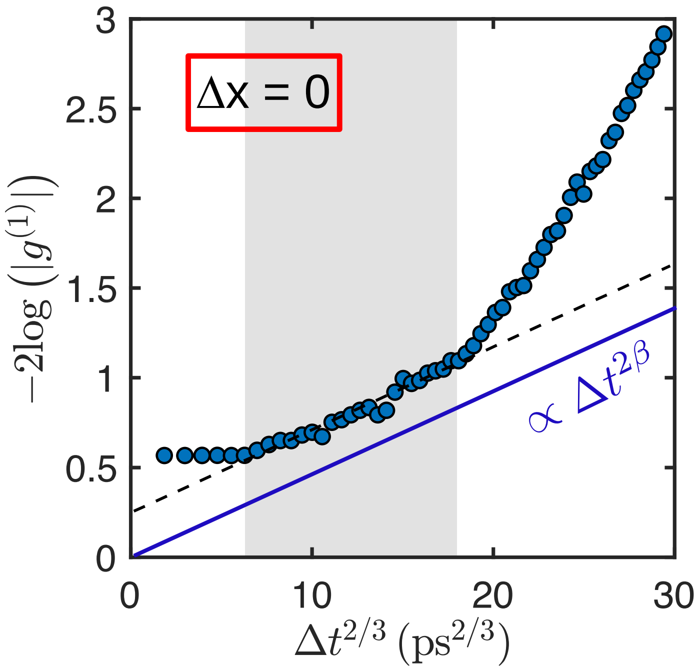
# KPZ physics in 1D polariton condensates

- "SURFACE ROUGHNESS" ⟷ $\mathrm{Var}\left[\Delta\theta\right] \simeq -2\log\left(g^{(1)}\right)$
- WE EXPECT:

$$-2\ln\{g^{(1)}(\Delta x, \Delta t)\} \sim \begin{cases} \Delta t^{2\beta} & \text{for } \Delta x = 0 \quad \beta = 1/3 \\ \Delta x^{2\chi} & \text{for } \Delta t = 0 \quad \chi = 1/2 \end{cases}$$



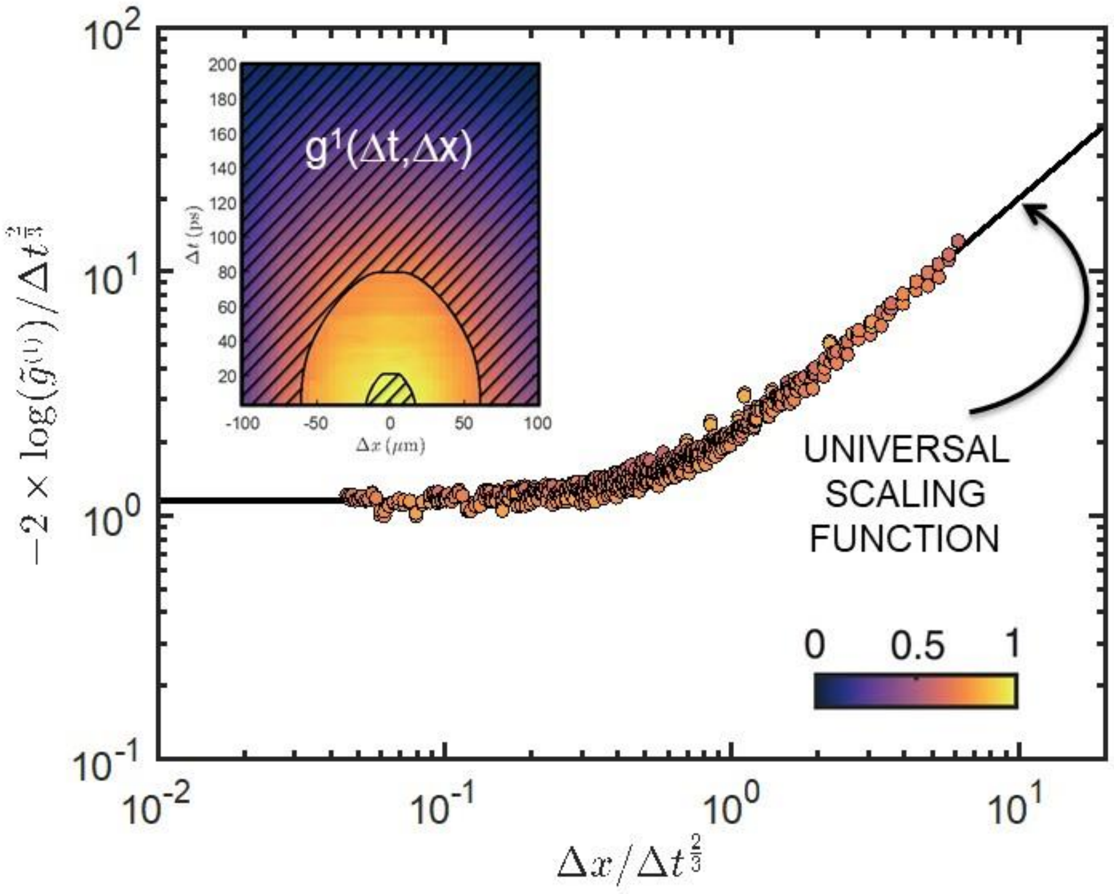



# KPZ scaling laws in 1D polariton condensates

KPZ scaling $$-2\ln\{g^{(1)}(\Delta x,\Delta t)\} = A\times\Delta t^{2\beta}\,\mathcal{F}\left[B\times\frac{\Delta x}{\Delta t^{1/z}}\right]$$

where $\mathcal{F}(y) = \begin{cases} c_0, & y\to 0 \\ y, & y\to\infty \end{cases}$ is the UNIVERSAL KPZ SCALING FUNCTION.

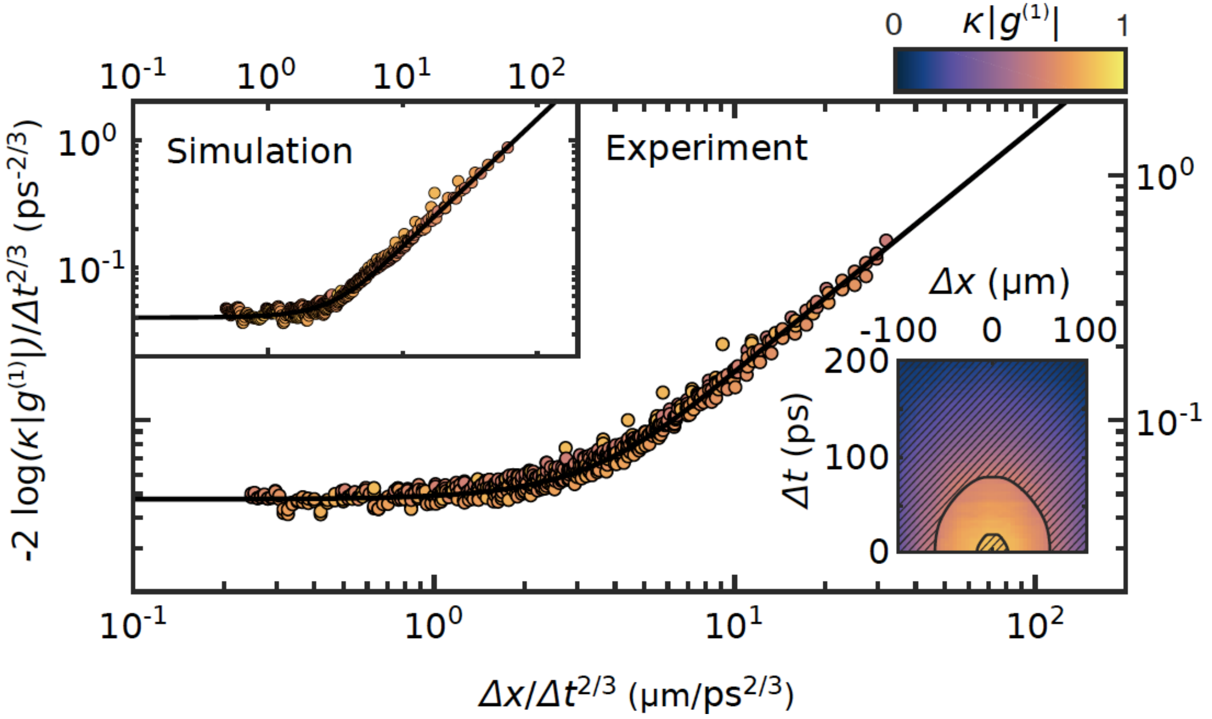



Q. Fontaine, *et al*, Nature 608, 687 (2022)

- Integrate numerically the two coupled equations model

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m^*}\nabla^2 + g\,|\psi|^2 + 2g_R n_R + i\frac{\hbar}{2}(Rn_R - \gamma(\mathbf{k}))\right]\psi + \xi$$

$$\frac{\partial n_R}{\partial t} = P - (\gamma_R + R\,|\psi|^2)n_R$$

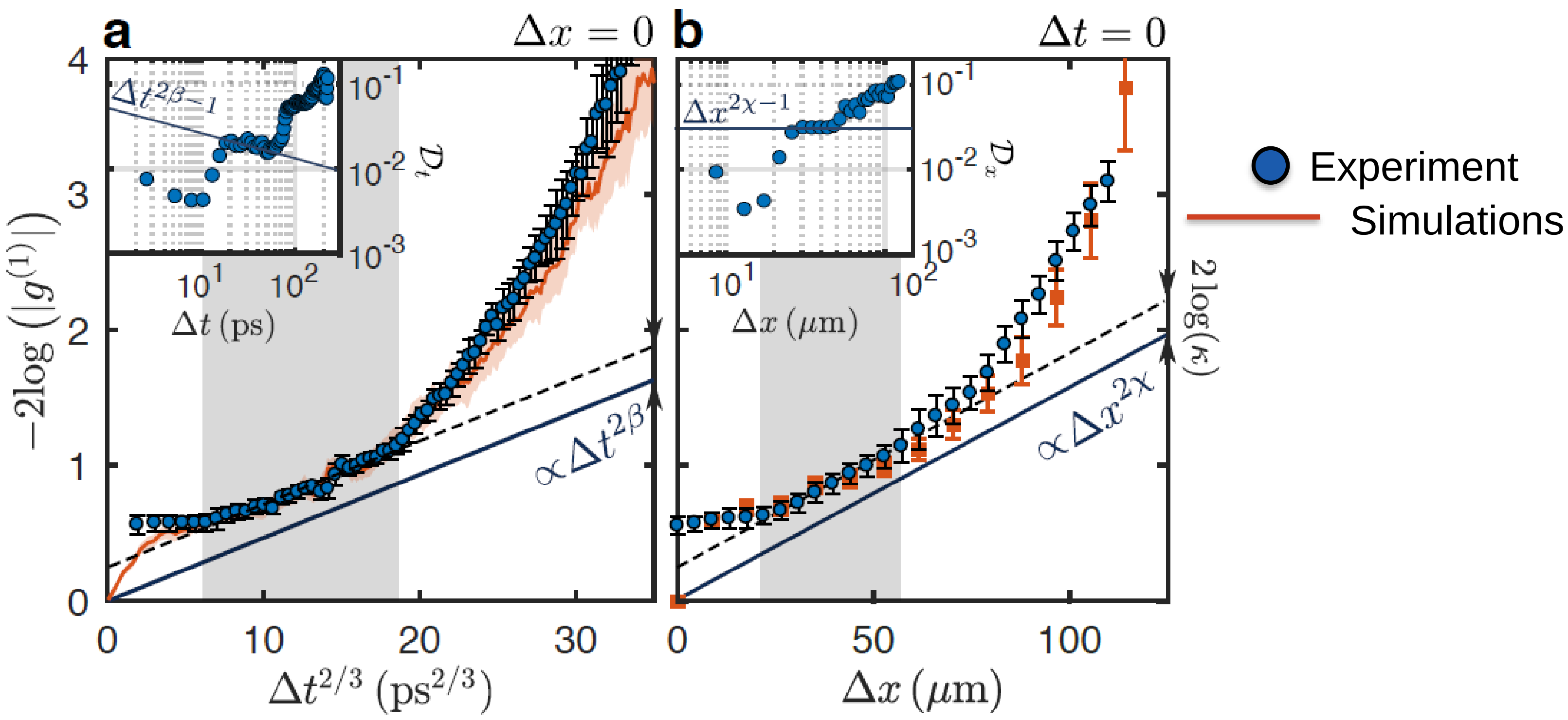



D. Squizzato

A. Minguzzi

L. Canet

Q. Fontaine, *et al*, Nature 608, 687 (2022)




$$\chi_{\text{exp}} = 0.51 \pm 0.08$$
$$\beta_{\text{exp}} = 0.36 \pm 0.11$$


D. Squizzato


A. Minguzzi

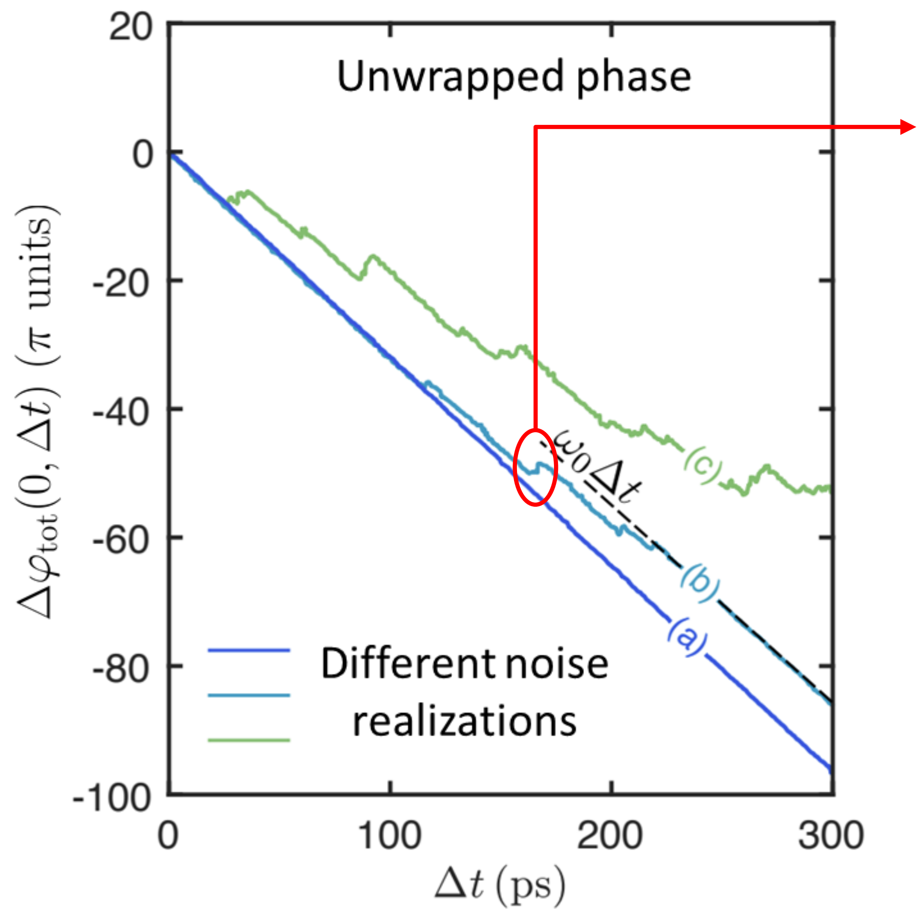

L. Canet

Q. Fontaine, *et al*, Nature 608, 687 (2022)

# Phase dynamics (simulations)

- Calculate total phase (unwrapped) difference for several noise realisations:

$$\Delta\varphi_{\rm tot}(0,\Delta t) = \varphi_{\rm tot}(0,\Delta t) - \varphi_{\rm tot}(0,0) = -\omega_0\Delta t + \Delta\theta(0,\Delta t)$$




- Occasional 2π phase jumps.

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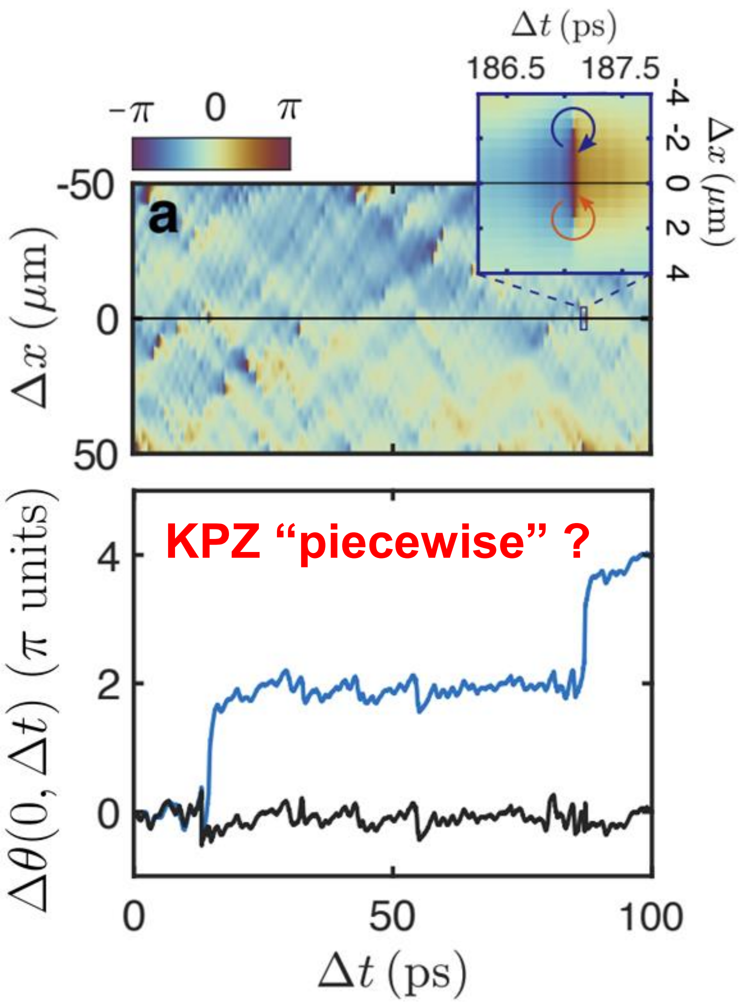


- Occasional 2π phase jumps.

- Pairs of vortex and antivortex appear in effective 2D space (Δx, Δt).

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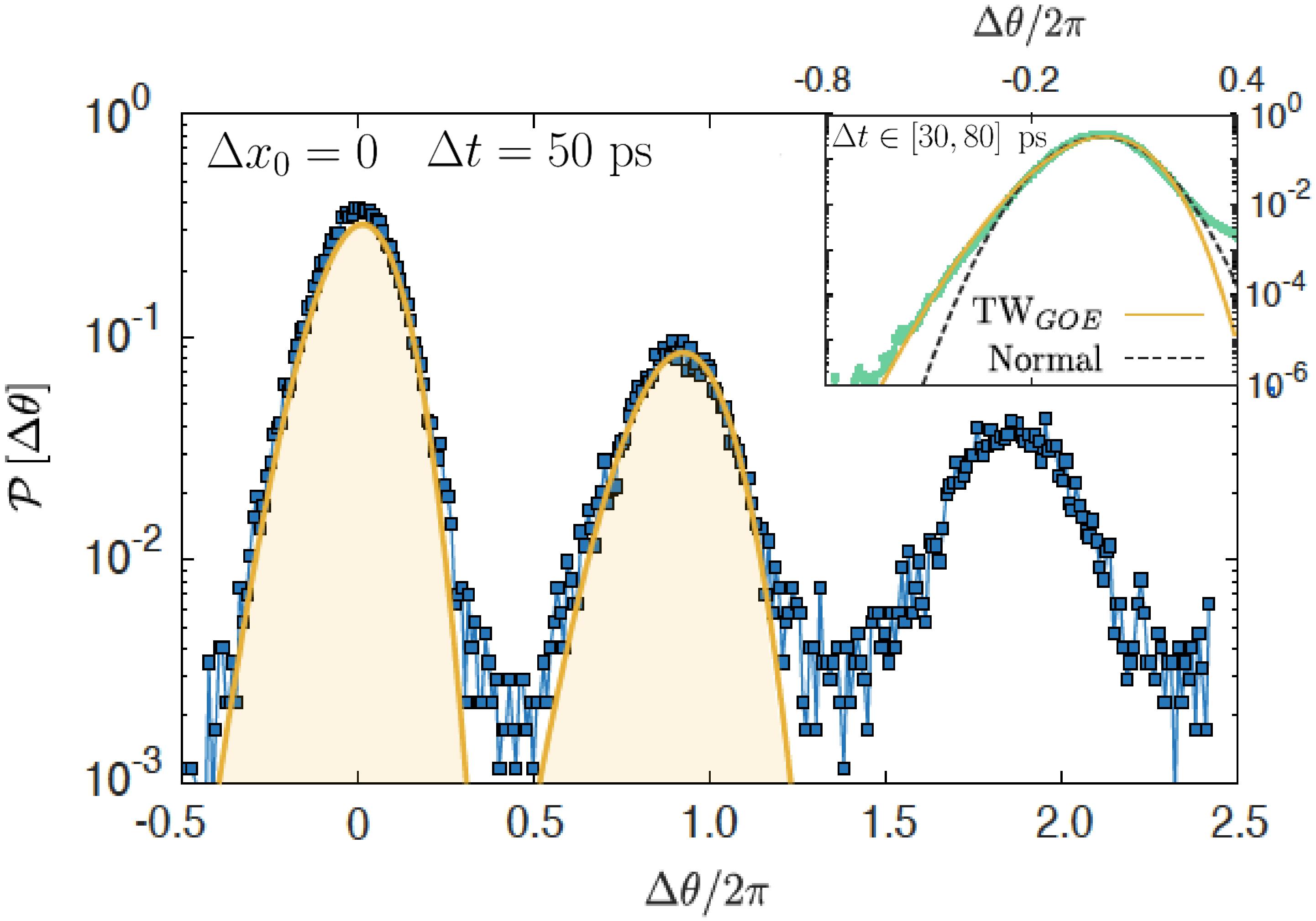
# Amplitude distribution of phase fluctuations

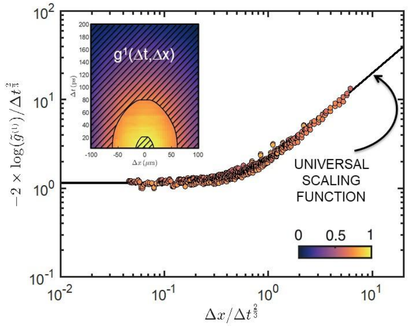
- For Δx and Δt within KPZ window $\Delta\theta(\Delta x_0, \Delta t)/(|\Gamma|\Delta t^{2/3})$ is a random variable expected to obey Tracy-Widom statistics (non-Gaussian).



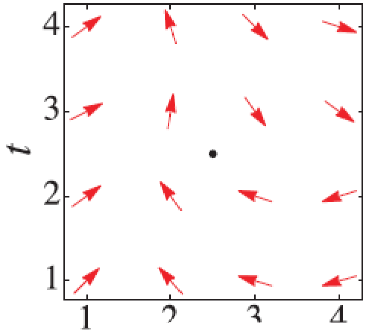

Q. Fontaine, *et al*, Nature 608, 687 (2022)

# Conclusion and prospects

- 1D driven-dissipative condensates belong to the KPZ universality class

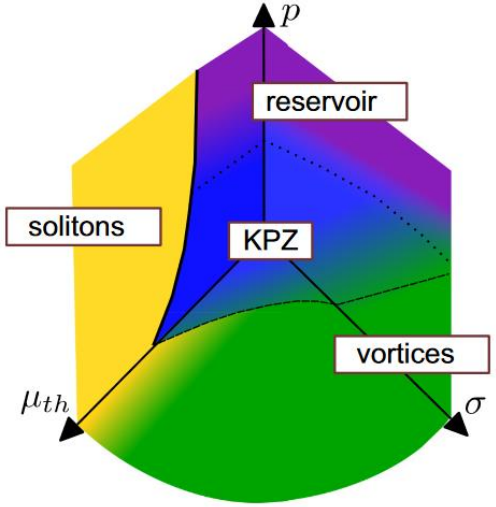



- Compact version of KPZ with a phase variable
  $\Longrightarrow$ topological defects



- KPZ scaling can be resilient to these defects

- Exploration of the phase diagram




F. Vercesi et al. Phys. Rev. Research 5, 043062 (2023)

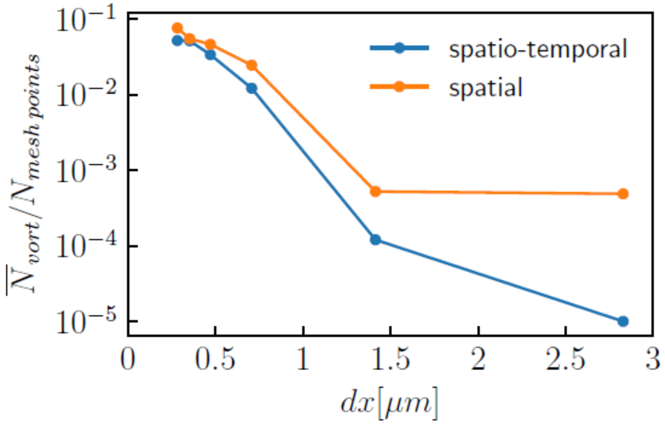
# Conclusion and prospects

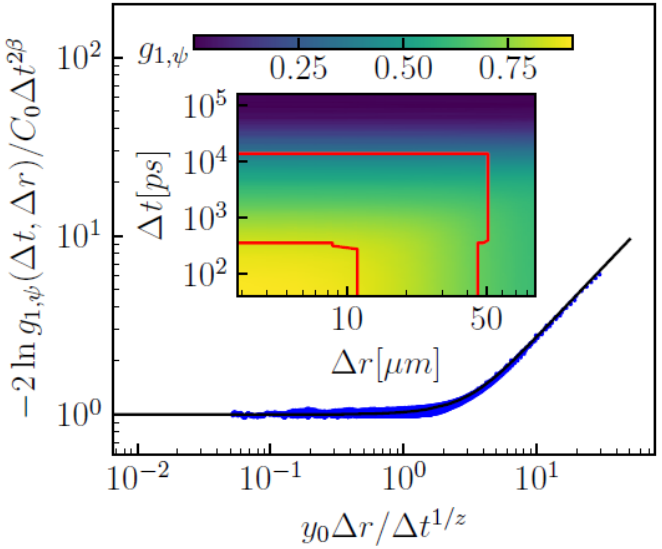
- In 2D: Space time **AND** spatial vortices
Vortex proliferation kills KPZ correlations?

**Debated topics!**

E. Altman, *et al.*, PRX **5**, 011017 (2015)
A. Zamora, *et al.*, PRX **7**, 041006 (2017)
Q. Mei, *et al.*, PRB **103**, 045302 (2021)
A. Ferrier, *et al.*, PRB **105**, 205301 (2022)

- KPZ predicted in recent simulations using 2D discrete (lattice) model

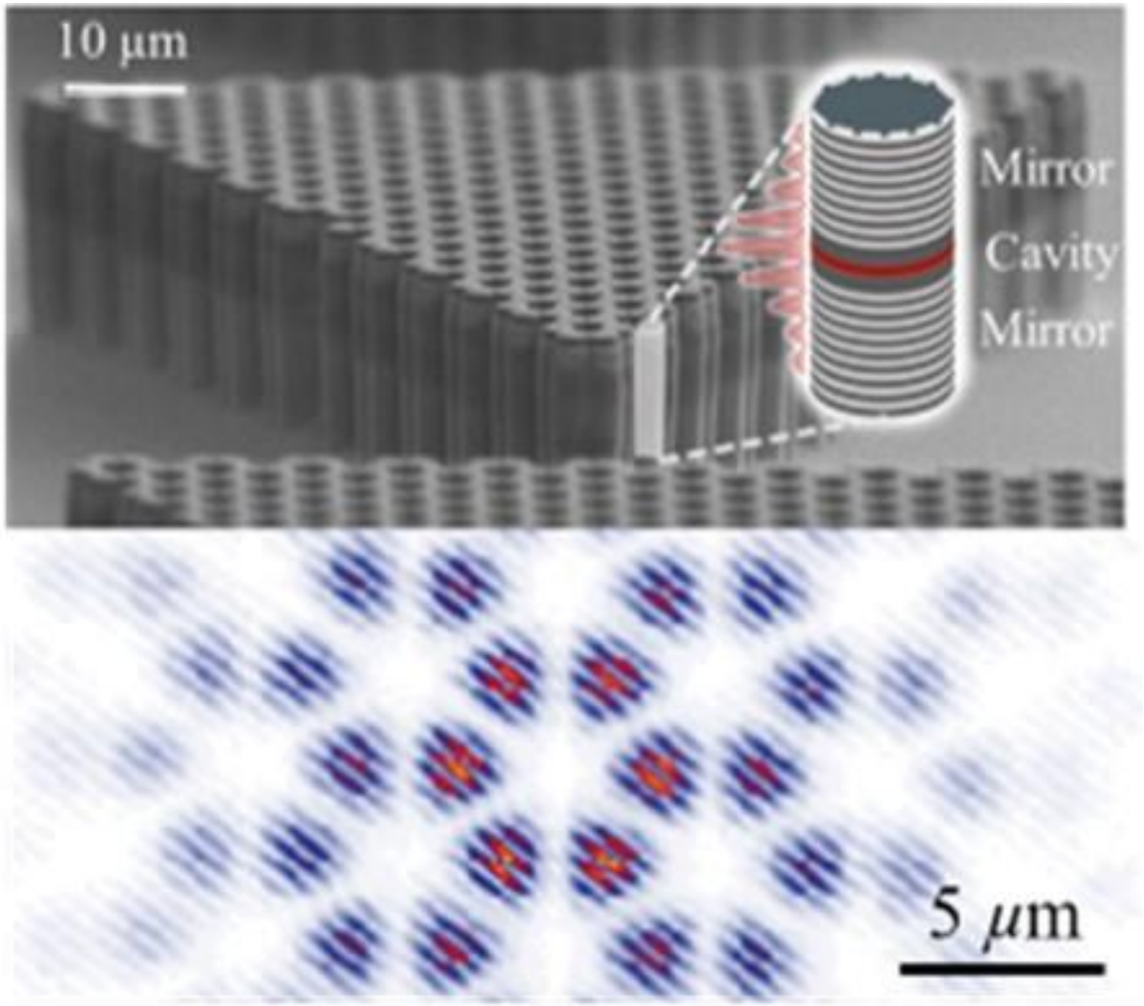



K. Deligiannis, *et al.*, Phys. Rev. Research 4, 043207 (2022)

# KPZ physics in 2D Polariton condensates?

Extended condensates in 2D




Daniela Pinto Dias

Quentin Fontaine

See Quentin's talk and

Daniela's poster

next week