Local conserved charges of Floquet XXX circuit

$$Q_1^+ = \sum_{n=1}^{N/2} q^{[1,+]}_{2n-2,2n-1,2n}, \quad Q_1^- = \sum_{n=1}^{N/2} q^{[1,-]}_{2n-1,2n,2n+1}$$

$$q^{[1,\pm]}_{1,2,3} = \frac{i}{2(1+\delta^2)}\left[\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3 + \delta^2\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3 \mp \delta\,\boldsymbol{\sigma}_1\cdot\big(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\big)\right]$$

$$Q_2^+ = \sum_{n=1}^{N/2} q^{[2,+]}_{2n-2,2n-1,2n,2n+1,2n+2}, \quad Q_2^- = \sum_{n=1}^{N/2} q^{[2,-]}_{2n-1,2n,2n+1,2n+2,2n+3}$$

$$q^{[2,\pm]}_{1,2,3,4,5} = \frac{i}{2(1+\delta^2)^2}[\mp 2\delta\,\boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_4 \mp 2\delta\,\boldsymbol{\sigma}_4\cdot\boldsymbol{\sigma}_5$$

$$\pm 2\delta\,\boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_5 - (1-\delta^2)\,\boldsymbol{\sigma}_3\cdot(\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5) - \boldsymbol{\sigma}_2\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4)$$

$$-\delta^2\,\boldsymbol{\sigma}_2\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5) - \delta^2\,\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4)$$

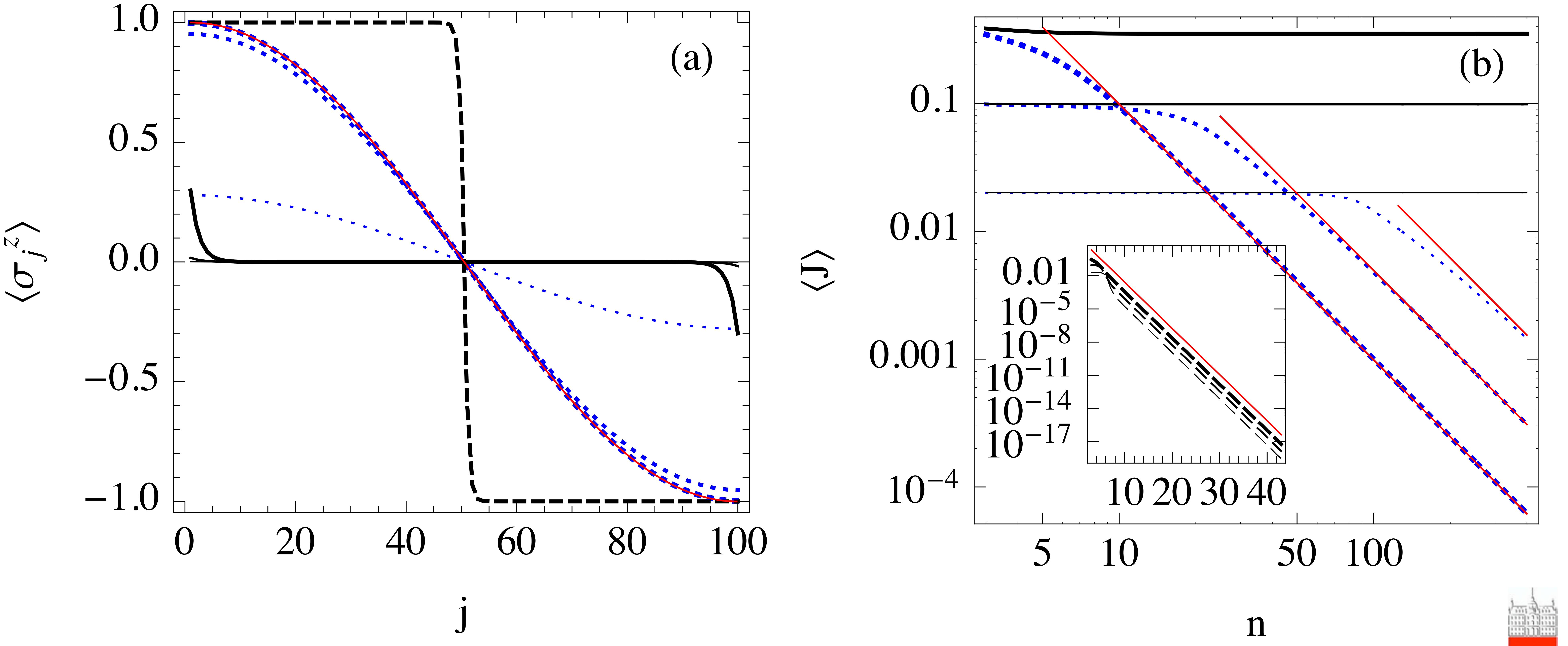
$$-\delta^4\,\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5) \pm \delta\,\boldsymbol{\sigma}_2\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5)$$

$$\pm\delta\,\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4) \pm \delta^3\,\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5)$$

$$\pm\delta^3\,\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_5) - \delta^2\,\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\times\boldsymbol{\sigma}_4\times\boldsymbol{\sigma}_5)].$$

NESS of boundary driven XXZ chain

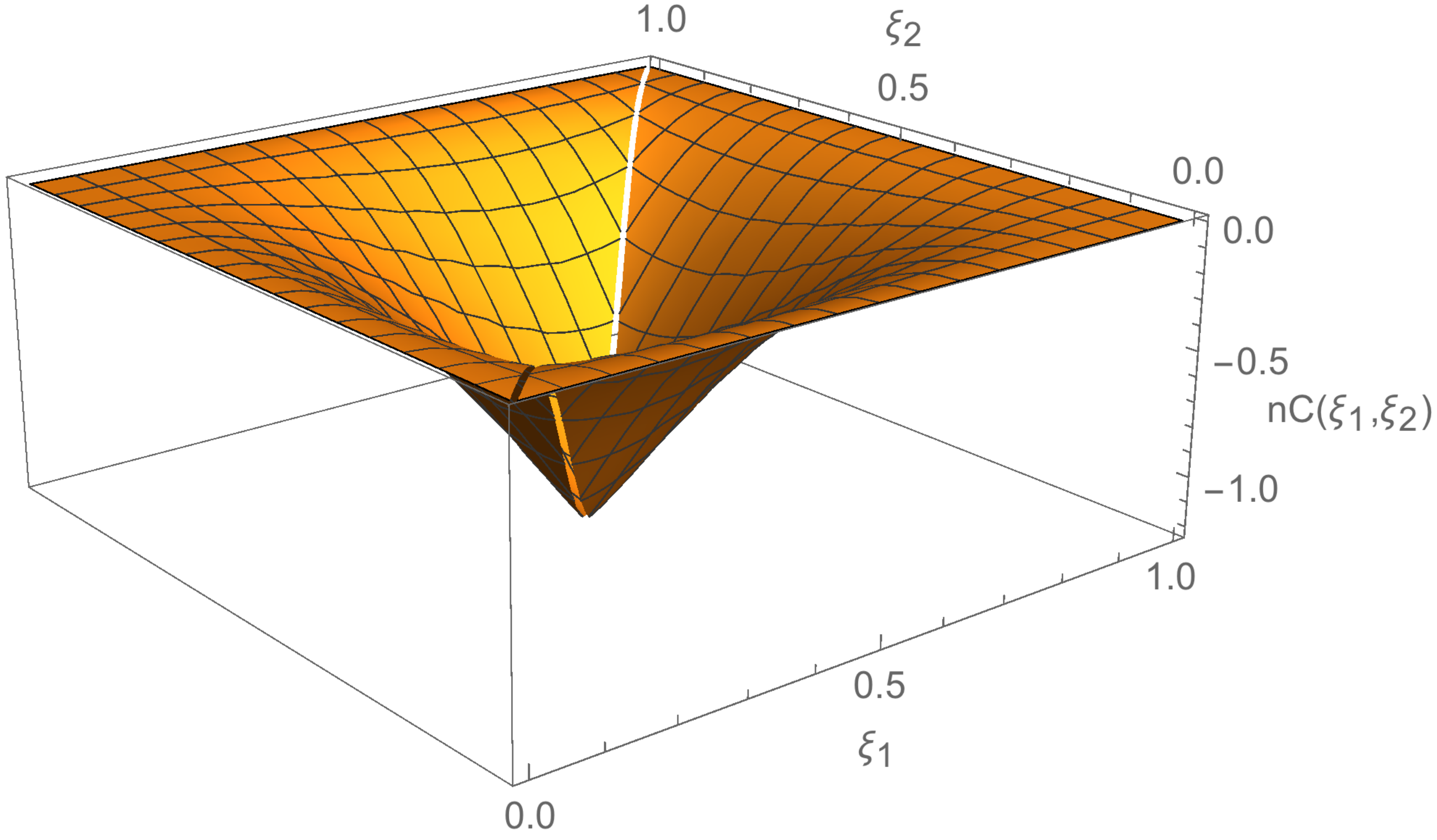
- For $|\Delta| < 1$, $\langle J \rangle \sim n^0$ (ballistic)
- For $|\Delta| > 1$, $\langle J \rangle \sim \exp(-\mathrm{const}\, n)$ (insulating)
- For $|\Delta| = 1$, $\langle J \rangle \sim n^{-2}$ (anomalous)



2-point correlation function in NESS

$$C\left(\frac{x}{n}, \frac{y}{n}\right) = \langle \sigma_x^z \sigma_y^z \rangle - \langle \sigma_x^z \rangle \langle \sigma_y^z \rangle$$

for isotropic case $\Delta = 1$ (XXX)



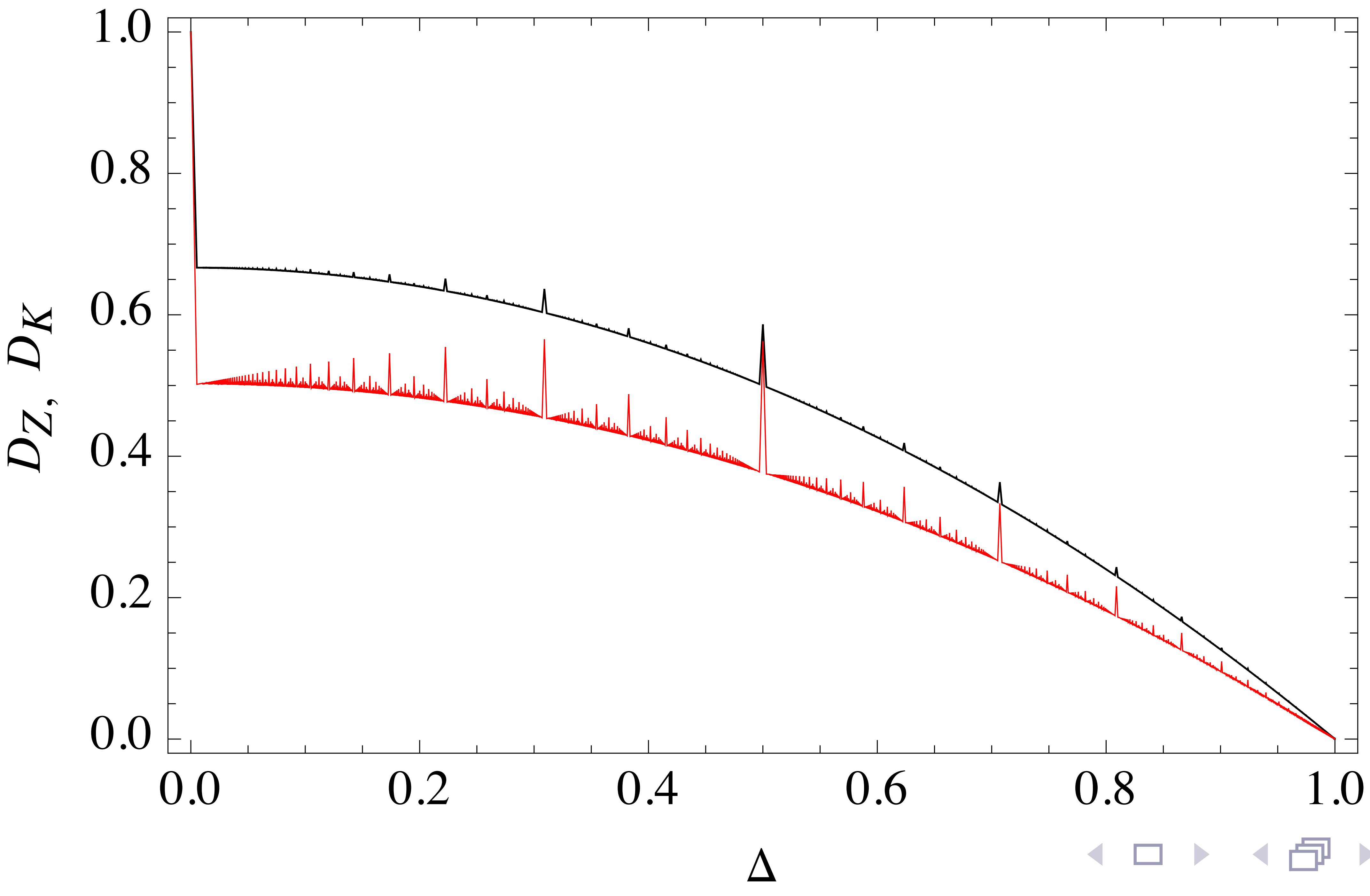
$$C(\xi_1, \xi_2) = -\frac{\pi^2}{2n}\xi_1(1-\xi_2)\sin(\pi\xi_1)\sin(\pi\xi_2), \quad \text{for} \quad \xi_1 < \xi_2$$

Fractal spin Drude weight

Fractal Mazur bound on Drude weight

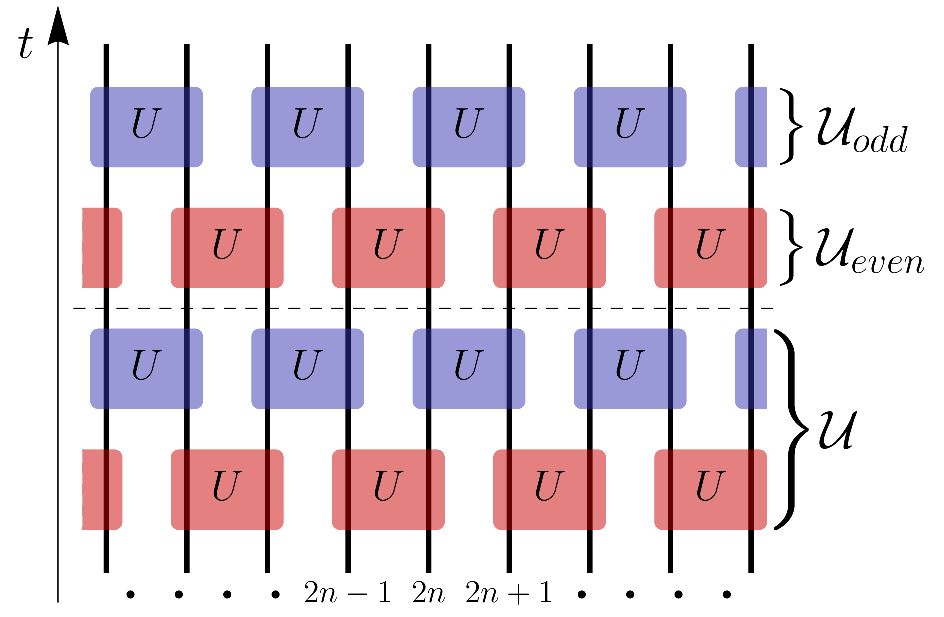
$$\frac{D}{\beta} \geq D_Z := \frac{\sin^2(\pi l/m)}{\sin^2(\pi/m)}\left(1 - \frac{m}{2\pi}\sin\left(\frac{2\pi}{m}\right)\right), \quad \Delta = \cos\left(\frac{\pi l}{m}\right)$$

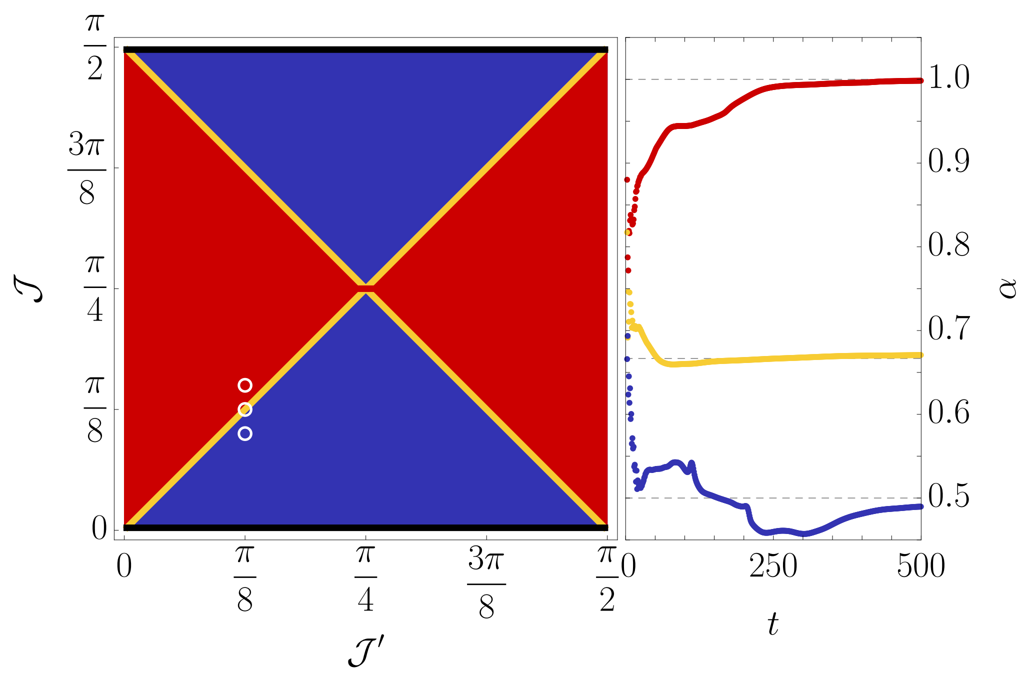
TP PRL **106** (2011); TP, Ilievski, PRL **111** (2013); TP, NPB **886** (2014); Ilievski, De Nardis, PRL **119** (2017)



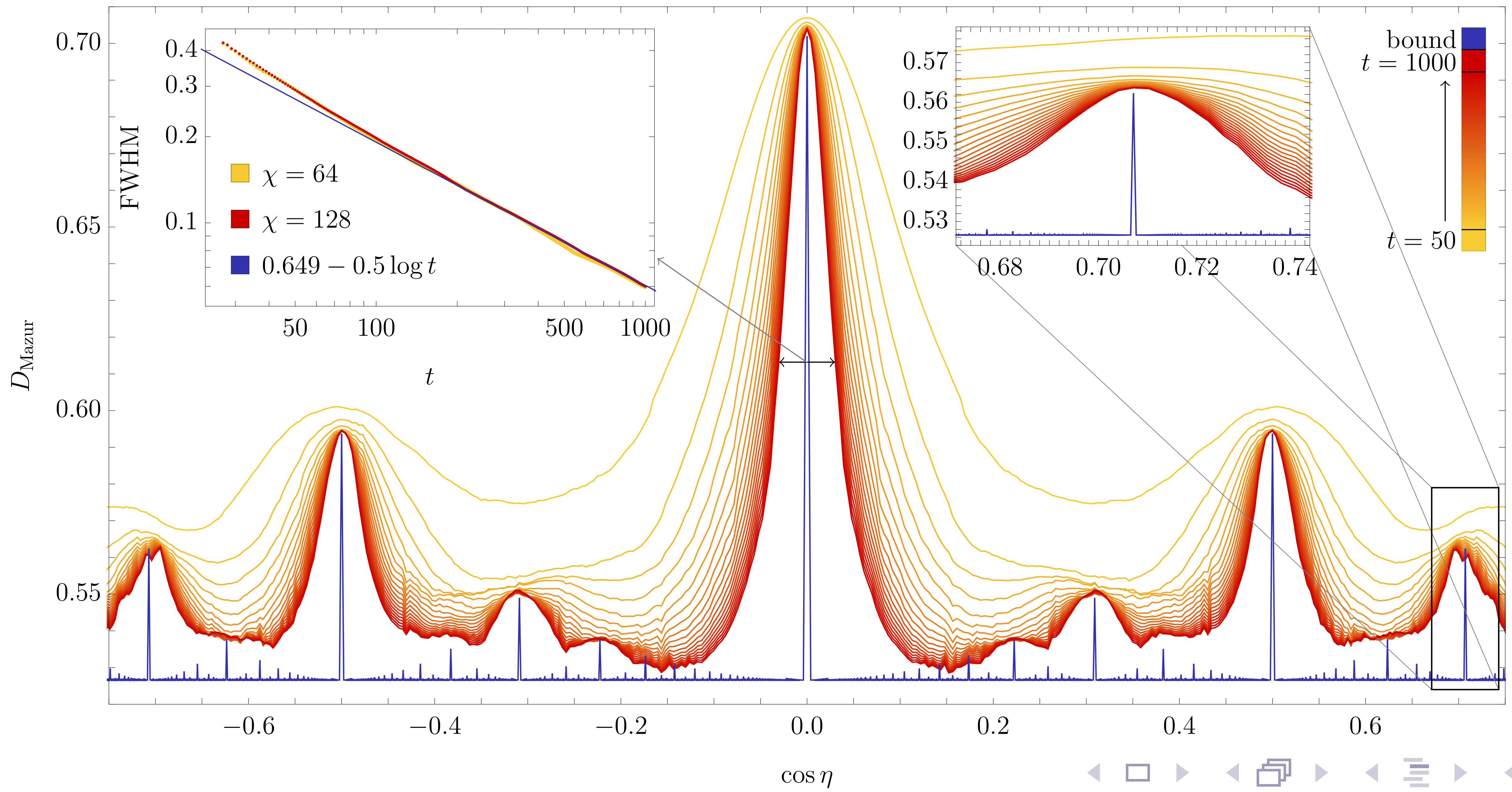
Fractal Drude weight in Floquet XXZ chains

[M. Vanicat, L. Zadnik and TP, PRL **121**, 030606(2018)]
[M. Ljubotina, L. Zadnik and TP, PRL **122**, 150605(2019)]

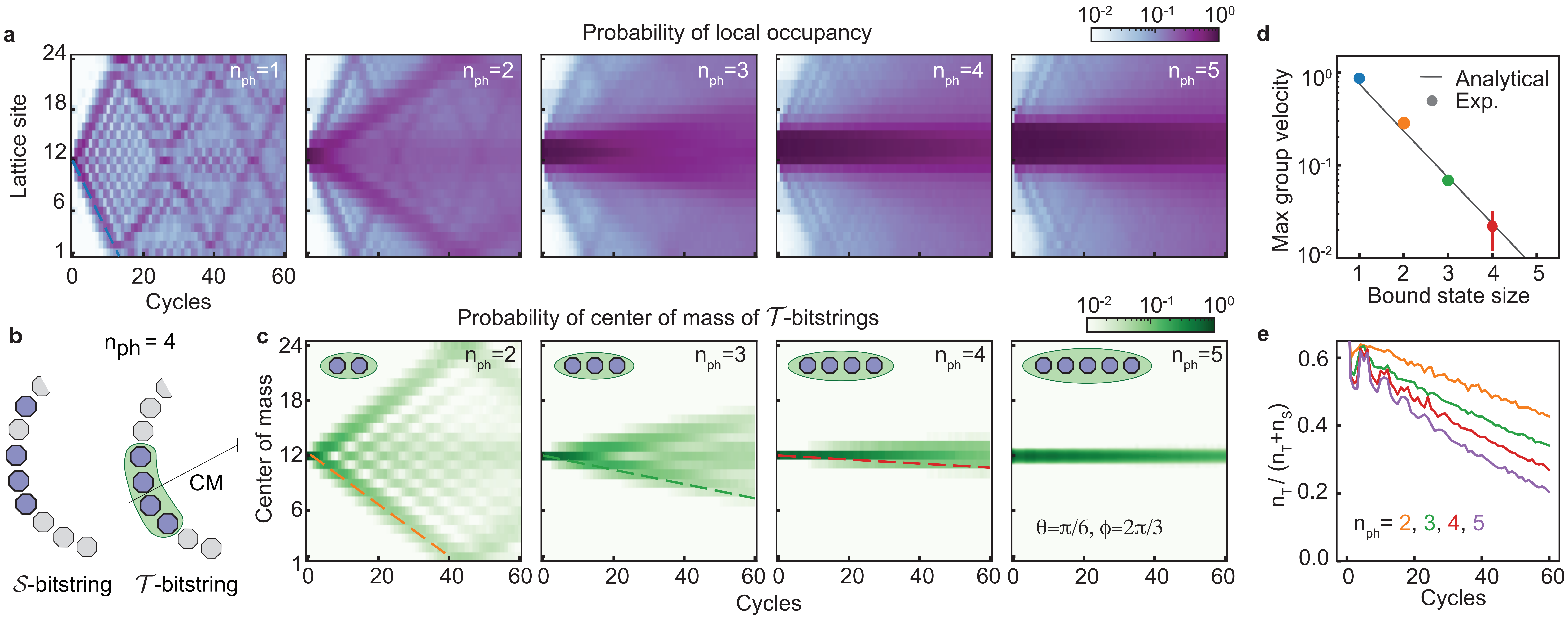




$$U = \exp\left(-i\mathcal{J}(\sigma^x \otimes \sigma^x + \sigma^y \otimes \sigma^y) - i\mathcal{J}'\sigma^z \otimes \sigma^z\right)$$



Integrability structure recently nicely demonstrated on a NISQ device!



[Google Quantum AI, arXiv:2206.05254],
see also [I. Aleiner, Ann. Phys. **433**, 168593(2021)]

ARTICLE

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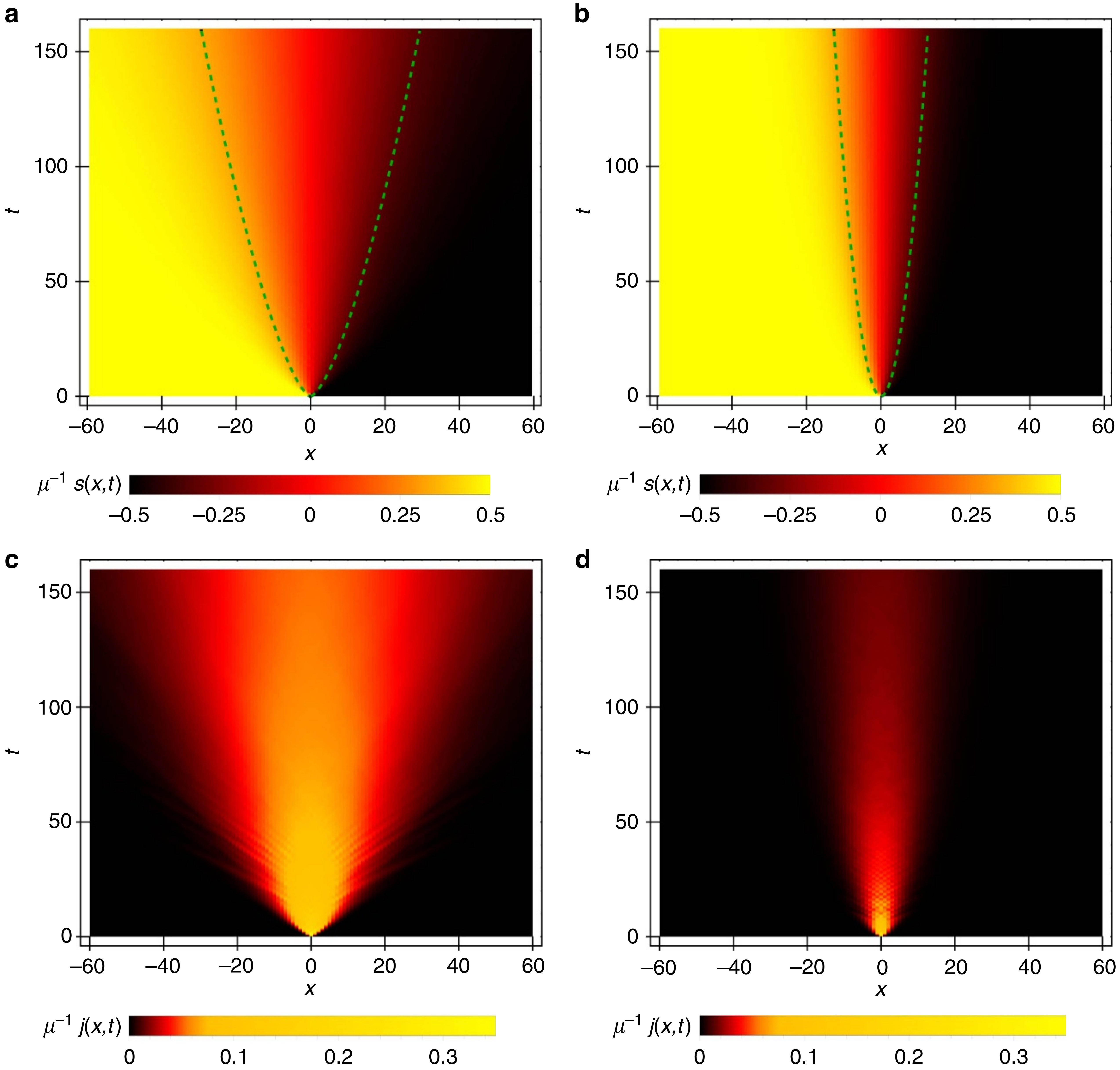
Spin diffusion from an inhomogeneous quench in an integrable system

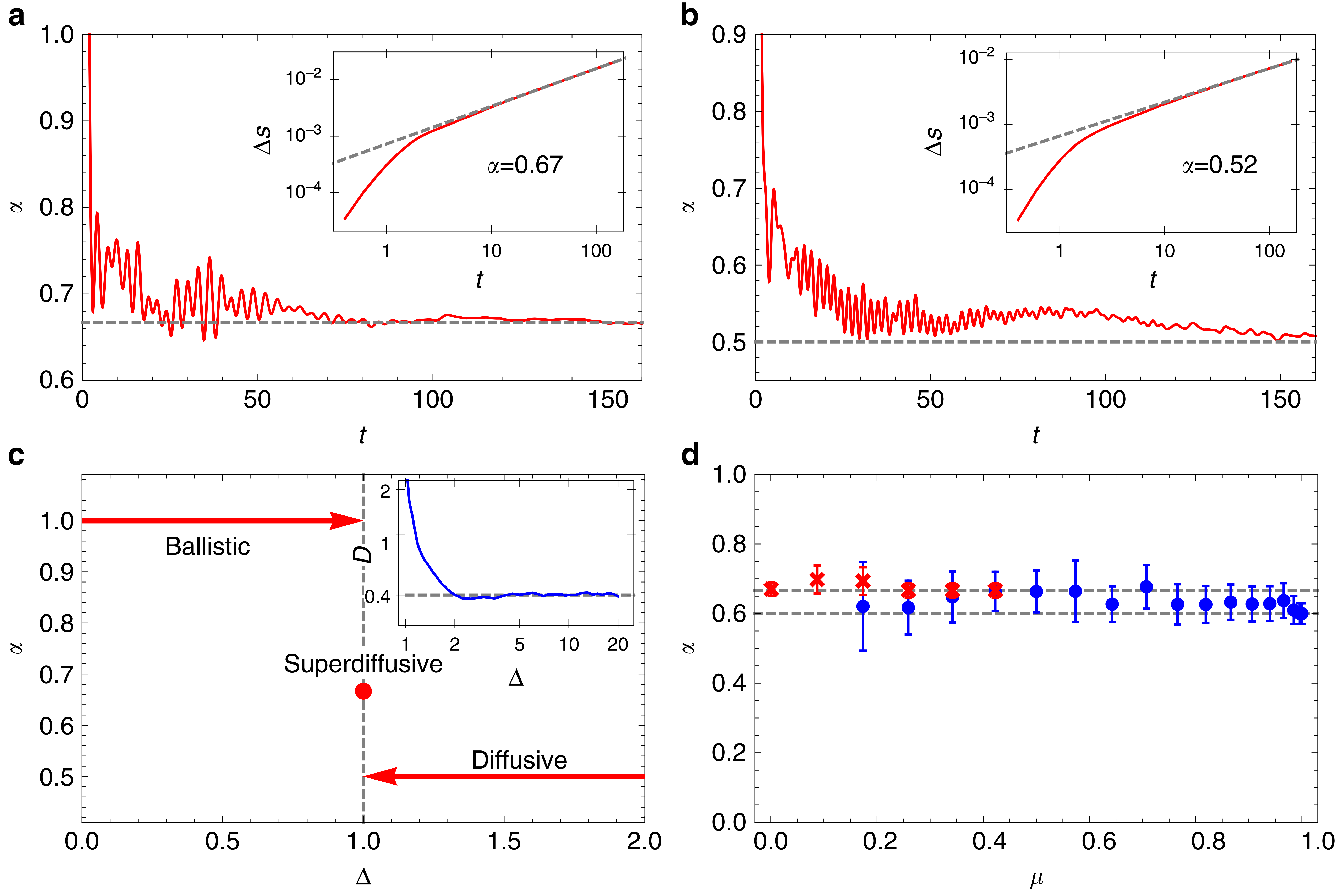
Marko Ljubotina[1], Marko Žnidarič[1] & Tomaž Prosen[1]

$$H = \sum_{j=1}^{L} (\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \Delta \sigma_j^z \sigma_{j+1}^z)$$

$$\rho(t=0) = (\mathbb{1} + \mu\sigma^z)^{\otimes L/2} \otimes (\mathbb{1} - \mu\sigma^z)^{\otimes L/2}$$

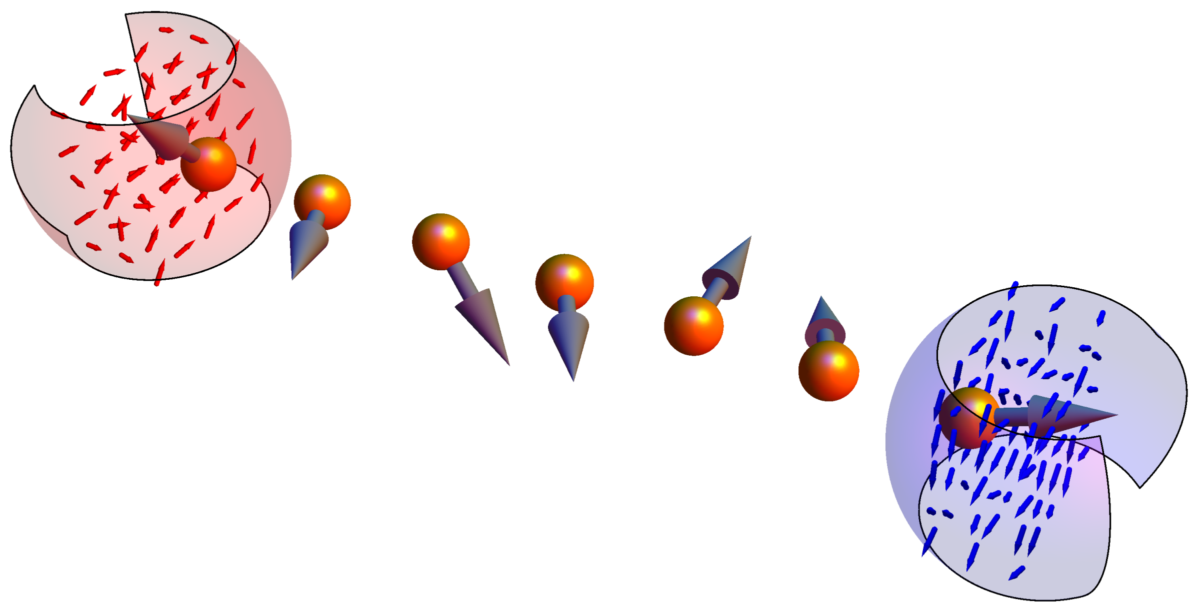


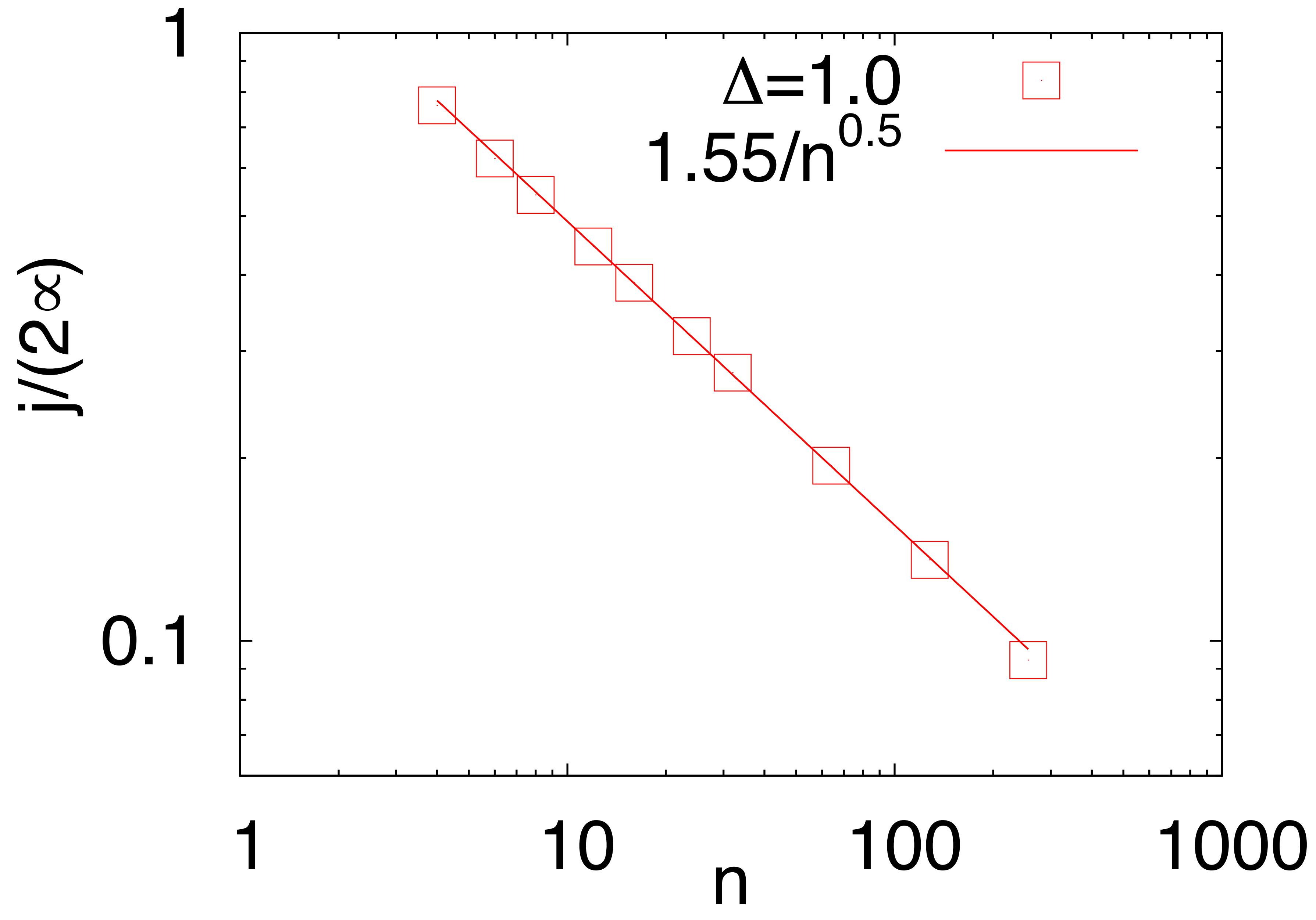
$$\Delta s^z(t) = \int_0^t \mathrm{d}t' j(x = L/2, t') \propto t^\alpha$$

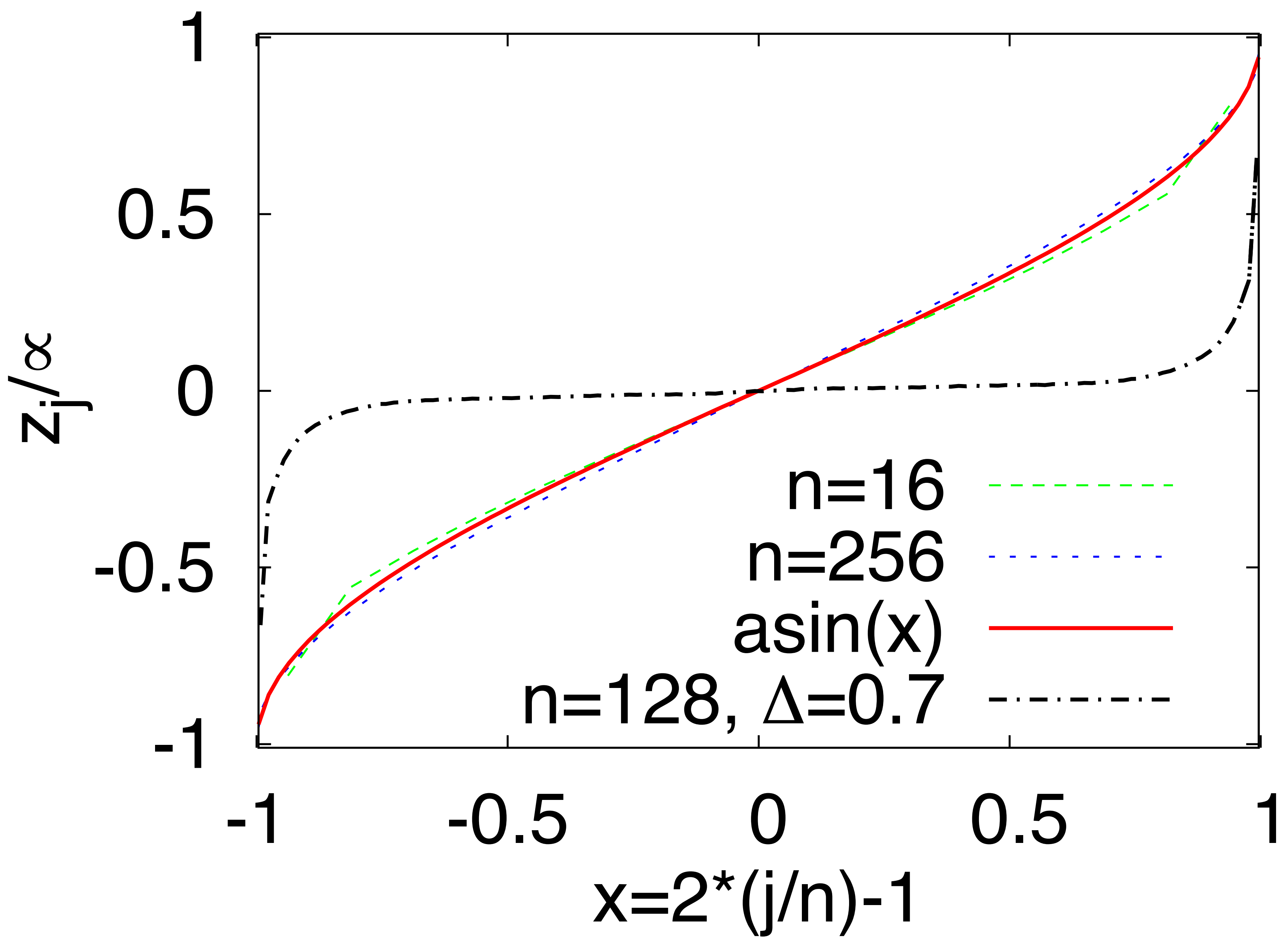


First data came from boundary driven Lindblad XXX chain

[Žnidarič, PRL (2011)]







Kardar-Parisi-Zhang Physics in the Quantum Heisenberg Magnet

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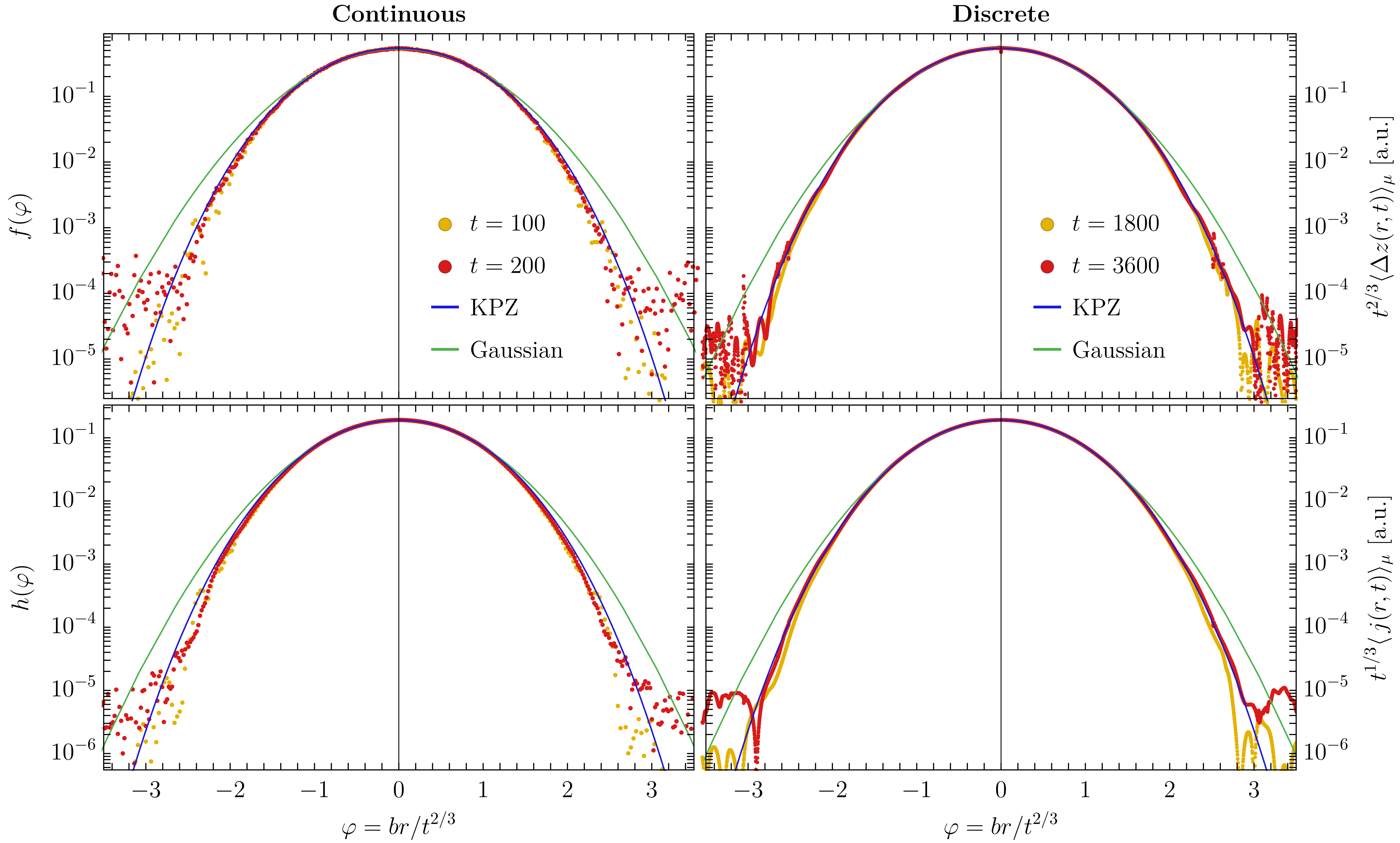
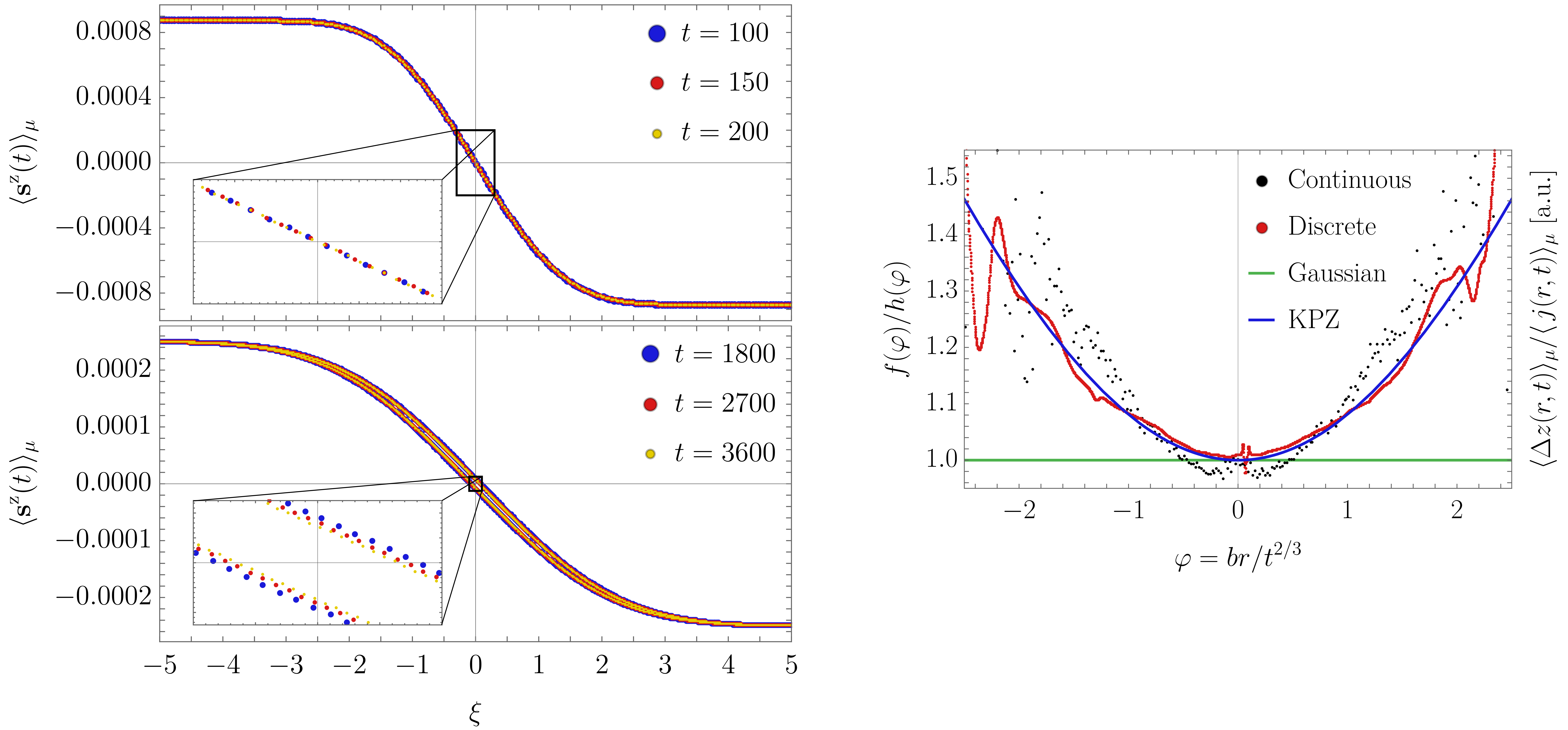
(Received 7 March 2019; published 31 May 2019)

Equilibrium spatiotemporal correlation functions are central to understanding weak nonequilibrium physics. In certain local one-dimensional classical systems with three conservation laws they show universal features. Namely, fluctuations around ballistically propagating sound modes can be described by the celebrated Kardar-Parisi-Zhang (KPZ) universality class. Can such a universality class be found also in quantum systems? By unambiguously demonstrating that the KPZ scaling function describes magnetization dynamics in the $SU(2)$ symmetric Heisenberg spin chain we show, for the first time, that this is so. We achieve that by introducing new theoretical and numerical tools, and make a puzzling observation that the conservation of energy does not seem to matter for the KPZ physics.

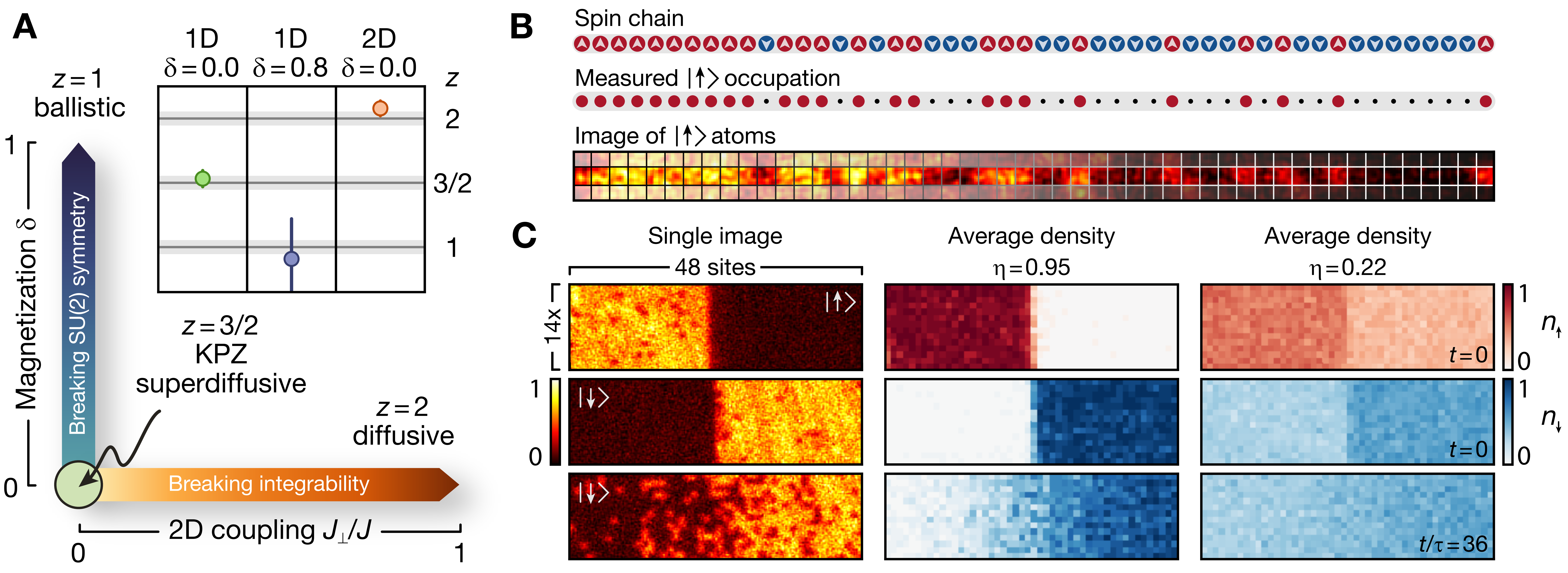




0.0008
0.0004
0.0000
−0.0004
−0.0008
⟨s^z(t)⟩_μ
t = 100
t = 150
t = 200
0.0002
0.0001
0.0000
−0.0001
−0.0002
⟨s^z(t)⟩_μ
t = 1800
t = 2700
t = 3600
−5 −4 −3 −2 −1 0 1 2 3 4 5
ξ
1.5
1.4
1.3
1.2
1.1
1.0
f(φ)/h(φ)
Continuous
Discrete
Gaussian
KPZ
−2 −1 0 1 2
φ = br/t^{2/3}
⟨Δz(r,t)⟩_μ/⟨j(r,t)⟩_μ [a.u.]

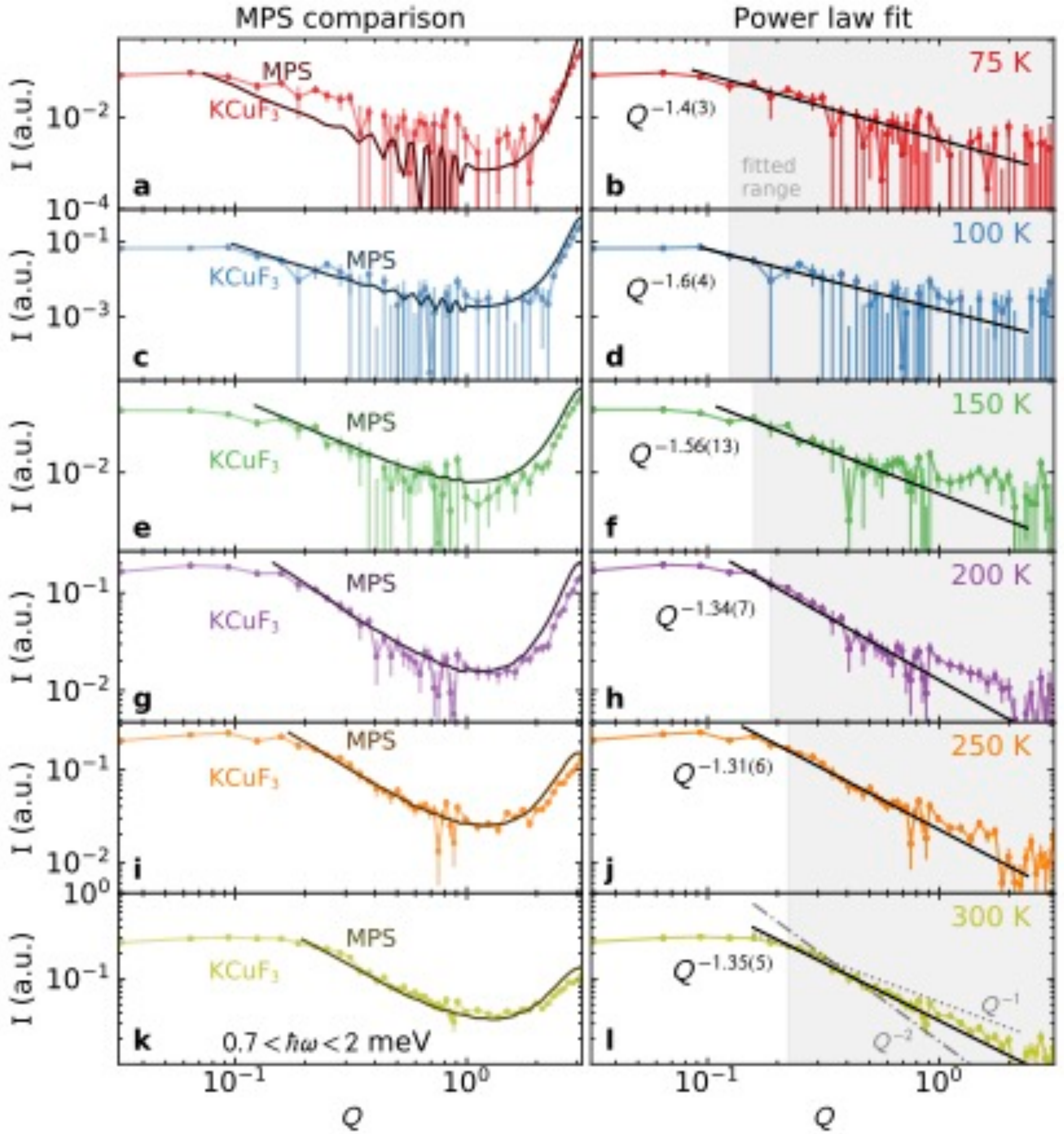
Experiments (cold atoms):

D. Wei, A. Rubio-Abadal, B. Ye, F. Machado, J. Kemp, K. Srakaew, S. Hollerith, J. Rui, S. Gopalakrishnan, N. Yao, I. Bloch, J. Zeiher, Science (2022)

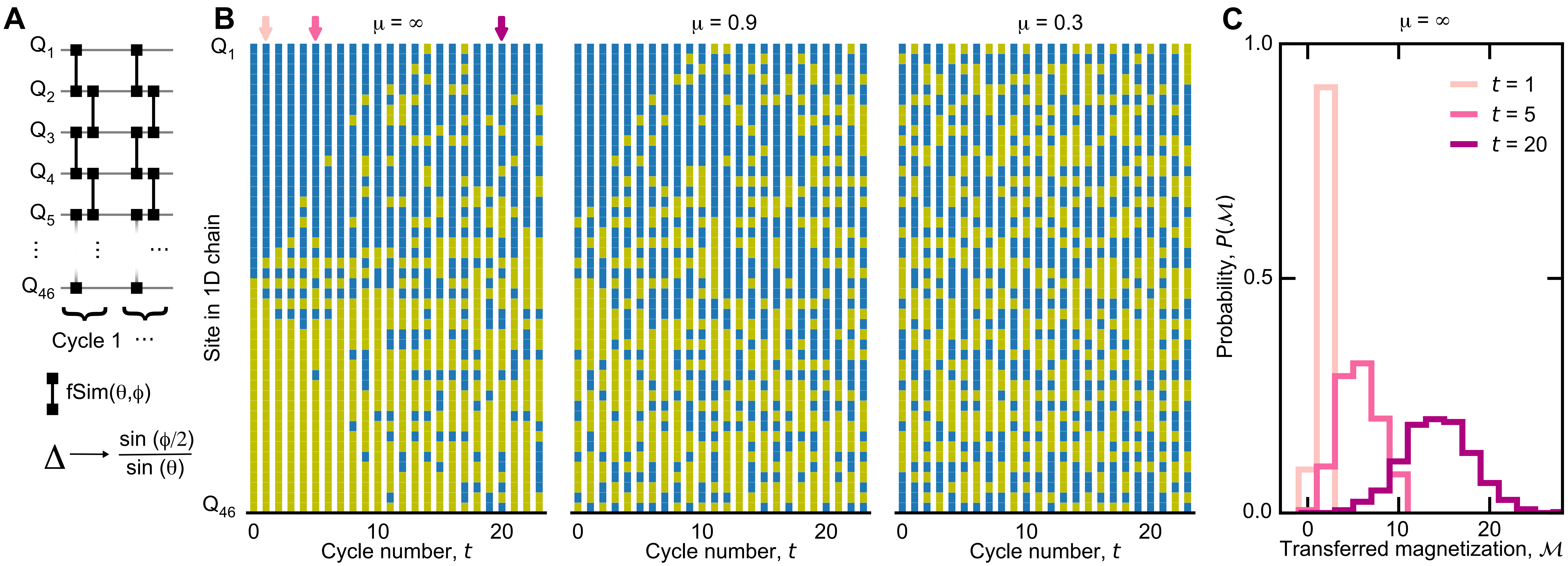


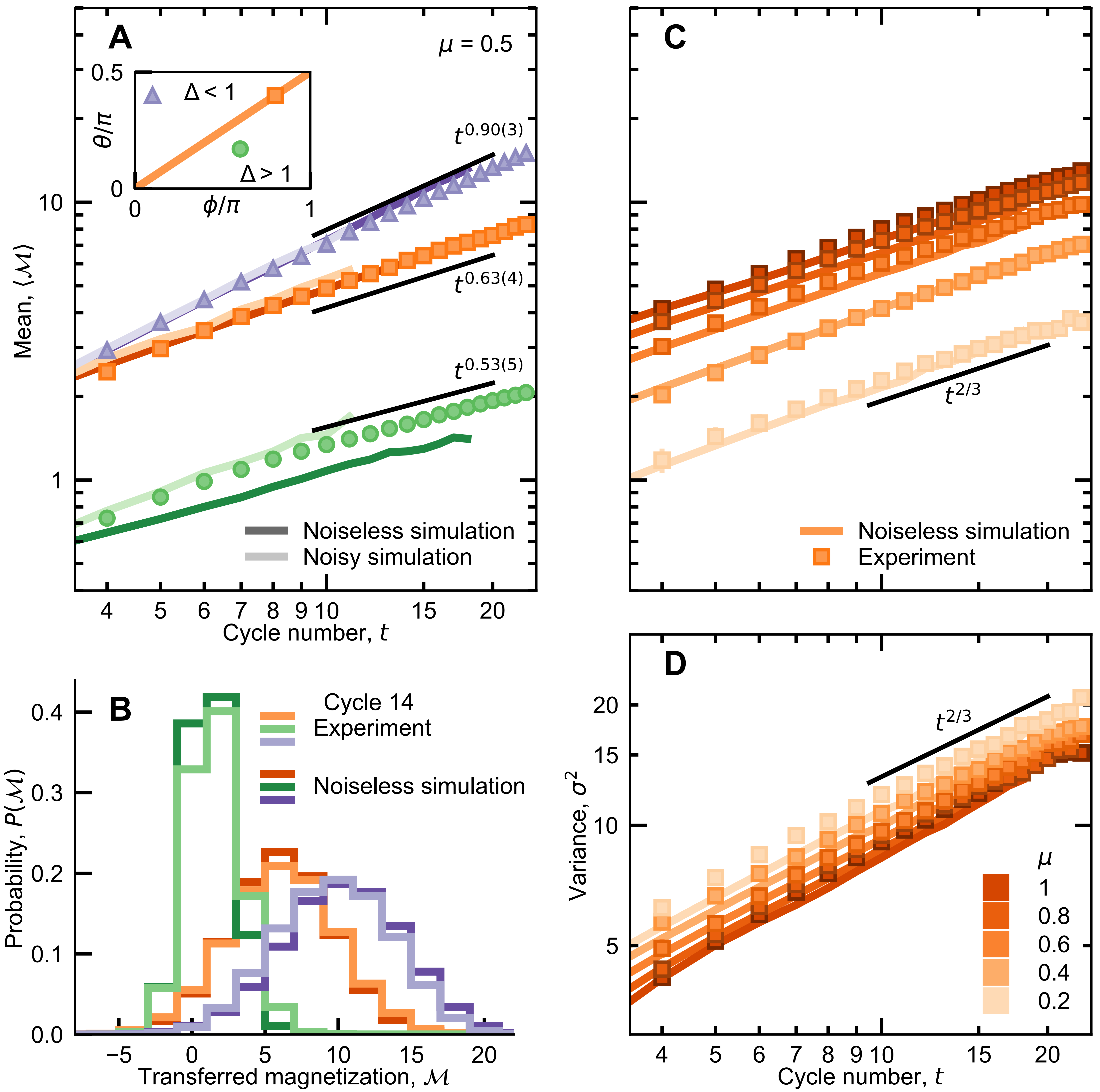
Experiments (solid state, neutron scattering):

A. Scheie, N. E. Sherman, M. Dupont, S. E. Nagler, M. B. Stone, G. E. Granroth, J. E. Moore, D. A. Tennant, Nature Phys. (2021)

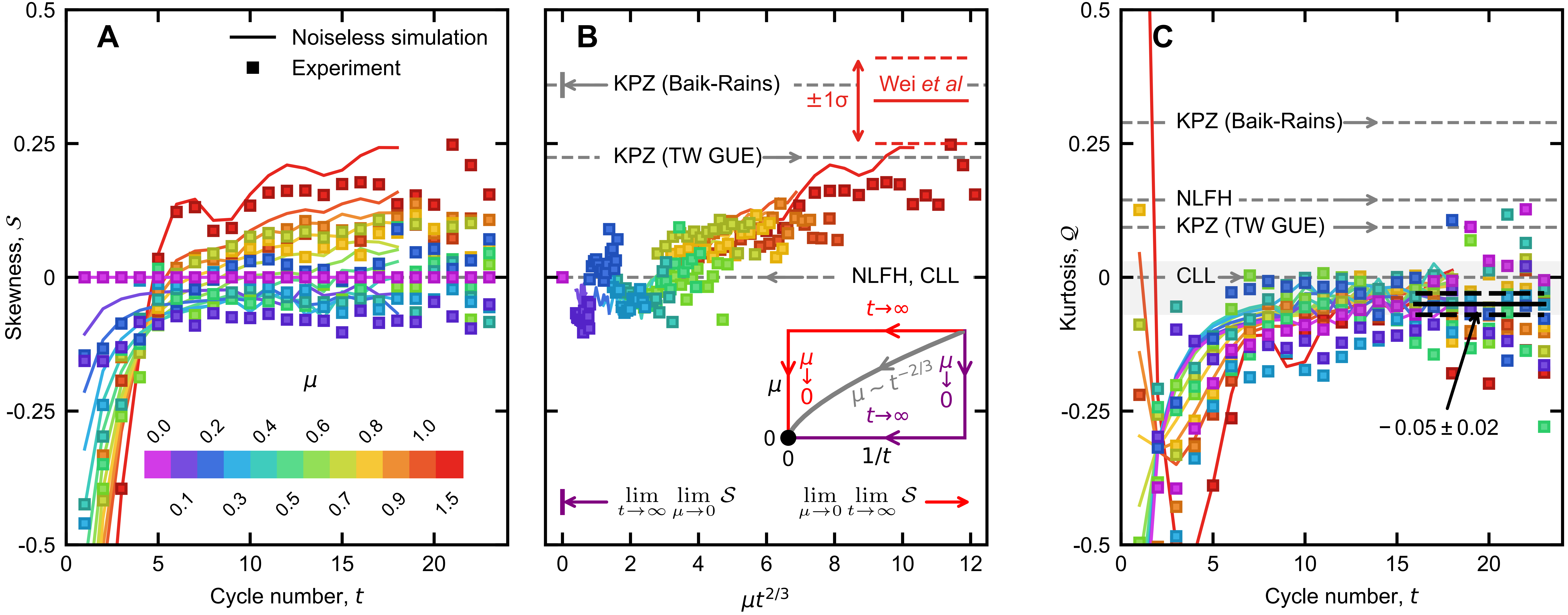


Transport and full counting in XXX/XXZ circuits on Sycamore chip [Google Quantum AI and collaborators, Science 2024]





A
μ = 0.5
0.5
Δ < 1
θ/π
Δ > 1
0
0
ϕ/π
1
10
t^0.90(3)
t^0.63(4)
t^0.53(5)
Mean, ⟨M⟩
1
Noiseless simulation
Noisy simulation
4
5
6
7
8
9
10
15
20
Cycle number, t
B
Cycle 14
Experiment
Noiseless simulation
Probability, P(M)
0.4
0.3
0.2
0.1
0.0
−5
0
5
10
15
20
Transferred magnetization, M
C
t^2/3
Noiseless simulation
Experiment
D
t^2/3
Variance, σ^2
20
15
10
5
μ
1
0.8
0.6
0.4
0.2
Cycle number, t



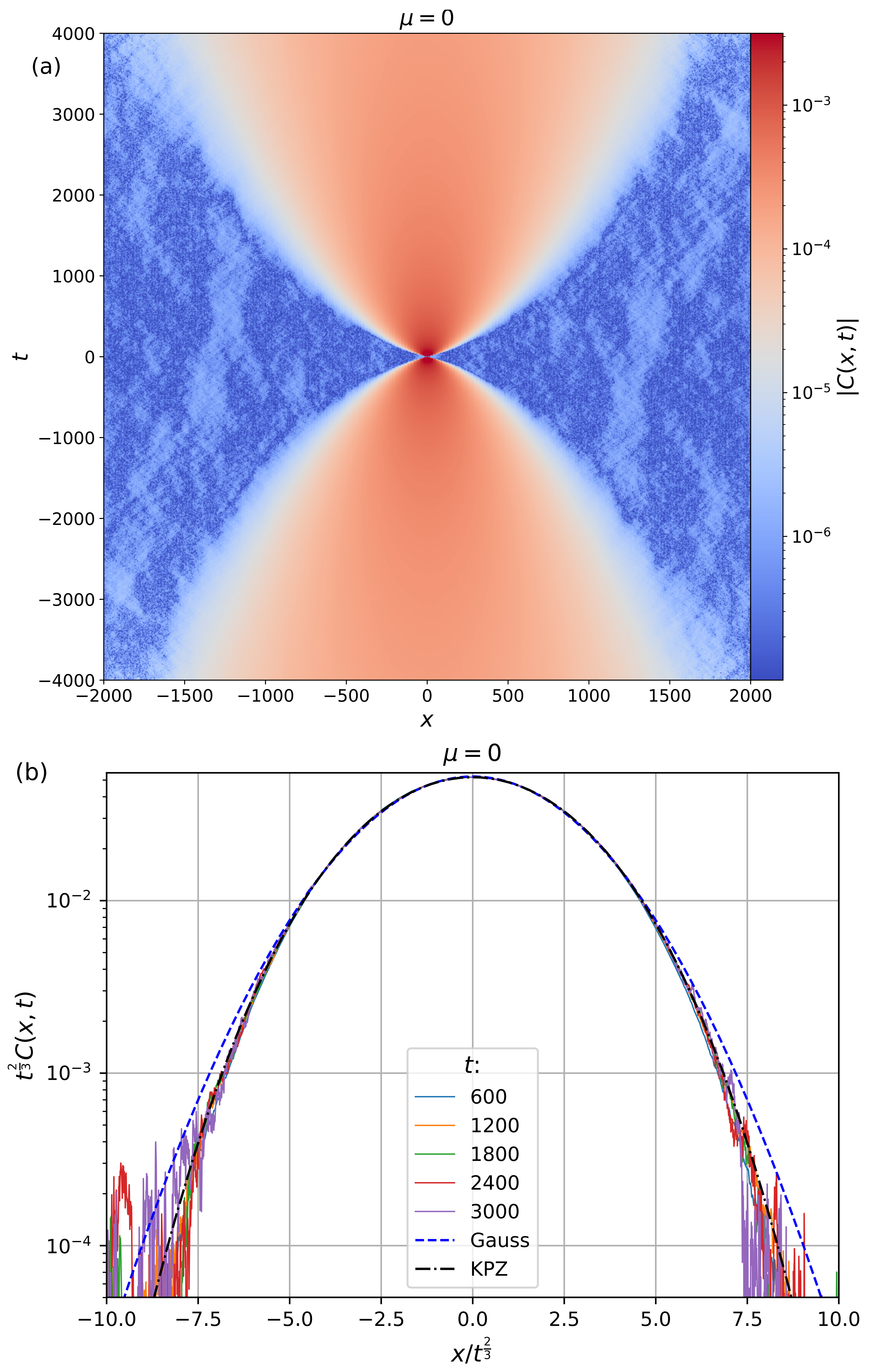
A
Noiseless simulation
Experiment
Skewness, $\mathcal{S}$
Cycle number, t
μ
0.0 0.2 0.4 0.6 0.8 1.0
0.1 0.3 0.5 0.7 0.9 1.5
B
KPZ (Baik-Rains)
KPZ (TW GUE)
±1σ
Wei *et al*
NLFH, CLL
$t\to\infty$
μ
$\mu \to 0$
$\mu \sim t^{-2/3}$
$1/t$
$\lim_{t\to\infty} \lim_{\mu\to 0} \mathcal{S}$
$\lim_{\mu\to 0} \lim_{t\to\infty} \mathcal{S}$
$\mu t^{2/3}$
C
Kurtosis, $\mathcal{Q}$
KPZ (Baik-Rains)
NLFH
KPZ (TW GUE)
CLL
-0.05 ± 0.02
Cycle number, t

Spin transport in classical SO(3) symmetric integrable symplectic lattice gas

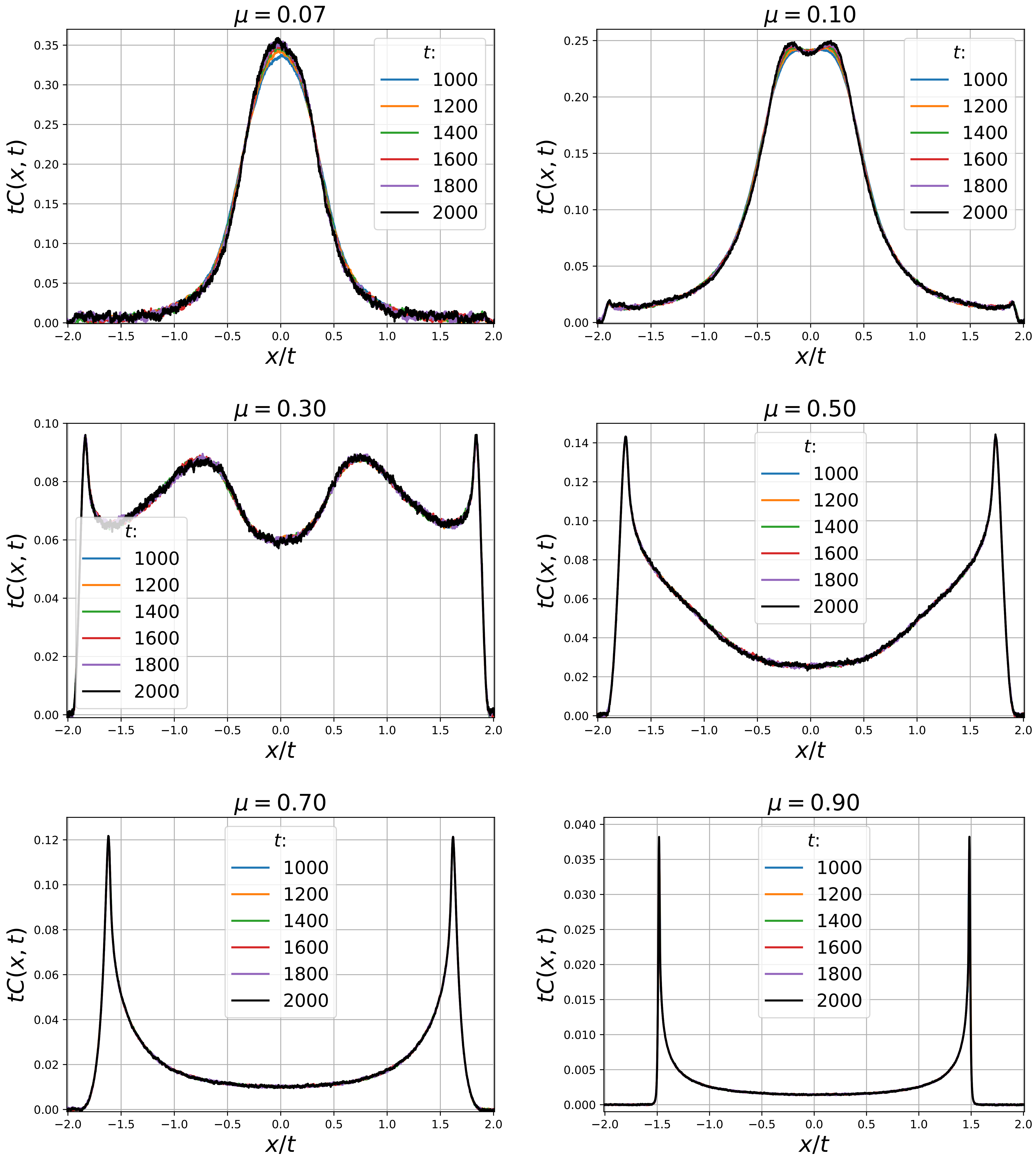
$$\Phi_\tau(\mathbf{S}_1,\mathbf{S}_2) \;=\; \frac{1}{\sigma^2+\tau^2}\Big(\sigma^2\mathbf{S}_1+\tau^2\mathbf{S}_2+\tau\mathbf{S}_1\times\mathbf{S}_2, \sigma^2\mathbf{S}_2+\tau^2\mathbf{S}_1+\tau\mathbf{S}_2\times\mathbf{S}_1\Big)$$

$$(\mathbf{S}^{2t+1}_{2x},\mathbf{S}^{2t+1}_{2x+1}) = \Phi_\tau(\mathbf{S}^{2t}_{2x},\mathbf{S}^{2t}_{2x+1}), \qquad (\mathbf{S}^{2t+2}_{2x-1},\mathbf{S}^{2t+1}_{2x}) = \Phi_\tau(\mathbf{S}^{2t+1}_{2x-1},\mathbf{S}^{2t+1}_{2x})$$

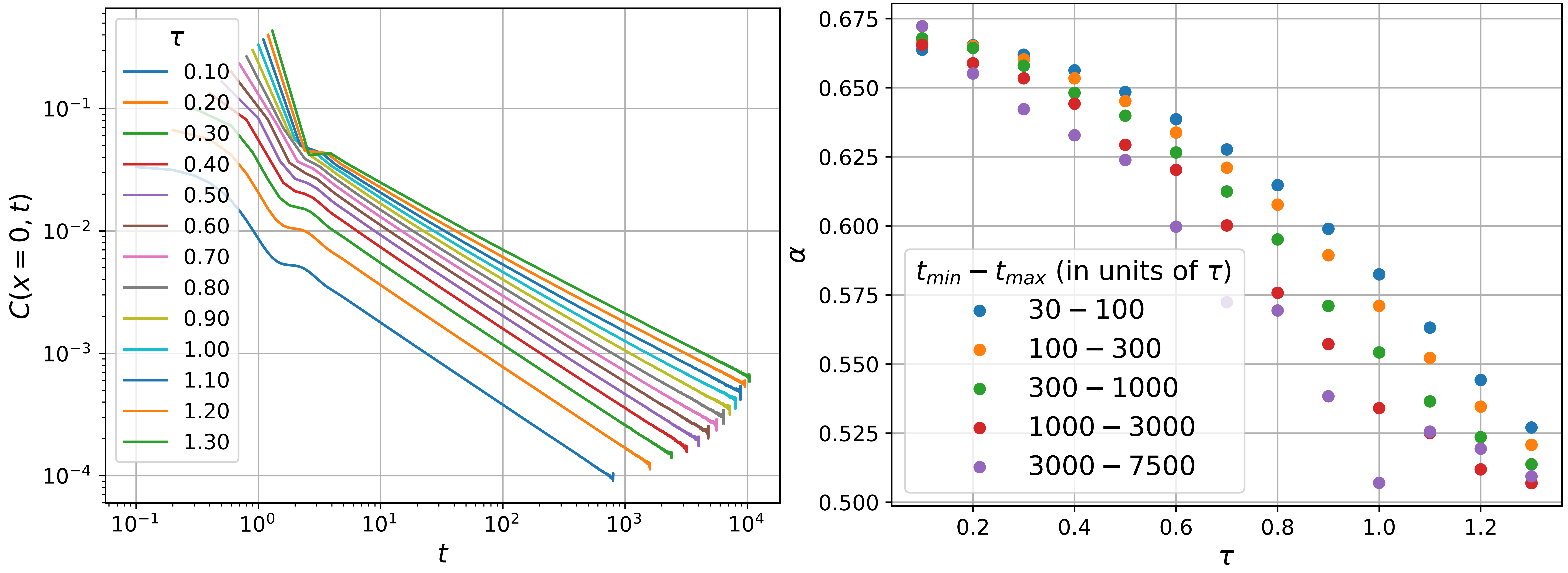
[Krajnik, P, JSP2020]



Breaking SO(3) symmetry - polarized equilibrium state



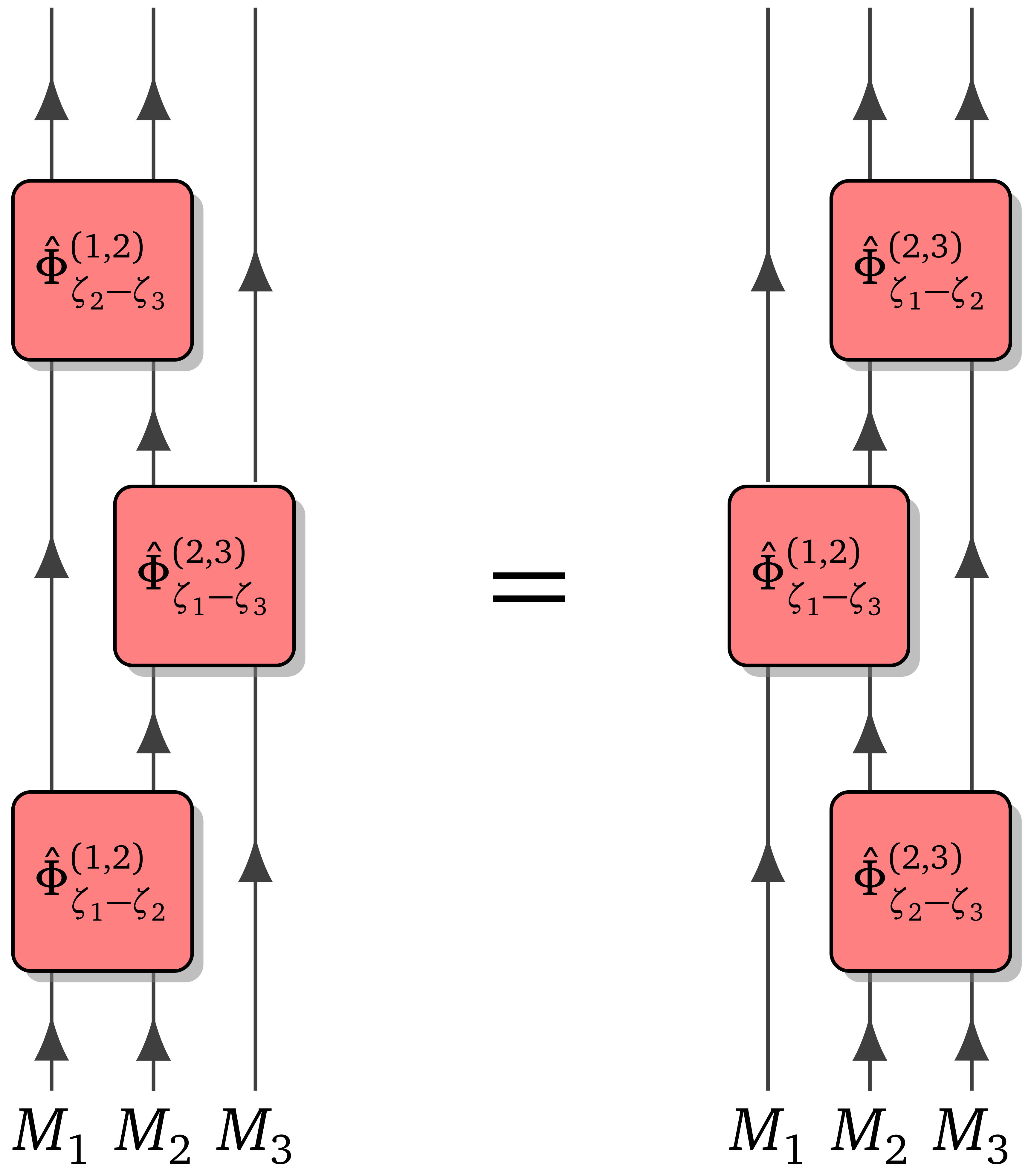
Breaking integrability



(a) (b)

Set theoretic Yang-Baxter

$$\hat{\Phi}^{(1,2)}_{\zeta_2-\zeta_3} \circ \hat{\Phi}^{(2,3)}_{\zeta_1-\zeta_3} \circ \hat{\Phi}^{(1,2)}_{\zeta_1-\zeta_2} = \hat{\Phi}^{(2,3)}_{\zeta_1-\zeta_2} \circ \hat{\Phi}^{(1,2)}_{\zeta_1-\zeta_3} \circ \hat{\Phi}^{(2,3)}_{\zeta_2-\zeta_3}$$



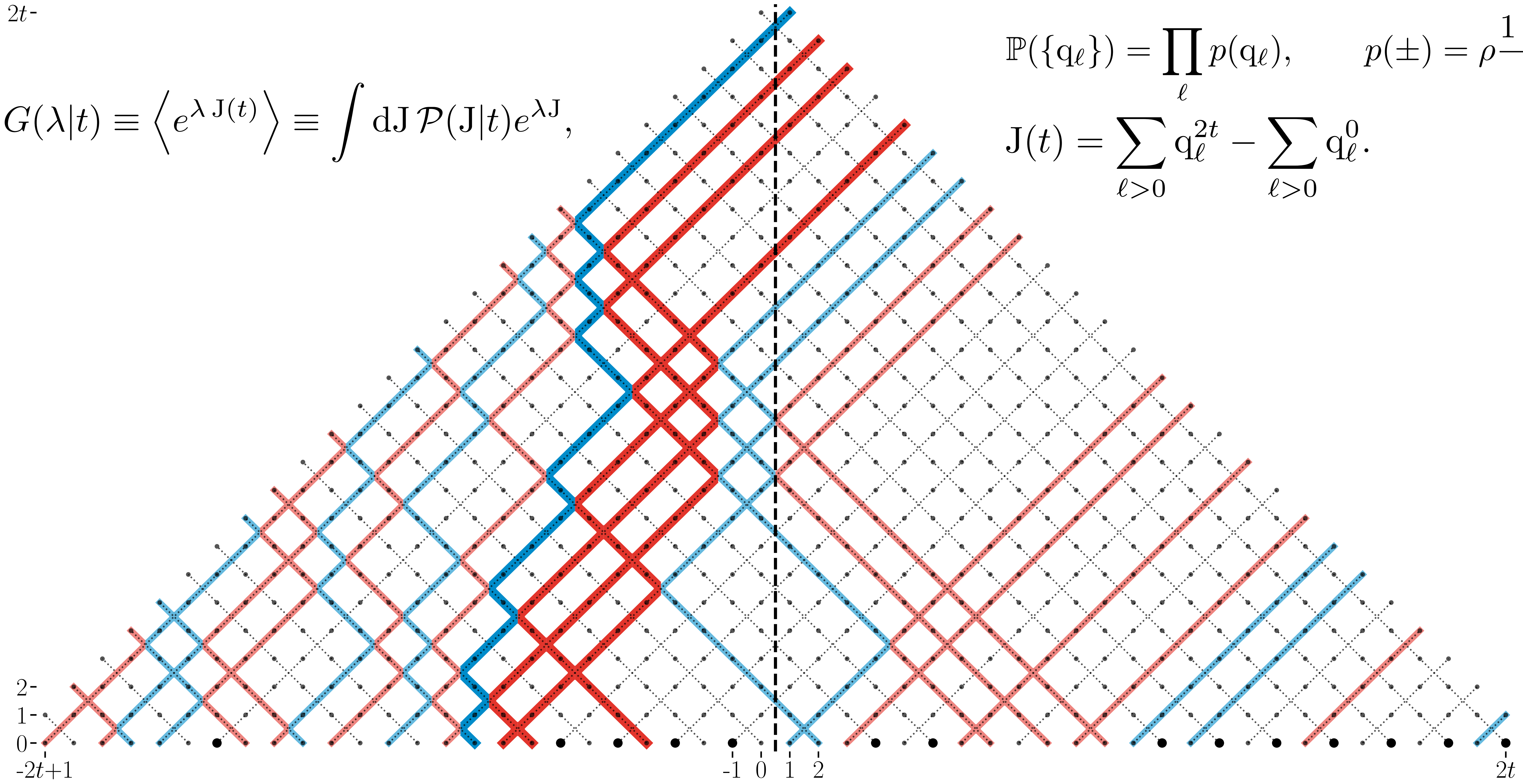
Full counting statistics in deterministic many body dynamics

Exactly solvable charged hard point cellular automaton

$$G(\lambda|t) \equiv \left\langle e^{\lambda\,\mathrm{J}(t)} \right\rangle \equiv \int \mathrm{dJ}\,\mathcal{P}(\mathrm{J}|t)e^{\lambda \mathrm{J}},$$

$$\mathbb{P}(\{\mathrm{q}_\ell\}) = \prod_\ell p(\mathrm{q}_\ell), \qquad p(\pm) = \rho\frac{1 \pm b}{2}, \qquad p(\emptyset) = \bar{\rho} = 1 - \rho,$$

$$\mathrm{J}(t) = \sum_{\ell > 0} \mathrm{q}_\ell^{2t} - \sum_{\ell > 0} \mathrm{q}_\ell^{0}.$$



$$G(\lambda|t;\Sigma) = \prod_{\ell \in \Lambda_-} \langle e^{\lambda q_\ell} \rangle \prod_{\ell \in \Lambda_+} \langle e^{-\lambda q_\ell} \rangle = \mu_-^{|\Lambda_-|}\mu_+^{|\Lambda_+|}, \qquad |\Lambda_\pm| = \frac{|l - r| \pm (l - r)}{2}. \qquad \mu_\pm = \cosh\lambda \mp b \sinh\lambda,$$

$$G(\lambda|t) = \rho^{2t} \sum_{l=0}^{t} \sum_{r=0}^{t} \binom{t}{l}\binom{t}{r} \nu^{l+r} \mu_-^{|\Lambda_-|}\mu_+^{|\Lambda_+|}$$

[Krajnik, Schmidt, Pasquier, Ilievski, P, PRL22]

Critical scaling of cumulants

$$c_n(t) \equiv \frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}\Big|_{\lambda=0} \log G(\lambda|t).$$

$$c_n^{[0]}(t) = \sum_{l=0}^{r} c_{n|l}^{[0]} t^{(n-2l)/4} + \mathcal{O}(t^{(n-2(r+1))/4}),$$
$$c_n^{[b]}(t) = \sum_{l=0}^{r} c_{n|l}^{[b]} t^{(n-l)/2} + \mathcal{O}(t^{(n-(r+1))/2}).$$

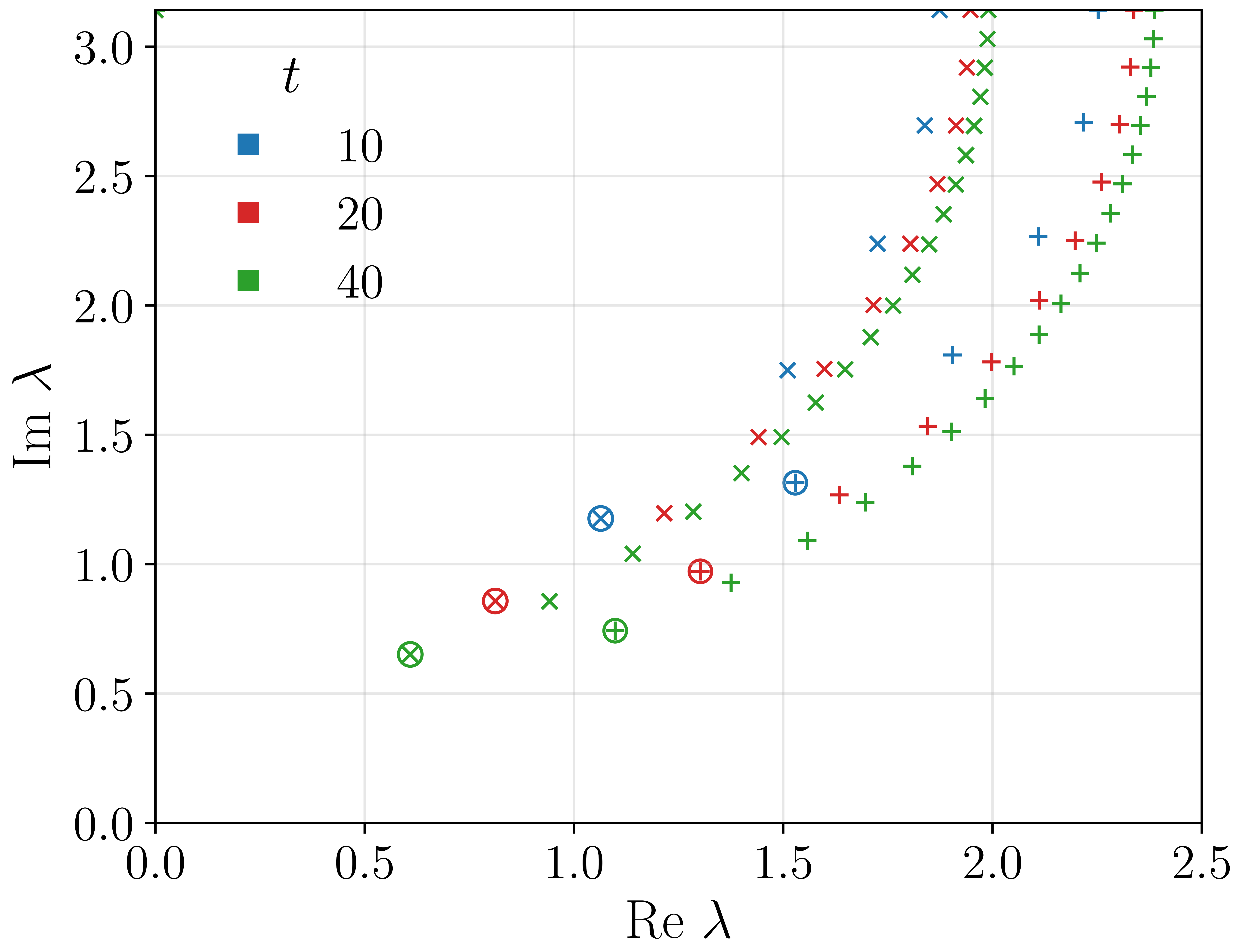
$$c_{2|0}^{[0]} = \frac{2\varDelta}{\pi^{1/2}},$$
$$c_{4|0}^{[0]} = \frac{6\varDelta^2}{\pi}(\pi-2),$$
$$c_{6|0}^{[0]} = \frac{60\varDelta^3}{\pi^{3/2}}(4-\pi),$$
$$c_{8|0}^{[0]} = \frac{3360\varDelta^4}{\pi^2}(\pi-3),$$
$$c_{10|0}^{[0]} = \frac{7560\varDelta^5}{\pi^{5/2}}(3\pi^2-40\pi+96),$$

$$c_{4|1}^{[0]} = -\frac{4\varDelta}{\pi^{1/2}},$$
$$c_{6|1}^{[0]} = \frac{60\varDelta^2}{\pi}(2-\pi),$$
$$c_{8|1}^{[0]} = -\frac{1680\varDelta^3}{\pi^{3/2}}(4-\pi),$$
$$c_{10|1}^{[0]} = \frac{201600\varDelta^4}{\pi^2}(3-\pi).$$

$$c_{2|0}^{[b]} = 2\varDelta^2 b^2 \qquad c_{n>2|0}^{[b]} = 0$$
$$c_{2|1}^{[b]} = \frac{2\varDelta}{\sqrt{\pi}}(1-b^2),$$
$$c_{4|1}^{[b]} = \frac{24}{\sqrt{\pi}}\varDelta^3 b^2(1-b^2)$$
$$c_{6|1}^{[b]} = -\frac{120}{\sqrt{\pi}}\varDelta^5 b^4(1-b^2)$$
$$c_{8|1}^{[b]} = \frac{1344}{\sqrt{\pi}}\varDelta^7 b^6(1-b^2)$$
$$c_{10|1}^{[b]} = -\frac{21600}{\sqrt{\pi}}\varDelta^9 b^8(1-b^2)$$

$$c_{4|2}^{[b]} = \frac{6\varDelta^2(1-b^2)}{\pi}\left[(1-b^2)(\pi-2)-\frac{8}{3}\pi b^2\right],$$
$$c_{6|2}^{[b]} = \frac{360 b^2\varDelta^4}{\pi}(\pi-2)(1-b^2)^2,$$
$$c_{8|2}^{[b]} = -\frac{13440\varDelta^6 b^4}{\pi}(1-b^2)^2,$$
$$c_{10|2}^{[b]} = \frac{483840\varDelta^8 b^6}{\pi}(1-b^2)^2.$$

Dynamical criticality via Lee-Yang zeros

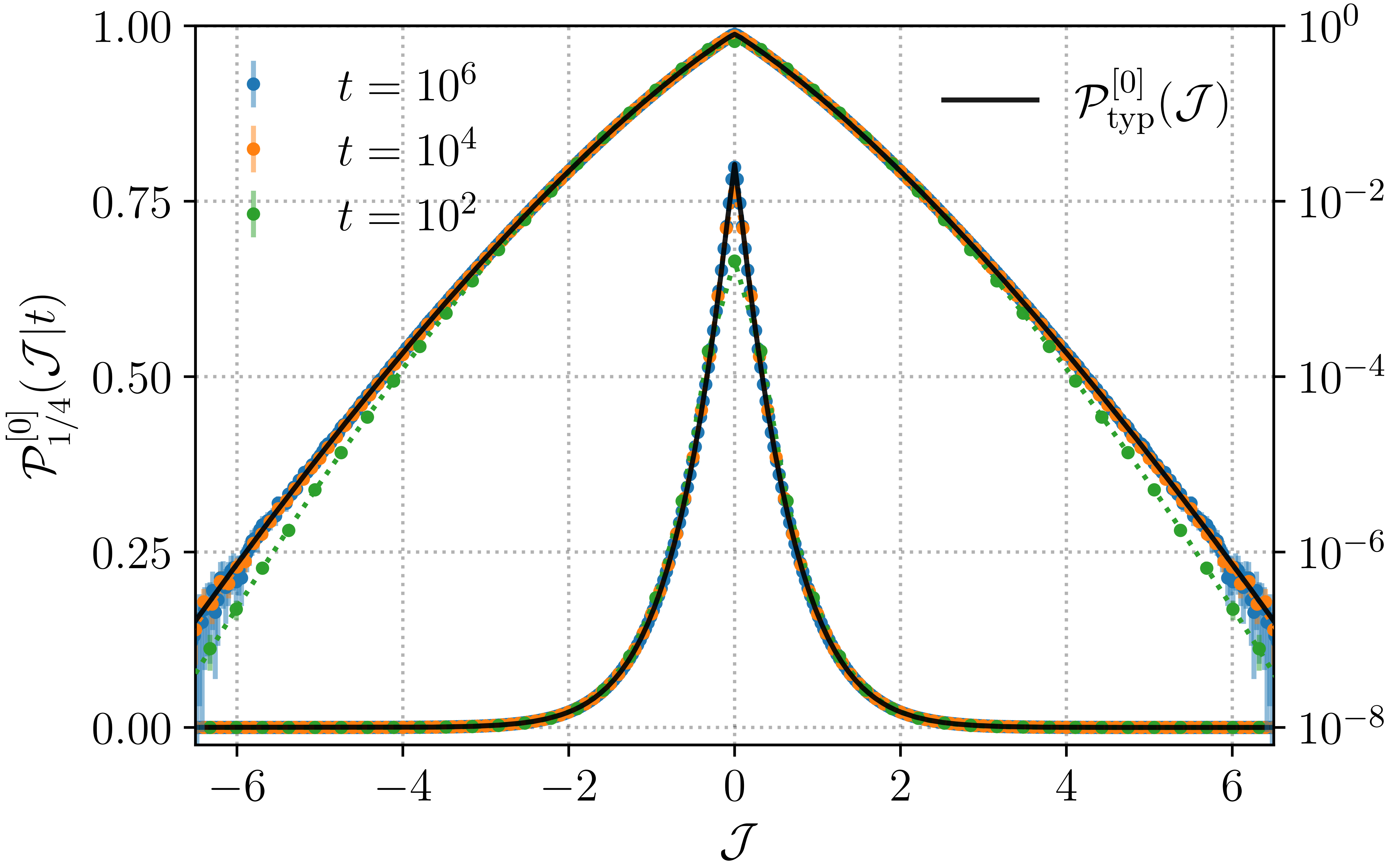


$$\log G(\lambda|t) = \log g(\lambda|t) + \sum_{i=1}^{2t} \log\left(1 - \lambda/\lambda_i\right). \qquad \lambda_i(t) \xrightarrow{t\to\infty} \lambda_c \in \mathbb{R}$$

Typical fluctuations

$$\mathcal{P}^{[b]}_{\text{typ}}(j) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{j^2}{2\sigma^2}\right], \quad \sigma^2 = 2(b\Delta)^2.$$

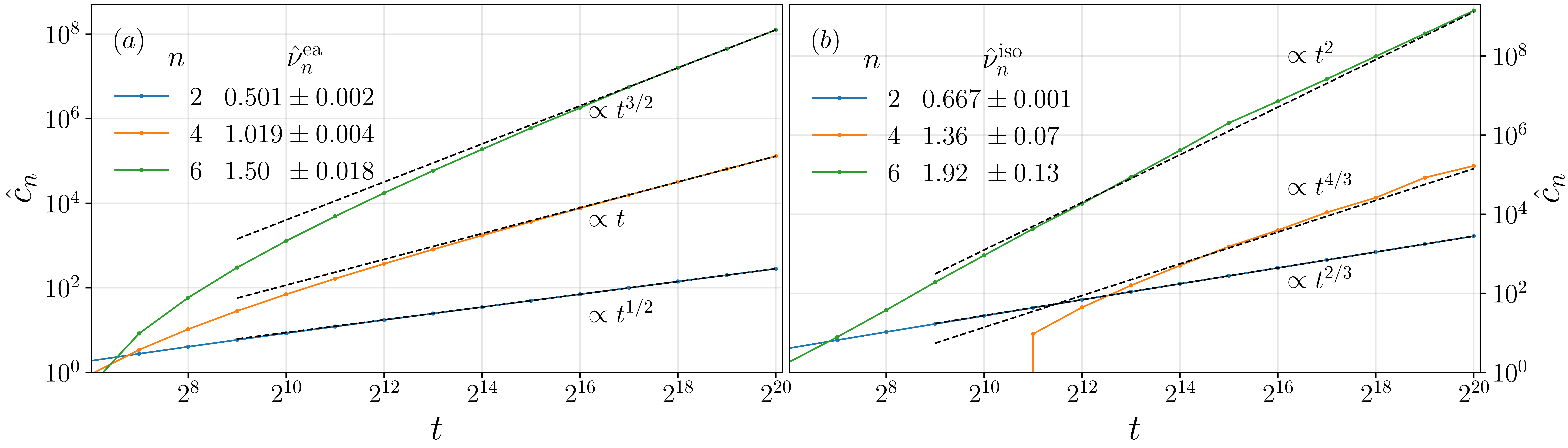
$$\mathcal{P}^{[0]}_{\text{typ}}(j) = \frac{1}{\sqrt{2}\pi\Delta} \int_{\mathbb{R}} \mathrm{d}u \exp\left[-\left(\frac{u^2}{2\Delta}\right)^2 - \frac{j^2}{2u^2}\right].$$



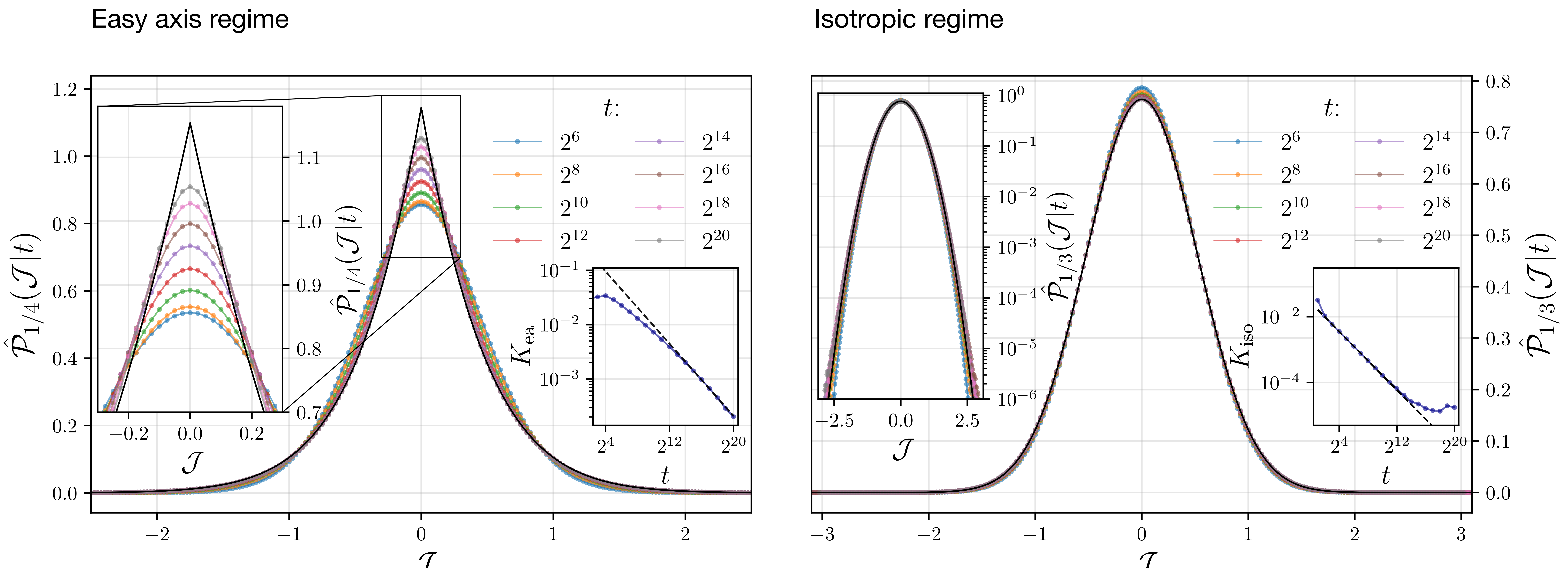
Dynamical Criticality of Magnetization Transfer in Integrable Spin Chains

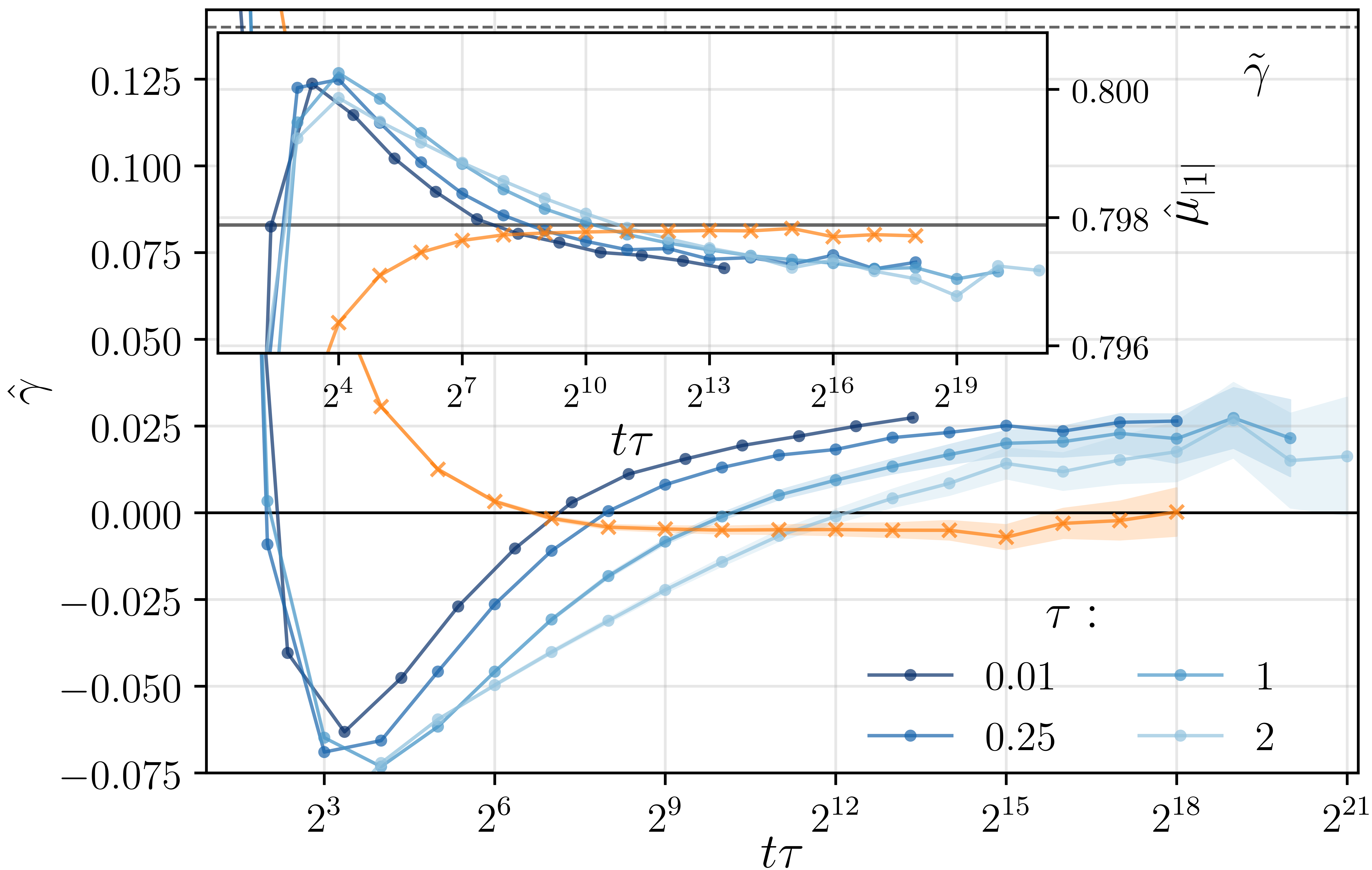
Easy axis regime

Isotropic regime



[Krajnik, Schmidt, Ilievski, P, PRL24]


Easy axis regime
Isotropic regime
$\hat{\mathcal{P}}_{1/4}(\mathcal{J}|t)$
$\hat{\mathcal{P}}_{1/3}(\mathcal{J}|t)$
$\mathcal{J}$
t:
2^6
2^8
2^{10}
2^{12}
2^{14}
2^{16}
2^{18}
2^{20}
K_{ea}
K_{iso}
t



$\hat{\gamma}$
0.125
0.100
0.075
0.050
0.025
0.000
−0.025
−0.050
−0.075
2^3 2^6 2^9 2^{12} 2^{15} 2^{18} 2^{21}
$t\tau$
$\tilde{\gamma}$
$\hat{\mu}_{|1|}$
0.800
0.798
0.796
2^4 2^7 2^{10} 2^{13} 2^{16} 2^{19}
$t\tau$
τ :
0.01
0.25
1
2